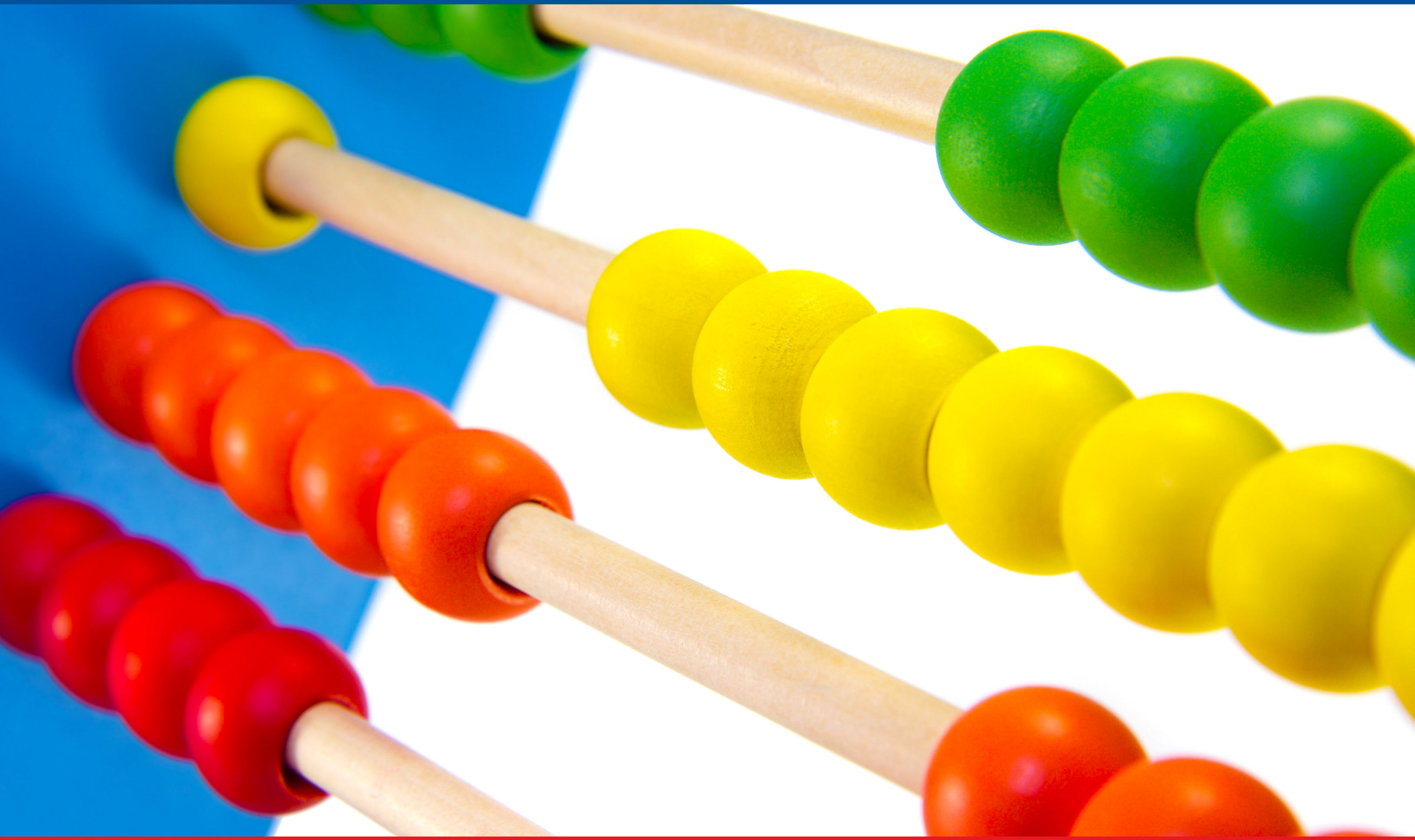

WHY MLS WORKS

Its Scientific, Theoretical, and Evaluation Research Base

Creative Education Institute®
Mathematical Learning Systems®

Why *MLS®* Works:

Its Scientific, Theoretical, and Evaluation Research Base

by

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Why *MLS* Works:
Its Scientific, Theoretical, and Evaluation Research Base
Executive Summary

Chapter I: Introduction

Mathematical Learning Systems (MLS) was designed by Creative Education Institute (CEI) approximately ten years ago, using to every extent possible what had been found to be effective in accelerating student achievement in *Essential Learning Systems (ELS)* approximately ten years prior. It serves as a therapeutic intervention in mathematics for learners who struggle, k-adult.

The purpose of this study is to document the scientifically-based research (SBR) that grounds the content, lesson design, instructional strategies, and implementation support that comprise *MLS*. SBR in mathematics is not as plentiful as it is in reading, and the federal government's insistence on scientific designs as criteria for quality research is in itself controversial. One thing that is clear is that the program that a school uses for struggling learners is considered to be based on scientific evidence if its component parts are themselves grounded in SBR, as *MLS* is. *MLS* features also correlate with the requirements of federal programs requiring SBR, including *NCLB* programs: Title I, Math Now, and Title III, as well as Section 504 (e.g., dyslexia), *IDEA*, and the new Response-to-Intervention.

Chapter II: Mathematics Difficulties

Researchers speculate that as many as 70 percent of the students currently being identified for special education services could be served well in general education since their learning difficulties are the result of factors other than brain disabilities. They would need, of course, an appropriate intervention, such as *MLS*, to help them overcome the causes of their difficulties and to accelerate their learning so that they achieve the grade-level standards.

Among the root causes of much low achievement in mathematics are cultural attitudes that place low values on mathematics and science knowledge and careers; the experience deficits of students from economically disadvantaged homes, as well as the sense of entitlement of students from high socio-economic homes; and the peculiarly American belief that high achievement is a result of inborn talent, in contrast to the Asian belief that it results from effort.

Other situations that result in lack of motivation to learn mathematics include stereotype threat, particularly for girls/women and minority males; mathematics anxiety/phobia, usually resulting from poor teaching and/or being embarrassed in the classroom; and general low motivation, especially students having a low sense of self-esteem or self-efficacy.

Motivation to learn mathematics is not necessarily a problem among English-language learners (ELLs), but there are several ways that the cultural and language backgrounds of ELLs can affect their achievement. A major problem is learning mathematics vocabulary since many of the words have different meanings in mathematics than they do in informal communication. Another problem is that they are frequently confused by the standard algorithms taught in American schools, which may differ from those taught in their native countries. American teachers do not necessarily recognize that the algorithms they use result in correct answers, so they penalize the students *and* take precious time to teach the American algorithm.

The greatest cause of mathematics difficulties, however, is inadequate or inappropriate instruction. Inadequate instruction includes lack of preschool cognitive development in the home or in a more formal setting; poor attendance; mobility resulting in time lost from school; wasted class time; and not having access to the necessary intensive instruction when needed. Inappropriate instruction occurs from inappropriate curriculum, curriculum materials, misinformation provided by teachers, instructional strategies, and lack of assessment used for decision-making. The so-called "math wars" are a long debate over what constitutes appropriate mathematics curriculum (e.g., coverage vs. mastery) and instruction (e.g., discovery learning vs. direct instruction). Much of the disagreement is the result of the participants not differentiating between the needs of general education students who make adequate progress—and the needs of lower achieving general education students who acquire difficulties in learning—and the needs of students who truly have learning disabilities. Educators are advised always to avoid generalizing research findings to populations other than the ones studied.

Chapter III: Mathematics Disabilities

Mathematics disabilities are sometimes lumped into one category termed *dyscalculia*. There are, however, many differences in the kinds of mathematics disabilities and how they are manifested. A model designed by Geary and Hoard (2005, p. 260) is very useful in understanding the variety of ways that mathematics achievement can be affected by learning disabilities.

First, it is important to note that the nature of mathematics disabilities is specific to the mathematical domain. That is, a learner who has a disability in the visuospatial system will, undoubtedly, have difficulty in learning geometry, but little difficulty in arithmetic. The two supporting competencies in mathematics, concepts and procedures, are also affected differently according to the mathematical disability. One student may have problems with the concept of place value, and another will struggle with regrouping in subtraction procedures.

A whole set of problems ensue when a learner has a disability relating to the central executive part of the brain. The central executive affects one's ability to stay focused, to inhibit distractions, to sequence, and to plan, for instance, and all are required for good mathematics achievement.

The root cause of most learning problems is, of course, faulty sensory processing. There is a whole body of research and theory available on information processing that explains how brains learn and remember. When there is a genetic defect, a lesion, or brain damage of some kind, the brain has difficulty processing in one or more modalities, making learning more difficult. (People without disabilities may appear to have them if there has been an absence or lack of appropriate instruction.) Research has found that people tend to retrieve learning in the same modality in which it was learned, so effective instruction for learners with mathematical difficulties or disabilities must be multi-sensory. Information processing research also provides the basis for knowing the importance of practice and repetition in developing fluent and accurate retrieval, in understanding how learning moves from short- to long-term memory, and how weak neural pathways can be strengthened or new neural pathways can be developed to bypass the problem areas in the brain.

Language system disabilities, which include problems in processing auditory (phonological) input, are manifested in varied ways in mathematics learning. Students with these problems are slow in processing, may have difficulty in decoding mathematics text, almost invariably have problems in learning mathematics facts (especially multiplication) and retrieving them, and, in general, have trouble in learning concepts and vocabulary. Many educators, used to definitions of dyslexia being confined to reading, writing, and spelling issues, are surprised to learn how negatively dyslexia (a language system disability) affects mathematics achievement.

Visuospatial system disabilities also result in processing problems and manifest themselves in various ways in mathematics learning. Students with these deficits lose their place, have problems lining up numbers in columns, write numbers illegibly, have great difficulties in geometry, sometimes confuse commas and decimals in numbers, and are slow in processing.

