



Mathematical Learning Systems' Correlation to the
NCTM CURRICULUM FOCAL POINTS (2006)



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The National Council of Teachers of Mathematics published in September 2006 an important new document, *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*. They explain in their introduction that “An approach that focuses on a small number of significant mathematical ‘targets’ for each grade level offers a way of thinking about what is important in school mathematics that is different from commonly accepted notions of goals, standards, objectives, or learning expectations” (p. 1). They continue:

The curriculum focal points presented here offer both immediate and long-term opportunities for improving the teaching and learning of mathematics. They provide ideas that may kindle fruitful discussions among teacher leaders and teachers about areas to emphasize as they consider the developmental needs of their students and examine a year’s program of instruction (p. 1).

NCTM makes it clear in this document that they believe that “When instruction focuses on a small number of key areas of emphasis, students gain extended experience with core concepts and skills. Such experience can facilitate deep understanding, mathematical fluency, and an ability to generalize” (p. 5). This position is exactly the position of Creative Education Institute (CEI) in its design of **Mathematical Learning Systems (MLS)**. The focus of MLS’ content has been on the “core concepts and skills” identified in research as areas in which students with learning difficulties or disabilities are most likely to have problems and the areas important to success in higher-level mathematics, such as algebra (See *Why MLS Works: Its Scientific, Theoretical, and Evaluation Research Base*, Chapter IV, CEI, 2006). Thus, MLS emphasizes both concepts and procedures, carefully integrating them so that an understanding in one area helps to build understanding in the other. Fluency is further emphasized, since without it students never develop good problem-solving strategies.

NCTM states that “A curriculum for pre-K-8 based on a connected set of such focal points could provide a solid mathematical foundation for high school mathematics” (p. 6). Again, CEI agrees. Its research indicates that there are two major reasons that students fail algebra: lack of fact fluency and lack of understanding of fractions. Peeling back the onion, so to speak, a major reason for students’ failure to understand fractions is that they never understood the concept of long division. MLS, therefore, devotes two of its five units to fractions, one level of another to division, and a whole strand of activities to develop rapid and accurate recall of math facts (fluency).

Another important point made in the NCTM document is that

These curriculum focal points should be considered as major instructional goals and desirable learning expectations, not as a list of objectives for students to master. They should be implemented with the intention of building mathematical competency for all students, bolstered by the pedagogical understanding that not every student learns at the same rate or acquires concepts and skills at the same time (p. 10).

Again, CEI agrees. MLS, a therapeutic intervention for struggling learners, is not designed or classified by grade-levels. The concepts and skills included are prerequisite knowledge and skills required for access to grade-level instruction. The lessons that an individual student may need depend not on the grade he/she is in, but on the developmental level. MLS can be used by any school to supplement the core curriculum to ensure mastery as topics are taught; as reinforcement to the classroom teacher’s instruction; as re-teaching for students who have not mastered; and/or as a Tier II or Tier III intervention for students who continue to struggle to learn mathematics.

The following correlation shows the relationship between NCTM’s curriculum focal points for grades prekindergarten-6 and MLS’ content. The correlation is strong for all the focal points having to do with Number and Operations, including those that prepare students for algebra. MLS does not include instruction on geometry, measurement (except as it pertains to time and money), or decimals (although the foundation for decimals in fractions is thoroughly covered). Information on MLS’ content, lesson design, instructional strategies, assessments, implementation fea-



tures, and results, as well as their research-base, can be reviewed in *Why MLS Works: Its Scientific, Theoretical, and Evaluation Research Base* (CEI, 2006).

Each set of MLS' concept lessons is taught using the concrete — semi-concrete — abstract (CSA) lesson sequence that is verified as effective, especially for students who struggle to learn mathematics. Manipulatives and a working mat are used in the concrete lessons; representations of those manipulatives and the mat appear on the computer screen for the semi-concrete lessons. At the abstract level, the students work with the numbers and words to solve real-world problems. As they are deciding how to solve a word/story problem, students also receive instruction on how to eliminate irrelevant information. Each set of lessons concludes with an assessment lesson, providing ongoing data for feedback to both the student and teacher.

MLS' comprehensive assessment system is another feature that is critical to its success. It includes one third-party assessment, CEI's automated and interactive placement test, continuous progress monitoring, an assessment lesson in each set of three lessons, and a summative assessment for use in student and program evaluation. Teachers have ample data to enable them to adapt/modify lesson settings so that the program is totally individualized and differentiated for each learner.

As of 2012-2013, a total of 45 states have entered the *Common Core State Standards Initiative* (CCSS), with the Mathematics Standards one of the first sets to be implemented. Using analyses of the CCSS and the NCTM *Focal Points*, schools using MLS can be assured that the *Focal Points* and the correlation between the two are still intact.

In terms of rigor, coherence, and focus, the mathematical content for the *Focal Points* was an important resource for the developers of the CCSS. Here are the findings by grade level:

Rigor

- + Elementary** **grades**
Through the end of grade 4, the CCSS and the *Focal Points* address much of the same content, although the CCSS introduce some content earlier than the *Focal Points*. By the end of grade 4, both documents expect students to understand the four basic operations with whole numbers, place value, and the meanings and uses of fractions. Both documents introduce statistical topics primarily as reinforcements for work with numbers and measurement in the early grades. The strong foundation in number sense allows students to progress quickly in middle and high school through data, probability and statistics, culminating in content that is generally more rigorous than found in many state standards. However, specific differences in grade placement are present in a number of instances. For example, the CCSS expect students to express whole numbers as fractions earlier than the *Focal Points*. Despite these differences, they do not lead one document to be more rigorous than the other since by the end of sixth grade, the two documents cumulatively describe substantially similar bodies of knowledge.
- + Middle** **grades**
With only a few minor exceptions in geometry and probability, the CCSS and the *Focal Points* describe similar content in the middle grades. Areas of overlap include the major hallmarks of algebra — proportionality, linear expressions, and inequalities, and using equations and inequalities to solve real-life and mathematical problems. This prepares students for more advanced mathematics in high school. There are some differences between the two documents. For example, the CCSS requires students to understand the role of the transformations of geometric shapes on a coordinate plane and the *Focal Points* does not.

Coherence and Focus

The Common Core State Standards and the *Focal Points* share some key traits of coherence and focus. For example, both documents develop an understanding of adding, subtracting, multiplying, and dividing whole numbers over an average of three years. Mathematical reasoning is addressed in both documents and both also highlight the importance of explaining and justifying solutions to problems.

Both sets of standards expect students to learn similar amounts of content in each grade level, and the *Focal Points* builds on the pioneering standards first developed by the NCTM in 1989 by articulating clear and realistic priorities for student learning in kindergarten through grade 8. The CCSS were designed to be similarly focused, although they extend through the end of high school. Both documents make clear that the curricula should be focused on a tightly defined set of content and skills each year in order for students to build a strong foundational understanding of mathematics before they begin high school coursework. In particular, both documents provide an in-depth treatment of numbers and operations, with

reinforcement from other content areas. This emphasis allows students to progress more quickly through later content, as the groundwork for more advanced mathematics has been laid.

Despite these similarities in coherence and focus, there are critical differences between the CCSS and the *Focal Points*:

- + The CCSS provide more detailed and specific expectations of students. For example, the CCSS indicate that fourth graders should compare base ten numbers. They should “Generalize place value understanding for multi-digit whole numbers” and in doing so, “Recognize that in a multi-digit whole number, a digit in the one place represents ten times what it represents in the place to its right.” On this topic, the *Focal Points* is less specific, indicating that students should be able to “... use place value and properties of operations to write, compare, and order multi-digit numbers.” The greater level of detail and specificity in the CCSS provides additional guidance to educators regarding learning expectations.
- + While both documents make connections among topics, the *Focal Points*’ presentation of the connections is particularly useful to educators. For example, the *Focal Points* layout includes a column in each grade that explicitly calls out the connections among the topics. In nearly all cases, these connections apply to CCSS as well, but they are not as explicit. As such, *Focal Points* can be an important tool in guiding the implementation of the CCSS, particularly with respect to drawing important connections and across the content.
- + Like the standards of many high-performing countries including Hong Kong and Japan, the CCSS begin with a focus on the properties of operations as a way of gradually preparing students for algebraic thinking. In contrast, *Focal Points* introduces algebraic thinking through the creation, description, extension, and explanation of patterns. The CCSS include this content somewhat later in Grades 4 and 5, after development of the core knowledge associated with numbers and operations in Grades 1-3. Because of this different progression, some states that have adopted the standards will need to be vigilant, as topics will be introduced at different grade levels.

References

- Creative Education Institute (2006). *Why MLS Works: Its Scientific, Theoretical, and Evaluation Research Base*. Waco, TX: Creative Education Institute.
- National Council of Teachers of Mathematics (2006). *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*. Reston, VA: National Council of Teachers of Mathematics.