There are several genetic defects that also result in mathematics disabilities. Turner syndrome, Fragile X syndrome, spinal bifida, and Gerstmann's syndrome are among those which usually result in at least some mathematical difficulties, if not more serious retardation.

More serious than mathematics difficulties and much more serious than dyscalculia or dyslexia alone are the problems that learners have when they carry both reading and mathematics disabilities (comorbidity). Almost every area of mathematics learning can be impacted, making it much more difficult and time-consuming to deliver interventions that improve achievement.

Knowing the reasons why students have low mathematics achievement, whether due to difficulties or to more serious disabilities, is important in order for educators to understand how to choose effective interventions to improve performance. This study documents the ways in which *MLS* addresses the manifestations of both difficulties and disabilities, as well as the research findings on the efficacy of each of the *MLS* components.

Chapter IV: Research Findings that Ground *MLS* Content

The content included in a mathematics intervention is very important. It must include the conceptual and procedural topics that are identified in research as problem areas, and it must include the foundational knowledge and skills that are identified as essential background for more advanced mathematical study, i.e., algebra. *MLS's* scope and sequence does precisely that. The five units included are as follows:

1. Understanding Numbers (Defining Numbers, Numbers 0-20, Numbers 21-99, and Numbers 100-999)
2. Number Operations (Addition, Subtraction, Multiplication, and Division)
3. Using Whole Numbers (Money, Time, and Estimation)
4. Understanding Fractions (Fraction Identification, Equivalent Fractions, Comparing Fractions, and Converting Fractions)
5. Fraction Operations (Addition, Subtraction, Multiplication, and Division).

The first unit carefully teaches to mastery fundamental concepts such as the base-ten system, place value, and counting before moving forward with more sophisticated concepts. A part of learning the concepts of number operations is extensive and varied practice in fact fluency, since research emphasizes that both concepts and procedures must be systematically taught to students who struggle and that exposure to concepts helps one learn the procedures, just as exposure to procedural practice helps one to internalize the concepts. A plethora of research findings document the wisdom of including and emphasizing these topics in an intervention, plus additional topics such as algorithms, problem solving, sequencing, direction, estimation, measurement, and fractions.

Interestingly, the research indicates that students who have difficulties in learning algebra almost always lack conceptual and procedural understandings of fractions. Once those are taught to mastery, most of algebra becomes easy to learn. The same is true for students who have problems with decimals and percents. More drill on those topics is not helpful unless the student thoroughly understands fraction concepts and procedures. Other research indicates that students have difficulty with fractions chiefly because they did not learn long division concepts and procedures.

The emphasis in *MLS* on fact fluency is also soundly grounded in scientific evidence. Students who lack fact fluency use up all their working memory in attempts to retrieve the facts and, thus, have no room left for analysis and problem solving. Too, what they do retrieve is frequently inaccurate, resulting in more problems. *MLS* students learn their facts to mastery through lessons employing multi-sensory processing strategies and through adequate and varied practice exercises that both engage and motivate students to keep working. A new *MLS* feature is a web-based fluency game called *Digit's Widgets*, which reinforces and provides practice relating to the lessons on whole number operations.

In Chapter II the barriers faced by English-language learners in mathematics was discussed. *MLS* incorporates a number of research-based strategies to help them work around those barriers:

- Emphasis on concepts, using consistent and academic vocabulary for mathematics
- Use of manipulatives in teaching concepts
- Use of modeling at the semi-concrete level and in problem solving lessons
- Auditory and visual instruction at the same time (multi-sensory)
- Modeling of English pronunciation of mathematical terms
- Use of visuals to illustrate meaning
- Explicit teaching of algorithms/procedures
- Adequate and varied practice to develop mastery and to develop fluency
- Instruction design to accelerate learning dramatically.

Chapter V: Research Findings that Ground *MLS'* Lesson Design

Not only must content be grounded in research in order to be effective, but so must the design of lesson structures and lesson delivery. Scientific research in these areas is some of the oldest educational research available, and the findings have been verified over and over. For instance, the stages/phases of lessons for struggling learners are well documented. A variety of researchers have learned that modeling, plus guided and independent practice are essential steps, and *MLS* incorporates all of them.

This research no doubt informed early research on several different lesson models, such as direct instruction, mastery learning, and one-on-one tutoring, all of which incorporate those research-based stages/phases, plus others such as corrective feedback and review lessons. Major differences in the three are that direct instruction is whole-class instruction; mastery learning depends on small-group instruction, based on diagnosed needs; and one-on-one tutoring is totally individualized. The research is very strongly in support of all three of these models, and *MLS* has elements of all three in its design. Computer-assisted instruction, of course, makes it possible to deliver one-on-one tutoring in a large class.

Another research-based feature of *MLS* lessons is its use of the concrete—semi-concrete—abstract (CSA) lesson sequence for all concept development instruction. Students begin to learn the lesson using concrete manipulatives provided with the program; they then explore the same concept in a semi-concrete or representational phase. In the third segment, the abstract phase, the student sees the number or the word and begins to use the concept in problem-solving applications. Researchers agree that the CSA sequence is effective with students experiencing difficulties in learning mathematics, and they almost unanimously endorse the use of manipulatives in the initial stages of teaching concepts.

A great number of studies were reviewed on the efficacy of computer-assisted instruction (CAI) in mathematics interventions. Findings indicate that CAI is effective for diverse reasons:

- facilitates more student-centered classrooms
- is more effective than traditional methods
- is more effective than use of printed materials alone
- permits individualization
- serves to mediate students in their zone of proximal development
- assists students with learning disabilities to learn better
- encourages more time on task
- actively engages students
- is motivating
- develops fluency in mathematics
- facilitates multi-sensory processing strategies
- provides opportunities for adequate and varied practice
- results in greater gains in a variety of basic skills
- facilitates learning for limited-English proficient students
- is effective with a variety of at-risk learners.

There is another body of research on what the screen design should be for programs intended for struggling learners. A synthesis of those findings includes:

- screens should be uncluttered
- screens should use simple illustrations that reinforce the instructional goal
- screens should use color sparingly and consistently, and
- screens should not place too much information on the screen at once.

MLS utilizes CAI in its lesson delivery to a great extent, and its screens reflect the research findings indicated for struggling learners.

Chapter VI: Research Findings that Ground *MLS'* Instructional Strategies

The instructional strategies used in *MLS* are those most often found through scientific research to be effective in teaching struggling learners so that they attain mastery. The following table indicates the *MLS* tasks, along with coding to indicate which instructional strategies are used in each:

***MLS* Tasks and Instructional Strategies**

MLS Task	Instructional Strategy
Concept Building Introduction	MSP, ID
Learn	MSP, ID, PR, TOT, SA
Solve	MSP, ID, PR, TOT, A
Help	MSP, ID, PR, TOT
Solve Intervention	MSP, ID, TOT
Let's Review	MSP, ID, PR, TOT
Word Problems Learn	MSP, ID, PR, TOT
Word Problems Solve	MSP, ID, PR, TOT, A
Word Problems Let's Review	MSP, ID, PR, TOT
Math Game	MSP, ID, PR, TOT
Printed Activities (7,8,9)	ID, PR, TOT, A
Math Magic	ID, PR, TOT, A
Drawing Conclusions	ID, PR, TOT, A
Fact Match	ID, PR, TOT, A
Flash Cards	ID, PR, TOT, A
Look, Listen, See and Say	MSP, ID, C, PR, TOT
See, Hear and Respond	MSP, ID, C, PR, TOT, A
Hear and Respond	MSP, ID, C, PR, TOT, A
See and Respond	MSP, ID, C, PR, TOT, A
Echo	MSP, ID, C, PR, TOT, A
Blank Out	MSP, ID, C, PR, TOT, A
Number Search	MSP, ID, C, PR, TOT, A
Quick Pick	MSP, ID, C, PR, TOT, A
Quick Answer	MSP, ID, C, PR, TOT, A

MSP=multi-sensory processing; ID=individualization/differentiation; PR=practice/repetition; TOT=time-on task and active engagement; C=chunking/clustering; A=assessment; SA=self-assessment

MLS' most effective and most unique instructional strategy is multi-sensory processing, and it is used in every lesson delivered by the computer software. It is through the use of multi-sensory processing, chiefly, that *MLS* addresses the faulty sensory processing cause of most learning problems. Multi-sensory processing is not a learning styles approach; rather, it delivers instruction in multiple modalities, not just the one preferred. In doing so, weak neural pathways are strengthened, and new neural pathways are developed, if needed. Also, multi-sensory processing gives the learner more flexibility in retrieval of learning since information most generally is retrieved in the modality in which it was learned. Deeper processing enables the learner to retrieve from multiple places in the brain. Multi-sensory processing strategies are a part of the Universal Design for Learning, making instruction accessible to a wider audience of learners. The scientific evidence that grounds the use of this strategy is found in cognitive psychology, biology, neuroscience, and other scientific journals, in addition to education journals.

One-on-one tutoring is the most powerful of instructional strategies, according to many research studies, and it is ideal for all those learners who struggle. CAI enables *MLS* to provide that high level of individualization and differentiation for every participating student, ensuring that at all times the learner is operating in what Vygotsky called the zone of proximal development (ZPD). The computer software allows the lab teacher/facilitator to individualize lesson levels, content, pacing, amount of practice, and lesson parameters so that every student gets exactly what he or she needs. *MLS* correlates perfectly with the new mandates in many states such as Arkansas and Texas for individualized learning plans for students failing the state assessments. It can also be the developmental education provided in the individual plans that many colleges are now required to offer all students who enter college without prerequisite knowledge and skills.