Mathematical Learning Systems' Correlation to the NCTM Curriculum Focal Points (2006)

NCTM's Curriculum Focal Points, Pre-K-6	CEI's Mathematical Learning Systems
<p>Pre-Kindergarten: Number and Operations: Developing an understanding of whole numbers, including concepts of correspondence, counting, cardinality, and comparison.</p>	<p>Unit 1: Understanding Numbers</p> <p>Level 1: Defining Numbers</p> <ul style="list-style-type: none"> Identification 0-10 Recognition 0-10 Identification 11-20 Recognition 11-20 <p>Level 2: Numbers 0-20</p> <ul style="list-style-type: none"> Patterns and Counting 0-20 Comparison 0-20
<p>Kindergarten: Number and Operations: Representing, comparing, and ordering whole numbers and joining and separating sets.</p>	<p>Unit 1: Understanding Numbers</p> <p>Level 1: Defining Numbers</p> <ul style="list-style-type: none"> Identification 0-10 Recognition 0-10 Identification 11-20 Recognition 11-20 <p>Level 2: Numbers 0-20</p> <ul style="list-style-type: none"> Patterns and Counting 0-20 Comparison 0-20 <p>Level 3: Numbers 21-99</p> <ul style="list-style-type: none"> Patterns and Counting 21-99 Comparison 21-99 <p><i>Drawing Conclusions</i></p> <p><i>Math Magic</i></p>



NCTM's Curriculum Focal Points, Pre-K-6	CEI's Mathematical Learning Systems
<p>Grade 1: Number and Operations: Developing and understanding of whole number relationships, including grouping in tens and ones.</p>	<p>Unit 1: Understanding Numbers</p> <p>Level 1: Defining Numbers</p> <ul style="list-style-type: none"> Identification 0-10 Recognition 0-10 Identification 11-20 Recognition 11-20 <p>Level 2: Numbers 0-20</p> <ul style="list-style-type: none"> Patterns and Counting 0-20 Comparison 0-20 <p>Level 3: Numbers 21-99</p> <ul style="list-style-type: none"> Patterns and Counting 21-99 Comparison 21-99 <p><i>Drawing Conclusions</i></p> <p><i>Math Magic</i></p> <p>Unit 2: Number Operations (Students use Base-Ten blocks in developing conceptual understandings of number operations.)</p>



NCTM's Curriculum Focal Points, Pre-K-6	CEI's Mathematical Learning Systems
<p>Grade 1: Number and Operations and Algebra: Developing understandings of addition and subtraction and strategies for basic addition facts and related subtraction facts.</p>	<p>Unit 2: Number Operations</p> <p>Level 1: Addition</p> <ul style="list-style-type: none"> Single Digits Addition Fluency <p>Level 2: Subtraction</p> <ul style="list-style-type: none"> Single Digits Subtraction Fluency <p>Digit's Widgets</p> <p><i>Flash Cards</i></p> <p><i>Fact Match</i></p> <p><i>Drawing Conclusions</i></p> <p><i>Math Magic</i></p> <p>Unit 3: Using Whole Numbers</p> <p>Level 2: Time</p> <ul style="list-style-type: none"> To the Hour In Hours and Minutes <p>Level 3: Estimation</p> <ul style="list-style-type: none"> Rounding to the Nearest Ten Rounding to the Nearest Hundred



NCTM's Curriculum Focal Points, Pre-K-6	CEI's Mathematical Learning Systems
<p>Grade 2: Number and Operations: Developing an understanding of the base-ten numeration system and place-value concepts.</p>	<p>Unit 1: Understanding Numbers</p> <p>Level 1: Defining Numbers</p> <ul style="list-style-type: none"> Identification 0-10 Recognition 0-10 Identification 11-20 Recognition 11-20 <p>Level 2: Numbers 0-20</p> <ul style="list-style-type: none"> Patterns and Counting 0-20 Comparison 0-20 <p>Level 3: Numbers 21-99</p> <ul style="list-style-type: none"> Patterns and Counting 21-99 Comparison 21-99 <p>Level 4: Numbers 100-999</p> <ul style="list-style-type: none"> Place Value 100-999 Patterns and Counting 100-999 Comparison 100-999 <p>Unit 2: Number Operations (Students use Base-Ten blocks in developing conceptual understandings of number operations.)</p> <p><i>Drawing Conclusions</i></p>
<p>Grade 2: Number and Operations and Algebra: Developing quick recall of addition facts and related subtraction facts and fluency with multidigit addition and subtraction.</p>	<p>Unit 2: Number Operations</p> <p>Level 1: Addition</p> <ul style="list-style-type: none"> Single Digits Double Digits Triple Digits Addition Fluency <p>Level 2: Subtraction</p> <ul style="list-style-type: none"> Single Digits Double Digits Triple Digits Subtraction Fluency <p><i>Digit's Widgets</i></p> <p><i>Fact Match</i></p> <p><i>Flash Cards</i></p> <p><i>Drawing Conclusions</i></p> <p><i>Math Magic</i></p>