Many researchers note that few curriculum programs have adequate practice exercises for struggling learners to master the concepts and procedures that they must learn. Also, teachers do not have the time to develop them in ways that individualize, engage, and motivate students—much less check them for mastery. *MLS* incorporates both more-than-adequate numbers of practice exercises to develop fact fluency, but they are also sufficiently varied to keep students interested and engaged. The research findings are clear that practice really does make perfect, and no mathematics intervention can be effective without plenty of it.

Another important instructional strategy used in *MLS* fact fluency lessons is chunking or clustering. Cognitive psychology research has verified the efficacy of this strategy. Chunking or clustering enables learners to learn more and remember it better since working memory is incapable of holding more than about seven to nine items at any one time. Grouping new facts into meaningful chunks is a useful strategy to overcome that barrier.

Educators have known for a very long time that the amount of time-on-task has a positive relationship to general mathematics achievement. Later research clarified that the time has to be "engaged" time in order to affect learning significantly. *MLS* is structured in such a way to ensure that students have high degrees of success as they work through the lessons, but with an adequate amount of challenge to keep them working. Schools should implement *MLS* as a supplement to core instruction, giving students additional time to learn (i.e., intensive instruction). Researchers consistently find that struggling learners simply have to have more time than general students to learn the required material to the proficiency level.

In recent years researchers have verified time and again that formative or ongoing assessment is critical to effective instruction, and especially so for instruction involving struggling learners. It is effective, however, only if the teacher uses the data to inform instructional decisions and makes necessary adaptations and modifications. *MLS'* comprehensive assessment system allows the lab teacher/facilitator to screen, diagnose, monitor, and evaluate student progress through the multiple assessments provided (two of them third-party). The *MLS* teacher/facilitator training focuses to a high degree on how to use assessment data to adapt and modify the lessons so that every student moves along at an optimal pace.

Another powerful, research-based strategy is corrective feedback. *ML*S allows the student to receive instantaneous auditory feedback after every response. Daily and periodic progress reports are available as a part of the software. Mastery lessons provide checks and feedback. And the teacher is trained to observe student performance and provide feedback and encouragement. The research findings concur that such feedback serves to make practice perfect (instead of allowing students to practice inaccurate information or procedures), as well to motivate students to continue their efforts toward mastery.

Chapter VII: Research Findings that Ground *MLS'* Implementation Support
CEI includes a variety of service/support features with every *MLS* program, doing everything that it can to ensure a school's smooth and effective implementation. These features help to maximize the kinds of results that students in an *MLS* lab can achieve. Just as in the design of content, lesson models, and instructional strategies, CEI investigated the scientific research in its design of implementation support activities.

Drawing on the research on the importance of the teacher in effective instruction, CEI developed the *MLS* program so that the lab teacher/facilitator maintains that important role. He or she is involved in providing student motivation, in diagnosing and placing students in the appropriate lessons, in monitoring student progress and making necessary adjustments, in coaching students and providing supplemental explanations, in establishing a quality environment, and in doing all he or she can to ensure student success. The sample job description that CEI provides for *MLS* lab teachers/facilitators reflects the existing research on effective teacher behaviors:

- Establish rapport with their students and provide a pleasant and orderly environment that is conducive to learning
- Maximize time on task using minimum class time for noninstructional routines
- Clearly define expected behavior
- Plan carefully and thoroughly for instruction
- Continually monitor learners' behaviors to determine whether they are progressing
- Heed the results of their monitoring and adapt their instructional strategies accordingly
- Require all learners to practice new learning while under direct teacher supervision
- Expect learners to practice skills without direct teacher supervision but only after guided practice has shown that the learners understand what is expected.

CEI provides adequate professional development for all those involved in *MLS* implementation, including the principal or other instructional leader, whole faculty or department awareness sessions, training for technical staff on deployment and trouble-shooting of software, and parent awareness sessions—in addition to the two days of intense training for *MLS* lab teachers/facilitators.

Ongoing and follow-up training is provided, as research indicates is essential, through access to CEI's webpage for users, CEI publications, the *SHARE* newsmagazine, and a one-day workshop in the spring. Additionally, on-demand training is available through e-mail and telephone consultations with CEI staff for educational issues or technical issues relating to the software. A much-valued service is the physical lab visits provided by CEI by its educational consultants, all of whom are certified teachers, and several of whom are former lab teachers/facilitators. At the end of each visit, the lab teacher/facilitator, plus the principal and/or other supervisor, receive a written lab report, which includes suggestions for improving implementation so that students achieve maximal results. At the end of the school year when labs do their post-testing of students, CEI provides a graphical analysis of their value-added gains in several different categories. Technical support is available as another form of critical support.

Student motivation is the third area of implementation support, included because of research findings indicating that struggling learners almost all suffer from a low sense of self-efficacy, and few believe that their efforts are related to improved learning. CEI provides in the program design a number of motivational strategies (e.g., ensuring high levels of success and providing auditory and written feedback), as well as lessons that are engaging and varied to maintain student interest. It also includes opportunities for individual student recognition through certificates of achievement signed by the president of the company and through stories published in *SHARE*, CEI's newsmagazine. Other rewards and incentives are available, and teachers are trained to provide motivational support and encouragement to their students.

Parental involvement is another area that research indicates is important for improving the academic achievement of children. CEI will provide, therefore, upon request, a parent awareness workshop. The content is a program overview, along with suggestions of ways they can support their children in the program, and with the kinds of growth they should be able to see.

Chapter VIII: Summary and Conclusions

There is a wealth of research available on the characteristics of effective mathematics interventions. *MLS* reflects that research and is correlated positively with the findings on appropriate content, lesson designs, instructional strategies, and implementation support. Its design further reflects the research on the manifestations of learning difficulties and disabilities and addresses them systematically, predicting the high levels of gains that ten years of CEI data indicate are achieved.

MLS also correlates with the requirements of three-tiered mathematics instruction, the model proposed by the United States Department of Education for response-to-intervention. It can be used as a supplement in tier one to reduce as much failure as possible; it can be used in a more intensive way as a tier two intervention; and it can even serve as a tier three intervention for students likely to require special education and an even more intense instructional program.

An effective *MLS* implementation also correlates very positively with W. Edwards Deming's "fourteen points" for total quality management or a continuous improvement model for school reform.

Schools implementing *MLS* effectively will, at a minimum, achieve the average gains that CEI has documented over time among its *MLS* labs. These scores include all those labs that did not implement as trained, and they include the scores of students who were not in the lab for a full academic year—those either arriving late or leaving early or having poor attendance. The average improvements in grade equivalents are as follows:

Basic Processes	**2.29 years**
Addition	**2.57 years**
Subtraction	**1.98 years**
Multiplication	**1.92 years**
Division	**2.69 years**

Interestingly, the most remarkable gains are in division. Division is an essential prerequisite concept and skill for learning fractions, and mastery of fraction concepts and procedures is essential for learning algebra. *MLS* can clearly not only improve academic achievement in elementary and middle school mathematics, but it can also prevent failure in algebra.

Several insights and conclusions emerged in the review of the scientific research that grounds each and every aspect of the *MLS* program:

Struggling Learners Are Diverse
Dyslexics Also Struggle with Mathematics
English-language Learners Also Struggle with Mathematics
Alignment Mandates Make no Sense for Struggling Learners
Math Wars Make No Sense if One Reads the Research
Content Matters Greatly
Lesson Models and Lesson Delivery Are Important
Instructional Strategies Can Be Powerful
Frequent Assessment Used to Inform Instruction Is Critical
Implementation Requires Leadership and Attention
Scientific Research Validates *MLS*' Pre/Post Scores
MLS Can Reduce the Dropout Problem and Improve Graduation Rates
MLS Is More than the Sum of Its Parts

In summary, *MLS* is proven to be an effective, scientifically-research based, therapeutic mathematics intervention for the diversity of struggling learners.

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Why *MLS* Works:
Its Scientific, Theoretical, and Evaluation Research Base

"We are who we are because of what we learn and what we remember"
(Kandel, 2006, 10).

Chapter I: Introduction

The Big Sister: Essential Learning Systems

In the beginning of Creative Education Institute's (CEI) work in the mid-1980's, there was only *Essential Learning Systems (ELS). ELS* is arguably the most effective literacy intervention available anywhere for the diversity of struggling learners, whether due to learning difficulties or disabilities. It incorporates highly effective multi-sensory processing, along with other scientifically-based instructional strategies, to teach all five components found by the National Reading Panel (2000) to be critical in early literacy programs: phonics, phonemic awareness, fluency, vocabulary development, and comprehension. *ELS*' grounding in scientifically-based evidence is documented in the 2005 publication of *Why ELS Works: Its Scientific, Theoretical, and Evaluation Research Base*, and a comprehensive descriptive of its lesson tasks is documented in the *ELS Teacher's Manual* (2006).

The story of almost 20 years of *ELS* history is the story of ongoing enhancements to ensure the product's continual updating to reflect latest research findings, to make the program as user friendly as possible, and to enhance its effectiveness with learners, k-adult. CEI's chief-executive officer, Mr. Terry Irwin (April 2006), states that "no company in the world knows more about how to address learning problems than CEI. Our team is a learning team on a daily basis, and we are long advocates that no child need be left behind, given what we know and can do." He added that "we are working continually to make our programs more accessible to all those who need help, and we are using technology to get that help to them."

Listening to Lead Users

According to Ashley Smith, one of the CEI employees with long tenure, "CEI was, even in its earliest days, engaged in action research—before that term became popular." She related that "we consultants interviewed users every time we made a lab visit to get their views on how *ELS* could be a better product for the learners. We learned from them," she said, "optimal time requirements for lab engagement, lesson sequences to address specific learning problems, where the software could save them time, and a host of other important things." When labs turned in their pre/post test scores at the end of the year, CEI's staff relentlessly studied them, compared them to documented lab practices, and grew more and more informed about effective implementations. That continuous improvement process continues, according to Smith.

Even in those first years of introducing *ELS* to schools throughout Texas and then spreading to other states, CEI's educational consultants began to hear from their best lab facilitators and from principals the request for a mathematics program "that does for math what *ELS* does for literacy."

As those suggestions came to the attention of CEI's management, the conversations and research began on how to develop such a program. Developers stayed in constant touch with teachers using *ELS* and incorporated as many of their suggestions as possible in the program's design. Their original premise was that *MLS* would use *ELS* methodologies, since they knew that they had effective strategies in that program. To a great extent, they remained faithful to that premise, although they did have to make some adjustments as they learned more and more about how the brain learns mathematics and about the inherent differences in the content and skills taught in the two domains. Ric Klein (March 2006), CEI's vice president for marketing, comments that "the best companies in the world listen to their customers as sources for good ideas. We at CEI have always done that."

MLS' Birth and Subsequent Development

In 1994 the real development work on *Mathematical Learning Systems (MLS)* began, and shortly afterwards the first modules were "born" and released to schools. Development continued, and *MLS 2.0* was released in 1998. *MLS 3.0*, which was totally rewritten in the latest programming language to take advantage of new technology, includes networking, as well as other user-friendly enhancements, plus access to a web-based game, *Digit's Widgets*. It was beta-tested in spring 2006 and released for implementation in fall 2006, according to David Merryweather, CEI's vice president for technology, and the leader of the development team.

Status of Mathematics Achievement in the United States

Research findings and government statistics document clearly the need for programs such as *MLS* to assist schools in their quest for improved performance, especially for learners who struggle to learn mathematics. Table 1 provides evidence of low achievement in mathematics, using several different measurements to document the status of current mathematics performance. Table 2 provides evidence of the critical need to improve mathematics achievement in the United States—for economic growth, for homeland security, for equity reasons (closing the achievement gap), and for quality-of-life benefits for individuals and their families.

Table 1: Evidence of Low Mathematical Achievement

Researcher(s)	Findings/Conclusions
National Research Council, 2001, xiii	"Apparently, there has never been a time when U. S. students excelled in mathematics, even when schools enrolled a much smaller, more select portion of the population."
Ed. Trust, 2002, 5	"But 31%--nearly one third—of all fourth graders perform at the below basic level, indicating that they cannot perform this relatively straightforward mathematics [on NAEP—the National Assessment of Educational Progress]."
Silver, 1998, 1	"In general, the TIMSS [Third International Mathematics and Science Study] results indicate a pervasive and intolerable mediocrity in mathematics teaching and learning in the middle grades and beyond. At grades 7 and 8, and also at grade 12, U.S. students achieve poorly in mathematics compared to students in much of the rest of the world."

Researcher(s)	Findings/Conclusions
Whitehurst, n.d., 1	"In the most recent NAEP, only 26% of grade 4 students, 27% of grade 8 students, and 17% of grade 12 students were judged 'proficient' in mathematics. At the same time 31% of grade 4 students, 34% of grade 8 students, and 35% of grade 12 students scored below the 'basic level'."
Viadero, 2005, 2	"On the 2000 National Assessment of Educational Progress in math, 17 percent of high school seniors scored at the 'proficient' level—just under half the percentage scoring at that level on the NAEP reading test. Twenty-two percent of college freshmen . . . are identified as needing remedial math, according to the National Center for Education Statistics."
Viadero, 2005, 3	"On 12^{th} grade NAEP math tests given in 2000, black and white students were separated by a gap of 34 scale-score points—about the same as in 1990."
Ed. Trust, 2002, 1	"More than 90% of mathematics enrollment in higher education are in courses also taught in high school."
Sousa, 2001, 139	"About 6 percent of school-age children have some form of difficulty with processing mathematics. This is about the same number as children who have reading problems."
Geary, n.d.,1-2	". . . the studies in number and arithmetic are very consistent in their findings: Between 6 and 7% of school-age children show persistent, grade-to-grade, difficulties in learning some aspects of arithmetic or related areas. These and other studies indicate that these learning disabilities are not related to IQ, motivation, or other factors that might influence learning."
Kroesbergen, 2002, 2.1	"About five to ten percent of the students in schools for general elementary education have difficulties with mathematics (Rivera, 1997)."
Brodesky, Gross, McTigue, & Tierney, Oct. 2004, 146	"On the national level, 13.2 percent of students have identified disabilities. This translates to 6,195,113 students, a jump of 30 percent from 1990 to 2000 (National Center for Educational Statistics, 2001)."
Miller & Mercer, 1997, 1-2	Research findings: • 8- and 9-year olds with learning disabilities performed at about a first-grade level on computation and application (Cawley and Miller, 1989). • Sixth graders with learning disabilities solved basic addition facts no better than third graders without disabilities (Fleischner, Garnett, and Shepherd, 1982). • Fifth graders with learning disabilities solved one third as many multiplication problems as their peers without disabilities on timed assessments (Fleischner, Garnett, and Shepherd, 1982). • Secondary students with mild disabilities attain math proficiency at the fifth to sixth grade level and perform poorly on required minimum competency tests (Cawley, Baker-Kroczynski, 1992). • Mathematical knowledge of students with learning disabilities tends to progress approximately 1 year for every 2 years of school attendance (Cawley and Miller, 1989). • Adolescents with learning disabilities reached a mathematics plateau after seventh grade. Students made an average of 1 year's growth during grades 7-12 (Warner, Alley, Schumaker, Deshler, and Clark, 1980). • The mean math scores of 12^{th} grade students was high fifth grade (Cawley and Miller, 1989, Warner et al., 1980). • Adolescents with learning disabilities plateaued at the fourth grade level and did not progress to higher stage problem solving (Greenstein and Strains, 1977).

Researcher(s)	Findings/Conclusions
Ontario Ministry of Education, 2005, 36	". . . research has shown that young children (in kindergarten and grade 1) with high levels of inattention symptoms are at significant risk for academic underachievement in reading and mathematics (Merrill & Tymms, 2001; Rabiner & Coie, 2000; Rabiner, Malone, & Conduct Problems Prevent Group, 2004)."
Cawley, Parmar, Foley, Salmon, & Roy, 2001, 312	"The background literature of special education has long shown that students with mild disabilities (a) demonstrate levels of achievement approaching 1 year of academic growth for every 2 or 3 years they are in school (Cawley & Miller, 1989); exit school achieving approximately 5th to 6th-grade levels (Warner, Alley, Schumaker, Deshler, & Clark, 1980); and demonstrate that on tests of minimum competency at the secondary level, their performance is lower for mathematics than it is for other areas (Grise, 1980). Warner et al. showed that students with learning disabilities attained only one-grade equivalent level in mathematics from Grade 7 through Grade 12. The data presented by Frise show that on a test of minimum competency for students in the 11th grade, 48% of students with learning disabilities passed the language/reading component, but only 16% of the students passed the mathematics component."
Dowker, 2004, 2	". . . Cockcroft (1982) reported that an average British class of eleven-year-olds is likely to contain the equivalent of a seven-year range in arithmetical ability."
Mercer & Mercer, 2005, 404	"The mathematical knowledge of students with learning problems progresses about one year for every two years of school attendance, and the mean math scores of students with learning disabilities in the 12th grade are at the high-5th-grade level (Cawley & Miller, 1989)."
Miller & Mercer, 1997, 4	"With regard to academic progress (a primary concern for students with learning disabilities, other educators . . . have reported that mainstreaming has not resulted in a high level of academic effectiveness. A 5-year longitudinal study involving over 500 adolescents with learning disabilities revealed that these students were significantly more likely to fail in general classes than in special classes."
Viadero, 2005, 1	"Researchers from the United Negro College Fund went to West Virginia last year and asked 62 high school dropouts in the federal Job Corps program a simple, open-ended question. 'What was it about school,' they wanted to know, 'that caused you to quit?' With surprising consistency, a majority of the participants, most of whom were African American or Hispanic, gave the same answer: 'Math'."
McEwan, 2000, 54	"Between 20% and 25% of students who complete high school are actually mathematics dropouts (Dossey, Lindquist, & Chamber, 1988)."
Darling-Hammond & Falk, 1997, 191	"Studies comparing the learning gains of students who were retained with those of academically comparable students who were promoted have found that retained students actually achieve less than their comparable peers who move on through the grades. Students appear not to benefit academically from grade retention regardless of the grade level or the student's initial achievement level. As Lorrie Shepard and Mary Lee Smith conclude in their review of research, 'contrary to popular beliefs, repeating a grade does not help students gain ground academically and has a negative impact on social adjustment and self-esteem'."
Darling-Hammond & Falk, 1997, 191	"One study found that children fear grade retention so much that they cite it third on their list of anxieties, following only the fear of blindness and death of a parent."
Fuchs & Fuchs, 2001, 85	"For students with learning disabilities (LD), mathematics problems are widespread and serious. More than 50% of students with LD have Individual Education Program goals in mathematics (Kavale & Reece, 1992), and research demonstrates the severity of mathematics difficulties for this population."
Fuchs & Fuchs, 2001, 85	"As demonstrated by Cawley et al. (1998), only 85% of normally achieving 12-year-olds have mastered computational addition; 81%, subtraction; 54%, multiplication; and 54%, division."