NCTM's Curriculum Focal Points, Pre-K-6	CEI's Mathematical Learning Systems
<p>Grade 3: Number and Operations and Algebra: Developing understandings of multiplication and division and strategies for basic multiplication facts and related division facts.</p>	<p>Unit 2: Number Operations</p> <p>Level 3: Multiplication Single Digits</p> <p>Level 4: Division Single Digits</p> <p>Digit's Widgets Fact Match Flash Cards Drawing Conclusions Math Magic</p>
<p>Grade 3: Number and Operations: Developing an understanding of fractions and fraction equivalence.</p>	<p>Unit 4: Understanding Fractions</p> <p>Level 1: Fraction Identification Less than One or Equal to One</p> <p>Level 2: Equivalent Fractions Using Larger or Smaller Denominators</p> <p>Drawing Conclusions</p>
<p>Grade 4: Number and Operations and Algebra: Developing quick recall of multiplication facts and related division facts and fluency with whole number multiplication.</p>	<p>Unit 2: Number Operations</p> <p>Level 3: Multiplication Single Digits Single and Double Digits Double Digits Multiplication Fluency</p> <p>Level 4: Division Single Digits Division Fluency</p> <p>Digit's Widgets Fact Match Flash Cards Drawing Conclusions Math Magic</p>
<p>Grade 4: Number and Operations: Developing an understanding of decimals, including the connections between fractions and decimals.</p>	<p>Unit 3: Using Whole Numbers</p> <p>Level 1: Money Pennies, Nickels, and Dimes Pennies, Nickels, Dimes, and Quarters</p>



NCTM's Curriculum Focal Points, Pre-K-6	CEI's Mathematical Learning Systems
<p>Grade 5: Number and Operations and Algebra: Developing an understanding of and fluency with division of whole numbers.</p>	<p>Unit 2: Number Operations</p> <p>Level 4: Division</p> <ul style="list-style-type: none"> Single Digits Single and Double Digits Double Digits Division Fluency <p>Digit's Widgets Fact Match Flash Cards Drawing Conclusions Math Magic</p>
<p>Grade 5: Number and Operations: Developing an understanding of and fluency with addition and subtraction of fractions and decimals.</p>	<p>Unit 4: Understanding Fractions</p> <p>Level 3: Comparing Fractions</p> <ul style="list-style-type: none"> Common Denominators Different Denominators <p>Level 4: Simplifying Fractions</p> <ul style="list-style-type: none"> Simplified Numerators Equal to One Simplified Numerators Greater than One <p>Level 5: Converting Fractions</p> <ul style="list-style-type: none"> Improper Fractions to Mixed Numbers Mixed Numbers to Improper Fractions <p>Unit 5: Fraction Operations</p> <p>Level 1: Addition</p> <ul style="list-style-type: none"> Common Denominators Different Denominators <p>Level 2: Subtraction</p> <ul style="list-style-type: none"> Common Denominators Different Denominators <p>Drawing Conclusions</p>



NCTM's Curriculum Focal Points, Pre-K-6	CEI's Mathematical Learning Systems
<p>Grade 6: Number and Operations: Developing an understanding of and fluency with multiplication and division of fractions and decimals.</p>	<p>Unit 4: Understanding Fractions</p> <p>Level 3: Comparing Fractions Common Denominators Different Denominators</p> <p>Level 4: Simplifying Fractions Simplified Numerators Equal to One Simplified Numerators Greater than One</p> <p>Level 5: Converting Fractions Improper Fractions to Mixed Numbers Mixed Numbers to Improper Fractions</p> <p>Unit 5: Fraction Operations</p> <p>Level 3: Multiplication Whole Numbers and Fractions</p> <p>Level 4: Division Common Denominators Fractions and Whole Numbers Different Denominators</p> <p><i>Drawing Conclusions</i></p>
<p>Grade 6: Number and Operations: Connecting ratio and rate to multiplication and division.</p>	<p>Unit 5: Fraction Operations</p> <p>Level 3: Multiplication Whole Numbers and Fractions Fractions and Whole Numbers</p> <p>Level 4: Division Common Denominators Fractions and Whole Numbers Different Denominators</p> <p><i>Drawing Conclusions</i></p>



NCTM's Curriculum Focal Points, Pre-K-6	CEI's Mathematical Learning Systems
<p>Grade 6: Algebra: Writing, interpreting, and using mathematical expressions and equations.</p>	<p>Problem-solving lessons in the abstract phase of each lesson cycle involving concept development</p> <p>Unit 2: Number Operations</p> <ul style="list-style-type: none"> Addition Fluency Subtraction Fluency Multiplication Fluency Division Fluency <p>Digit's Widgets</p> <p><i>Fact Match</i></p> <p><i>Flash Cards</i></p> <p><i>Drawing Conclusions</i></p> <p><i>Math Magic</i></p>