Table 2: Needs for Higher Achievement in Mathematics

Researcher(s)	Findings/Conclusions
US Department of Education, n.d., 1	"The Challenge: America's schools are not producing the math excellence required for global economics leadership and homeland security in the 21st century. The Solution: Ensure schools use scientifically based methods with long-term records of success to teach math and measure student progress."
National Research Council, 2001, xiii	"The globalization of markets, the spread of information technologies, and the premium being paid for workforce skills all emphasize the mounting need for proficiency in mathematics."
Education Trust, 2002, 3	"In 1989, after a thorough examination of the state of mathematics education in America, a prestigious panel assembled by the National Academy of Sciences issued a dire warning to the American people. 'We are at risk,' they said, 'of becoming a nation divided both economically and racially by knowledge of mathematics.'"
Battista, 1999, 426	"The mathematical ignorance of our citizenry seriously handicaps our nation in a competitive and increasingly technological global marketplace."
National Research Council, 2001, 16	"Citizens who cannot reason mathematically are cut off from whole realms of human endeavor. Innumeracy deprives them not only of opportunity but also of competence in everyday tasks. *All young Americans must learn to think mathematically, and they must think mathematically to learn."*
Ball, Ferrini-Mundy, Kilpatrick, Milgram, Schmid, & Schaar, 2005, 2	"All students must have a solid grounding in mathematics to function effectively in today's world. The need to improve the learning of traditionally underserved groups of students is widely recognized; efforts to do so must continue."
Enriquez, 2006, 1	"About one-third of the United State's PhDs in science and math are awarded to Asians and Asian-Americans—only 3 percent go to African-Americans and Hispanics. Within a few decades, 40 percent of the total U.S. population is likely to be Hispanic and African-American. Already 70 percent of the kids in the Los Angeles county school district are Hispanic. If large segments of the population do not become digital- and life science-literate, the engine of growth of the economy could begin to slow or stall. And there could be growing tensions between large ethnic islands."
RAND, 2002, 9	"The harsh reality is that our system produces starkly uneven results. Although some students develop mathematical proficiency in school, most do not. And those who do not have disproportionately been children of poverty, students of color, English language learners, and, until recently, girls. Recent NAEP results show the overall gap in mathematics achievement by social class and ethnicity has not diminished."
Romberg, 2001, 6	"First, all students need to have the opportunity to learn important mathematics regardless of socio-economic class, gender, and ethnicity."
Whitehurst, n.d.,1	"Low levels of achievement are more likely among minority groups and children from low-income backgrounds than among children from advantaged circumstances."
Schoenfeld, 2002, 1	"To fail children in mathematics, or to let mathematics fail them, is to close off an important means of access to society's resources."
Schoenfeld, 2002, 1	"Robert Moses . . . argues that children who are not quantitatively literate may be doomed to second class economic status in our increasingly technological society. . . ."
Schoenfeld, 2002, 3	"In purely functional terms, mathematics has long been recognized as a 'critical filter' (Sells, 1975, 1978). Course work in mathematics has traditionally been a gateway to technological literacy and to higher education. On such grounds alone, one could argue that there is a national obligation to insure that all students have access to high quality mathematics instruction."

Researcher(s)	Findings/Conclusions
Schoenfeld, 2002, 7	"Myriad data document disproportionate dropout and low-performance rates for students of color. . . ."
Friedman, 2006, 256	"Nearly 40 percent of the 18,146 people at NASA are age fifty or older. . . . NASA employees over sixty outnumber those under thirty by a ratio of about three to one. . . ."
Friedman, 2006, 256-257	". . . two-third of the nation's mathematics and science teaching force will retire in 2010."
Friedman, 2006, 257	"The NSB [National Science Board] report found that the number of American eighteen-to-twenty-four-year-olds who receive science degrees has fallen to seventeenth in the world, whereas we ranked third three decades ago. It said that of the 2.8 million first university degrees (what we call bachelor's degrees) in science and engineering granted worldwide in 2003, 1.2 million were earned by Asian students in Asian universities, 830,000 were granted in Europe, and 400,000 in the United States. In engineering specifically, universities in Asian countries now produce eight times as many bachelor's degrees as the United States."
Friedman, 2006, 258	"The number of jobs requiring science and engineering skills in the U.S. labor force, the NSB said, is growing almost 5 percent per year."
Friedman, 2006, 260	"Chinese applications to American graduate schools fell 45 percent his year, while several European counties announced surges in Chinese enrollment.

The negative consequences to the United States of not adequately addressing quality mathematics and science education for all its students are clearly very serious—for innovation, for business, for economic health, for quality of life, for military strength, and for homeland security. Friedman (2006) noted that "The brain gain started to go to brain drain around the year 2000" (p. 259) in the United States and that

> If action is not taken to change these trends, we could reach 2020 and find that the ability of U.S. research and education institutions to regenerate has been damaged and that their preeminence has been lost to other areas of the world (p. 258).

Purpose of This Study

The purpose of this study is to provide educators with the scientifically-based and other research evidence in which *MLS* is solidly grounded—at the (a) content, (b) lesson design, (c) instructional strategy, and (d) implementation feature levels. Given that the *No Child Left Behind (NCLB)* act stated more than 100 times that intervention programs and strategies must be based on "scientifically-based research," it is a responsibility of every program and strategy provider to provide that evidence. A McREL (Mid-continent Research for Education and Learning) official (2002, Summer) stated that "The onus for branding a product or program 'scientifically based' should rest first on the developers and distributors of the programs themselves" (p. 9). One is reminded of a comment by Pogrow (1996): that expecting teachers to do all of their instructional planning, to gather and vet and refine their own materials entirely on their own is akin to expecting actors to not just act, but write all their own scripts (p. 663).

What is presented, then, in this study is the scientific, theoretical, and evaluation research evidence that schools and districts need for assurance that *MLS* as a whole, its content, and its individual strategies and practices will result in improved achievement for a broad group of learners who have previously struggled with mathematics. Positive results—gains in student achievement—are the gold standard, of course. Southwest Educational Development Lab (n.d.) explained on their website that “When . . . programs are tested, the outcome that is measured is student achievement, and any program that increases student achievement significantly is considered to be an effective, research-based program.” The therapeutic nature of *MLS*, the knowledge and skills taught in the individual tasks, as well as the instructional strategies and other program features, have all been studied under experimental conditions, and the *MLS* program reflects the positive findings of many, many empirical studies, as well as additional theoretical and program evaluation studies.

Definitions of Scientifically-based Research (SBR)

Scientifically-based research is not a new educational term, but the attention it is currently receiving is certainly new due to its emphasis in *NCLB*, in the 2004 reauthorization of the *Individuals with Disabilities Education Act (IDEA),* and in other programs designed for struggling learners at the federal and state levels. Not only does the federal law now mandate that teaching strategies and programs be “scientifically based,” but it also defines in Title I (2001) what that is for reading:

> The term “scientifically based reading research” means research that (A) applies rigorous, systematic, and objective procedures to obtain valid knowledge relevant to reading development, reading instruction, and reading difficulties, and (B) includes research that (i) employs systematic, empirical methods that draw on observation or experiment; (ii) involves rigorous data analyses that are adequate to test the stated hypotheses and justify the general conclusions drawn; (iii) relies on measurements or observational methods that provide valid data across evaluators and observers and across multiple measurements and observations; and (iv) has been accepted by a peer-reviewed journal or approved by a panel of independent experts by a peer-reviewed journal or approved by a panel of independent experts through a comparably, rigorous, objective, and scientific review [*NCLB*, 2001, Sec. 1208(6)].

A similar definition will soon be disseminated for mathematics. President George W. Bush announced in February 2006 his new *NCLB* initiative: *Math Now: Advancing Math Education in Elementary and Middle School*. The language in that announcement made it clear that the mathematics initiative would also reflect the mandate that schools implement programs and practices that are grounded in scientifically-based evidence. He announced in April 2006 the appointment of a National Mathematics Panel to do a meta-analysis of mathematics research studies similar to that done by the National Reading Panel in 2000 for reading. He stated that “the National Mathematics Panel will convene experts to empirically evaluate the effectiveness of various approaches to teaching math, creating a research base to improve instructional methods for teachers.” In his April 16, 2006, press release, President Bush indicated that the report from the National Mathematics Panel would be submitted to him by January 31, 2007. Among the topics to be addressed are the following:

- The skills needed for students to learn algebra and be ready for higher levels of mathematics.

- The appropriate design of systems for delivering math instruction that combine elements of learning, curricula, instruction, teacher training, and standards, assessments, and accountability. And,
- Research needs in support of mathematics education (1).

On May 15, 2006, Secretary of Education Margaret Spellings announced the 17 members and six ex-officio members who will comprise the National Math Panel. CEI looks forward to the findings and recommendations from their report and the continued alignment of *MLS* with the best research available.

In the guidance provided by the United States Department of Education (2003) on how to identify effective programs and practices, they note: "By intervention, we mean an education *practice, strategy, curriculum*, or *program*" [emphasis added]. In another *NCLB* guidance document (Jan. 7, 2004), published by the United States Department of Education, SBR was defined as follows:

> Strategies grounded in scientifically based research are those that have demonstrated over time and in varied settings, an effectiveness that is documented by high-quality educational research. . . . For example, scientifically based research has shown that explicit instruction in (1) phonemic awareness, (2) phonics, (3) vocabulary development, (4) reading fluency, and (5) reading comprehension is effective in teaching reading to students in grades K-3. *Strategies that apply this research in a classroom setting would be grounded in scientifically based research* [emphasis added] (p. 10).

Again, although this guidance pertains to reading, it is easy to predict that the federal government will define SBR similarly for mathematics. In other words, a school or district may choose programs that include in their design the practices or strategies that have been verified as effective through scientifically-based research. Shaywitz (2003), one of the leading authorities on teaching learning disabled students, says that she recommends "total 'off-the-shelf' comprehensive programs rather than so-called eclectic ones that are stitched together by a child's teacher" (p. 262). She further notes that "programs are constantly changing, but the instructional principles remain the same (p. 263). *MLS* is the kind of evidence-based comprehensive program that she advocates.

Shanahan (2002) interprets "research based" in a similar way to the Department of Education. He suggests that the term should be "reserved for those instances when there was strong evidence that a particular type of instruction intervention—although not necessarily this particular version of it—had worked in the past" (p. 12). Deschler (2003) adds another dimension to the meaning of "research based." He states the following:

> I would submit that unless a so-called 'scientifically-based practice' has been shown to get results in a scaled-up and sustained fashion, it can't be said to be scientifically based. . . . Unless an innovation has been proven to be effective and usable in front line settings, researchers cannot legitimately claim their innovation to be scientifically-based (p. 1).

MLS meets these criteria. It has been used for almost a decade in all kinds and all levels of schools and other educational institutions, and it has consistently achieved improved learning for the diversity of students using it.

Stanovich and Stanovish (2003) further define SBR in describing the ways in which educators might gather evidence that new programs (whether purchased or designed in-house) are effective:

- Demonstrated student achievement in formal testing situations implemented by the teacher, school district, or state;
- Published findings of research-based evidence that the instructional methods being used by the teachers lead to student achievement; or
- Proof of reason-based practice that converges with a research-based consensus in the scientific literature. This type of justification of educational practice becomes important when direct evidence may be lacking. . ., but there is a theoretical link to research-based evidence that can be traced (p. 1).

The reader will find all three kinds of evidence documented throughout this study of *MLS'* effectiveness.

An example of how one can infer scientific evidence is provided by Mercer and Mercer (2005). They describe, for instance, several available research-based remedial programs, along with their features (pp. 304-306). Since these programs are proven to improve student learning, one can infer that similarly constructed programs grounded in the same research findings are also scientifically based, although not themselves directly studied. Reyna, the deputy of the Office of Educational Research and Improvement, stated in 2002 at the Department of Education's Working Group Conference (Neuman, 2002) the following: ". . . if we have a tested theory, we can sometimes extrapolate beyond just the limited group that was originally studied. . . . the boundary conditions for when an intervention is likely to be effective" (pp. 8-9). Reason requires the acceptance of the validity of such inferences; otherwise the cost in time and money to study directly every single strategy, procedure, curriculum, or program would be prohibitive and would paralyze all schools.

Mathematics SBR Controversies

Just as there continue to be controversies surrounding the findings of the National Reading Panel and of individual studies reviewed by them or recently published, so too are there controversy and concern about the existing scientifically-based evidence relevant to teaching mathematics, and especially as it relates to teaching mathematics to struggling learners. The first of these concerns is the paucity of mathematics research. At the Working Group Conference (Neuman, 2002) sponsored by the United States Department of Education in February 2002, Russell Gersten of the University of Oregon commented that "there isn't a lot of scientific research in math" (p. 11). The RAND 2002 Mathematics Study Panel in its proposal for a national mathematics research agenda concludes similarly:

> Mathematics education is an area of vital national interest, but also an object of considerable controversy. Claims and counterclaims abound concerning the value of

> distinctive curricular strategies and specific curricula, requirements for teacher knowledge, and standards that students should meet. For the most part these debates are poorly informed because *research evidence is lacking* [emphasis added] (p. xx).

Later in their study introduction, they reiterate the need for an adequate research base:

> The absence of cumulative, well-developed knowledge about the practice of teaching mathematics and the fragile links between research and practice have been major impediments to creating a system of school mathematics that works. These impediments matter now more than ever. The challenge faced by school mathematics in the United States today—to achieve both equity and mathematical proficiency—demands the development of new knowledge and practices that are rooted in systematic, coordinated, and cumulative research (p. 4).

The National Research Council's 2001 publication of a major synthesis of existing mathematics research, *Adding It Up: Helping Children Learn Mathematics,* also confronts the need for additional research on mathematics teaching and learning:

> One problem in weighing the evidence on a given issue in education is that a fully convergent database that speaks directly to the issue and yields unequivocal findings is seldom, if ever, available. The findings from experimental studies of mathematics learning often conflict. Data from non-experimental studies of relationships generally are ambiguous with respect to causality. Descriptive data can help frame an issue but usually do not address the question of which processes might lead to which learning outcomes. Ostensibly comparable studies can differ in key features, making it difficult to decide whether data are really comparable (p. 24).

A second concern, just as it has been for reading, is that educators are sometimes unaware that there is a different set of research findings for students with difficulties or disabilities than there is for the mainstream student. A major frustration of reading intervention providers has been the insistence of some states and districts in using the University of Oregon criteria for core reading programs for students in general education (for example, in Reading First programs) also as the criteria for choosing programs for students who need Tier II—III interventions. If those criteria did not work for struggling learners in Tier I, it is reasonable to determine that different criteria leading to more effective interventions might be required for Tier II-III, but objections were greeted with stone faces, and the sacredness of "standards" and "the NRP" were cited as the only evidence required. As early as 1997, Miller and Mercer comment on the harm of such thinking in their review of the latest curriculum standards published by the National Council of Teachers of Mathematics (NCTM) the following:

> Numerous educators have expressed concern regarding the application of the [NCTM] Standards to students with disabilities. . . . Among the concerns are the lack of references to students with disabilities in the Standards document, lack of research related to the Standards, and overall vagueness of the document (p. 3).

"Unfortunately," said Miller and Mercer, "none of these [national] reform movements produced good results. Instead, they neglected some of the basic psychological aspects of learning (e.g., attention, metacognition, memory, perception) and compounded the math problems of students with learning disabilities" (p. 2). Their analysis of the failure of recent reform movements in improving education for children with disabilities ends with this statement: "Reform in mathematics should be guided by replicable, validated programs that demonstrate effectiveness with targeted populations" (p. 9).

Deshler (2003) similarly expressed the following hesitations about embracing the new federal push for scientifically-based research:

> If neither practice nor outcomes improve on a large scale, sustainable basis, it is reasonable to question either the value of the specific line of research or the way that research programs in general are conceptualized and operated within a given funding agency. In short, federal investments in research programs for children, including those with individuals with disabilities, are defensible only if they lead to practices that improve the quality of services and outcomes for these individuals and their families (p. 2).

The problem of inadequate research or inappropriate research being applied to struggling learners is not limited to the United States. One finds familiar concerns in the Ontario Ministry of Education's 2005 publication of *Education for All: The Report of the Expert Panel on Literacy and Numeracy Instruction for Students with Special Education Needs, Kindergarten to Grade 6:*

> . . . research can provide teachers with a roadmap that highlights effective teaching techniques for all students. It is a roadmap we should pay attention to, because one of the most sobering findings is the evidence demonstrating the significant lack of progress that students with special needs in literacy or numeracy exhibit when not receiving a program based on research-supported instructional components. It is critical that instructional practice for all students reflect the best of what is available (p. 59).

A third issue is concerned with the difficulty of translating research into practice. Slavin (March 2005) is an advocate for the use of best practices by teachers: "the evidence-based policy movement remains the best hope for genuine reform of education in the U.S." (p. 5). And he adds, "Genuine, lasting progress will come in education (as in medicine) when practitioners have available effective methods, are expected to use them with intelligence and skill, and use data to monitor outcomes and benchmark these outcomes against those of practitioners in similar circumstances" (p. 7). According to Deshler (2003), that will not be easy:

> As encouraging as it is to have legislation espousing and even requiring the use of scientifically-based practices, there is no guarantee that results in our nation's classrooms will change unless we seriously confront the broad array of issues involved in effectively translating promising research findings into practice (p. 1).

Siegler (2003) agrees:

> Neither controlled scientific experimentation nor theoretical analyses automatically translate into prescriptions for classroom instruction. They can provide useful frameworks for thinking about teaching and learning, can indicate sources of difficulty that children encounter in learning particular skills and concepts, and can demonstrate potentially effective instructional procedures. However, a process of translation into the particulars of each classroom context is necessary for even the most insightful frameworks and the most relevant findings to be utilized in ways that improve learning (p. 230).

Schoenfeld (2002) concludes: "The best proof of the importance of research is documentation of its effects" (31).

Woodward (2004) goes a step further and expresses his concern about the value of the kind of research now required by the government to fund studies or to fund practice and programs:

> . . . research designs alone will not yield a satisfactory answer to the question, "what works?" *Scientific Research in Education* (National Research Council, 2002), which was commissioned by the National Research Council during the growing political debate over education research and practice, makes this clear. This report attempts to reconcile the tensions between different types of research methods by noting that different questions require different methodologies. What makes quasi-experimental and experimental designs important isn't their epistemological superiority as much as their value to politicians and decision makers who fund education and educational research. Large scale experimental research has face validity to politicians because the methodology is similar to what is used in medical research. However, the extent to which educational research can ever resemble medical research is a highly controversial assumption (Feuer et al., 2002).

Others temper their concerns. The RAND panel (2002) writes that "Tackling the problems of school mathematics obviously depends on much more than research, but research is necessary if energies and other resources are to be invested wisely" (p. 4). "Decisions about procedures," states the National Research Council (2001), "can be made with greater confidence when high-quality empirical evidence is available, but decisions about educational practice always require judgment, experience, and reasoned argument, as well as evidence" (p. 25).

In summary, educators find themselves in a perfect storm: one powerful wind mandates the use of empirical evidence in making decisions about curricula, programs, and strategies; another wind blows in with evidence that there is not yet enough research to guide decision-making and not enough funding to ensure its availability; another insists that the evidence used does not serve well all those students who struggle due to learning difficulties or disabilities; another wind whirls with the observation that evidence alone is not enough, and that translating that evidence into practice is the real obstacle; and another wind proclaims that the scientific evidence demanded by the government is not the appropriate way to study teaching and learning.

Amidst the ensuing confusion and frustration emerges *MLS*. The United States Department of Education (n.d.) has defined America's challenge as follows: "America's schools are not producing the math excellence required for global economic leadership and homeland security in the 21st century." The solution, it states, is to "Ensure schools use scientifically based methods with long-term records of success to teach math and measure student progress" (p. 1). This study will document that *MLS* is thoroughly grounded in the best research available on teaching and learning mathematics, especially the research that is available on struggling learners. Translating it into practice is already done in the program design, and schools can use it with confidence that they are providing their students with a program that works. It will also establish *MLS* as a therapeutic intervention, documenting the comprehensive assessments, the content, the instructional strategies, and other program features that make it effective, according to scientific research. Also, of great importance is that *MLS* is now ten years old, and it has over time continued to improve and continued to help struggling mathematics students improve their performance.

Programs Requiring SBR

All programs and strategies funded through federal dollars, including those that are grant-funded, and, increasingly, through state initiatives and grants, must reflect SBR. *MLS* is correlated in this section with the major federal programs serving low-performing, economically disadvantaged, limited-English proficient, dyslexic, and special education learners:

- Title IA Schoolwide Projects and Targeted Assistance Programs
- *NCLB Math Now*
- Title III Programs for Limited-English Proficiency Students
- Programs for Section 504 Disabilities (includes some dyslexic students)
- *IDEA* Programs for Special Education

A brief description of each program, information about the population(s) it serves, and references to CEI correlations with the program mandates are provided in Table 3.

This section of the study shows the alignment of *MLS* with major federal programs that serve struggling learners and which require evidence of SBR in order to be funded. Such correlations position *MLS* within the larger picture of a school's curriculum and instruction programs, as well as within *NCLB, IDEA*, and other program mandates. They also are a secondary level of research evidence. If a federal or state program mandate or standard is in itself research-based, then it follows that *MLS* is also research-based to the extent it correlates with the mandate or standard. For instance, a Texas Education Agency publication (2001) states that "TEKS . . . is a comprehensive research-based instructional program for grades K-12" (p. 5). To the degree that *MLS* is correlated with TEKS, its content is research-based, using those criteria.

Table 3: *MLS* Correlations with Federal Program Mandates

Federal Program	*MLS* Correlations
Title IA—Serves educationally disadvantaged. Accountability requires adequate yearly progress (AYP) on state assessments and on high school graduation rates for all students and for subgroups (racial/ethnic, limited-English proficient, economically disadvantaged, and special education).	CEI has constructed, upon request, numerous correlations of the *MLS* content with local and state curriculum standards. See Chapter IV of this paper for *MLS* scope and sequence chart. See CEI's webpage for *Correlation of MLS with the Diagnostic Screening Test for Mathematics (DSTM) and the NCTM standards*. CEI's *MLS Correlation to Title I Schoolwide Project Requirements* is available upon request. See Chapter VI for *MLS Correlation to Texas's Accelerated Mathematics Initiative* (similar to many states' mandates for interventions for students who fail state assessments in mathematics). See Chapter VI for *MLS Correlation to Arkansas's Mandate for Individualization in Interventions.*
Math Now—Serves underachieving students in K-7 in the elementary component, similar to Reading First. Serves middle school underachievers in the middle school component, similar to Striving Readers.	*MLS'* correlation with the characteristics of effective intervention programs will qualify it as a Tier II-III intervention for Math Now, grades K-8. See CEI's webpage for flier on *MLS Results*.
Title III Limited-English Proficient Learners—Accountability requires LEPs to take state assessments in English their fourth year in US schools. States must also test annually every LEP-identified student to measure growth in English proficiency. Districts must meet Annual Measurable Achievement Objectives (AMAOs) including performance of LEPs on state assessments, percentage growing at least one proficiency level on state assessment, and percentage exiting the LEP program.	*MLS'* correlation with the characteristics of effective intervention programs qualifies it as an effective preparation program for LEP students in learning mathematics and in learning mathematics in English. Academic English is used throughout. Appropriate mathematics vocabulary is stressed. Instructions are both written and verbal. Fluency development exercises are adequate and varied. Graphics are used to illustrate. Manipulatives are provided for concept development. See Chapters II and IV for sections on English-language learners.
Section 504 Disabilities—Schools must provide support and accommodations to children not eligible for special education, but with disabilities that affect learning.	Available on website in *SHARE* archives: *Dyslexics Need ELS and MLS*, April/May 2006.
IDEA—Special Education. Accountability requirements include not only those in IDEA, but also those in NCLB. There is a percentage cap on number of special education students scoring proficient on alternative assessments that can be used to calculate AYP for special education students. Cap varies according to state negotiations with USDE.	All documents above apply also to special education.
IDEA—Response to Intervention. This alternative model for the identification of learning disabled students calls for early intervention in K-3 as a way to avoid students needing special education, but it can be used K-12. Emphasis is on literacy, but two of the eight categories of disability pertain to mathematics.	Available on website in *SHARE* archives: *CEI's Response to Response-to-Intervention,* April/May 2006

Methodology

Just as effective teachers strive to incorporate as many scientifically-based teaching strategies as possible into any single lesson and throughout the school year, so did CEI in designing and developing *MLS*, using to the extent possible the *ELS* methodologies. The first step in documenting the scientifically-based evidence that grounds the program, therefore, was to "deconstruct" the program; that is, each task was analyzed to determine, first, whether it is instruction, practice, and/or assessment and the nature of the lesson design (whether direct instruction, mastery learning, or one-to-one tutoring). Then each task's specific content was determined, as well as the specific instructional strategies employed. The assessments used to determine diagnosis, program placement, progress, and mastery were also listed. And, finally, other program features that support effective implementation were enumerated. Only then did the work begin to identify whether empirical/scientific research validated each component of *MLS*.

CEI offices are filled with files, notebooks, and shelves of books on learning disabilities and on reading and mathematics research—some dating back to its founding; some produced by members of the staff; some written by graduate students from universities and presented to CEI; some produced by individual schools and districts; some pulled from the Internet; and some reproduced from research journals, old and new. Staff members, according to Lesley Mullen, service manager, eagerly consume these studies, for they serve as a constant validation of their work. They help, as well, to answer educators' questions about individual students in their labs, and they guide thinking about future development. Research for this study started in the archives, just as it did for *ELS*, to document SBR for CEI's customers and clients by gathering all those documents and books. Additional searches were conducted in libraries and using the Internet to identify potential empirical studies that would predict the effectiveness of *MLS*. Special efforts were expended to review as many recent studies as possible.

Education research journals were not the sole source for studies. Also included were studies from medicine, biology, neurobiology, neuropsychology, cognitive science, psychology, optometry, and other relevant disciplines. Numeracy is a complex set of understandings and skills; therefore, understanding it well clearly requires the study of diverse experts. Caine and Caine (1991) state it this way: "Teaching to the human brain. . ., based on a real understanding of how the brain works, elevates teaching into a challenging field requiring the finest minds and intellects" (p. ix).

In many ways this study is similar to the one performed by the National Reading Panel (2000). A thorough search was conducted for the relevant research, and then it was summarized and synthesized. A statistical meta-analysis with calculations of effect sizes was not performed, for those had already been done by several researchers. In fact, although the bibliography includes scores of individual studies, this study relied most heavily on the syntheses already conducted by such reputable researchers as the National Research Council (1997, 1999, 2001), Alliance for Curriculum Reform (1995, 1999), Robert Marzano and his colleagues (1992, 1998, 2001, 2003) at McREL, Mercer and Mercer (2005), and others. The studies quoted in their research syntheses are not included in this study's bibliography since all the experiments in the meta-analyses were not inspected directly. Only the sources directly cited are included. The bibliographies of each of the cited sources are recommended to the reader as additional evidence.

The reader will find in subsequent chapters, beginning with Chapter IV, a pattern of organization. First, the topic is defined and the research provided, and, second, the way that the particular topic is descriptive of some feature of *MLS* is explained or described. A decision, then, was made to use some direct quotations from the research findings in this explanation, but, for the most part, the research findings are listed in tables without any filtering on the part of the writers. The reader can review the cumulative evidence and make his or her own inferences about their meaning and application, assuring as much objectivity as possible in this presentation of scientifically-based evidence.

The writers also felt that it was important to include a wide sampling of the available research and then to invite the reader to use the specific sources and findings that they deem most important and relevant to their decision-making. Different districts and different states have varying expectations about how to document the scientifically based research behind an intervention program, so the flexibility of this document will make it easier for educators to cite what they determine will best meet their needs in making a decision to include *MLS* as an intervention for struggling students or in justifying to funding sources a prior implementation of *MLS*.

Description of *MLS* and Its Uses

Mathematical Learning Systems (MLS) is a supplementary, therapeutic intervention program for students, K-adult, who are experiencing difficulty in learning mathematics, regardless of the cause of the difficulty or disability, whether inherited, acquired, or environmental. Given the diversity of learners that the program serves, it is individualized and differentiated in many ways and therefore has a complex architecture. Subsequent chapters of this study describe its many components and strategies.

MLS is not in itself a comprehensive mathematics program, not at any grade level. It is, rather, a learning system in that it provides cognitive therapy to address the root causes of mathematics failure, and it focuses on the areas of mathematics where learning difficulties and/or disabilities are most likely to be manifested, according to scientific evidence. It correlates with curriculum standards only at the basic skill level, and it includes only the most critical prerequisite knowledge and skills found in research to be problem areas for students and/or to be critical for students being able to move to higher-level mathematics, such as algebra. *MLS*, in other words, teaches the prerequisite knowledge and skills that make it possible for all those learners currently failing to learn how to learn mathematics and to learn what they need to know to access the grade-level curriculum in order to meet the standards of proficiency on state examinations. CEI program expert, Joann Price (March 2006), points out that “It makes no sense for a school to require students to be tutored on grade-level curriculum standards if they cannot retrieve their math facts fluently or if they have no deep understanding of the base-10 system or place value.” It is important to attend to “first things first.”

Schools at all levels typically adopt *MLS* as an intervention program to be used by any student (K-adult) who is failing mathematics courses or failing the state assessments in mathematics. Sometimes funding determines the target population—Title I, dyslexia, at-risk, special education, limited-English proficiency, or Response-to-Intervention. Some schools use *MLS* as a component of their Comprehensive School Reform model. Others use it in after-school or in-school tutoring programs. Some use it to teach mathematics in English to English-language learners. And some use it in adult education. *MLS* is also used as intensive instruction in second mathematics periods and/or in summer school for remediation and/or acceleration. Some elementary schools integrate its use with general classroom instruction, using *MLS* lessons to reinforce their own lessons, using *MLS* fluency exercises to develop faster and more accurate fact retrieval, and even using *MLS* to accelerate students who are ready to move ahead. Many schools adopt *MLS* for special education, finding its progress monitoring to be helpful in documenting progress for Individual Education Plans (IEPs), and others use it to satisfy state mandates for academic improvement plans for students who fail the state assessment. CEI is identifying increasing numbers of schools interested in using *MLS* as a Tier II or Tier III intervention in the new Response-to-Intervention model (see Chapter VIII).

MLS includes five units in its concept building scope and sequence: (1) Understanding Numbers; (2) Number Operations; (3) Using Whole Numbers; (4) Understanding Fractions; and (5) Fraction Operations. Each unit includes one to four levels, each with lesson phases moving from simple (concrete) to more complex understandings (semiconcrete and abstract). There are 14 tasks related to the concept building lessons, and the fluency development lessons include another nine tasks that emulate those found in *ELS*. Tasks will be further described and discussed in the chapters on *MLS* content (Chapter IV), lesson models (Chapter V), and instructional strategies (Chapter VI). Detailed descriptions of the individual tasks, as well as supplementary and resource materials, are provided in the *MLS Teacher's Manual*.

Organization of Study

The documentation of research findings begins in Chapter II with studies relating to various causes of mathematics difficulties (as opposed to mathematics disabilities), such as cultural attitudes, race/gender issues, mathematics phobia, low self-esteem and other motivational issues, and problems manifested by English-language learners. Chief among the reasons for difficulties, as determined by researchers, is the issue of inadequate or inappropriate instruction. The debate on appropriate emphasis in mathematics programs—the "math wars"—will be explored, as well as CEI's position on that debate. An understanding of the causes of mathematics difficulties is very important for the educator choosing an effective intervention.

Chapter III will focus on mathematics disabilities—both those learners with a mathematics disability only (e.g., dyscalculia) and those with both reading and mathematics disabilities—and the effects of those disabilities on mathematics achievement. The research findings relating to the manifestations of those with mathematics disabilities will be reported. Again, an understanding of these is critical for educators searching for an effective intervention.

Chapter IV begins with an overview of the research defining mathematics cognition—as background to understanding the research on what kinds of content are needed in an effective mathematics intervention, relating back to the manifestations of mathematics difficulties and disabilities discussed in Chapters II-III. The majority of Chapter IV will be a description of *MLS'* content emphases and a discussion of the research studies that ground the design decisions. *MLS'* scope and sequence will be included.

A description of the structure of *MLS* lessons will begin Chapter V, followed by the research findings that validate those structures, especially lesson phases and the emphasis on direct instruction, with elements of mastery learning and one-on-one tutoring strategies in the various lesson tasks. This chapter will also include the research on the importance of the concrete-semiconcrete-abstract lesson sequence and the use of manipulatives in developing mathematical concepts. Chapter V concludes with a review of the efficacy of computer-assisted instruction, including the research on computer screen graphics and how they impact learning.

Chapter VI begins with the research on the instructional strategy for which both *ELS* and *MLS* are best known: multi-sensory processing. The discussion will then move to other scientifically-based methodologies: individualization/differentiation, practice/repetition or fluency development, chunking/clustering, and engaged time-on-task. Each section will include information about the ways in which these strategies are used in *MLS*. Because assessment is critical to effective instruction, the *MLS* comprehensive assessment system is then described, followed by the research on assessment, corrective feedback, informed instruction, and self-assessment—all strong features in the *MLS* design.

Because CEI sees support for effective implementation as one of its major responsibilities, Chapter VII will focus on the research behind some of *MLS'* implementation program features, such as the role of the lab teacher/facilitator in implementing *MLS*, professional development, student motivation, and parental involvement. Salient research on each of these topics will be included, as well as descriptions of how they are a part of the *MLS* program.

In Chapter VIII there will be an analysis of CEI's statistics on achievement gains over multiple years in schools using *MLS*. This chapter will include a research synthesis of the characteristics of effective mathematics interventions and the ways in which *MLS* reflects those characteristics. Lastly, Chapter VIII ends with a summary of previous chapters, and a discussion of insights and conclusions.

At the conclusion of the paper, the reader will find four documents: (1) *MLS* Bibliography, (2) Dictionary of Acronyms, (3) Index of Terms, and (4) Index of References.

Creative Education Institute <u>knows</u> that *MLS* works. Its effectiveness is documented annually in the accelerated growth that learners achieve in labs serving diverse ages and needs. It is further documented in the case studies of individual students and labs—many of which appear in *SHARE*, CEI's bimonthly newsmagazine. CEI also points to its very high rate of service contract renewals each year as evidence of customer satisfaction relating to lab results in individual schools. This study documents the scientific research and how it is applied in the *MLS* design to provide the necessary evidence of <u>why</u> it works.

Chapter II: Mathematics Difficulties

"Many adults believe they have a clear sense of what mathematics is and why they despise it."
(Zemelman, Daniels, and Hyde, 1998, 83)

Overview

Chapter I provided a general introduction to the study. Chapters II and III summarize current research studies that define both mathematics difficulties and mathematics disorders, as well as the research on the manifestations of each—very helpful information for educators attempting to diagnose and prescribe interventions for students who struggle. The typical report that teachers receive after state or local assessments lists the students and reports a score on his or her performance. With a little work, one can do an item analysis to discover which questions presented difficulties and which ones had accurate responses—for an individual student or for the class, school, or district as a whole. This information is, of course, helpful, especially for curriculum developers and supervisors, if there is enough information available from the items to link them to the established curriculum standards. Schools' data analyses typically stop here, however, assuming that information is disaggregated by all the *NCLB* subgroups, plus other groups such as gender and length of time in the school, or, perhaps, in United States schools. Two responses are in order at this point: curriculum alignment (or mapping) and/or tutoring for individuals or groups on their areas of weaknesses.

When the analyses are complete, unfortunately, all the educators know is which kinds of questions and which topics indicate strengths and which indicate weaknesses—for individuals or groups. Nothing is known at this point about "why" there are weaknesses. The analysis does not reveal any of the potential reasons for low achievement, making diagnosis and prescription a guessing game, a game of trial and error. Nor does it reveal whether the reason is related to a "difficulty" or to a "disability." Students who fail year after year are the victims. And the complexity of "why" is what makes education "rocket science."

"Developing profound understandings of difficulties and disabilities in reading and mathematics are a major part of the work of CEI," according to its President, Bonnie Lesley. "Without those understandings, our software design, publications about the products, professional development for lab facilitators, on-going coaching, *SHARE* news magazine, and training of CEI staff would indeed be spurious."

Definitions

Researchers frequently make a distinction between learners with mathematics difficulties versus learners with mathematics disabilities. In fact, searching for a scientifically-based method to distinguish them is the theory behind the new initiative in the 2004 reauthorization of the *Individuals with Disabilities Education Act (IDEA),* Response-to-Intervention. Students with mathematics difficulties should, it is reasoned, respond positively to scientifically-based interventions that accelerate learning so that they can achieve at the level and rate of their peers. These students comprise as much as 70 percent of those referred to special education, according to Lyon (1996), so they can be served well in general education, and the costs of special education could be dramatically reduced. Those who do not respond positively to interventions are candidates for identification for special education since they likely have disabilities.

Mathematics difficulties are those that result generally from one or more of the following situations:

- lack of motivation to learn mathematics, which is linked in many cases to cultural attitudes about the importance of mathematics proficiency;
- fear of mathematics, sometimes termed mathematics phobia, and/or sometimes linked to stereotype threat (both gender and race);
- general lack of motivation, especially issues of low self-esteem;
- learning interferences or confusion caused by the structure of one's home language; and
- most frequent and most controversial--inadequate or inappropriate instruction in mathematics.

Pennington (1991) makes the following observation: "A child can have poor school performance without having a learning disorder, when the poor school performance is due entirely to emotional, motivational, or cultural factors" (p. xii). Geary, Hamson, and Hoard (2000) agree:

> . . . it appears that children who show low achievement levels in one grade but average or better achievement levels in another grade . . . do not have the cognitive deficits associated with MD/RD *[math disability/reading disability]* and MD. Rather, the intermittent academic difficulties of these children appear to be related to other factors (e.g., emotional difficulties) (p. 256).

This chapter will synthesize the research on each of these sources of difficulty. Subsequent chapters on the appropriate and scientifically-based content (Chapter IV) and methodologies (Chapters V and VI) of an effective intervention (Chapter VIII) will include documentation of how *MLS* can be a school's solution for students with mathematics difficulties.

Cultural Effects on Motivation to Learn Mathematics

There is no doubt that cultural attitudes and values have a great deal to do with school achievement. What is valued most tends to be what is encouraged and developed among the young. Miller, Kelly, and Zhou (2005) write that "There is a fairly clear relationship between parental beliefs and child academic abilities" (p. 174). Gardner (1985) says that everyone, absolutely everyone, has multiple intelligences, but that we tend to develop most those areas that are valued in the culture, with the most important cultural influence being the home (p. 26). Parental attitudes about the value of mathematics are a cause of great concern among many mathematics educators in the United States:

> "Our culture has a built-in distaste for math, which I hope we can change," says Johnny Lott, president of the National Council of Teachers of Mathematics. "The non-unusual quip heard from parents at parent-teacher conferences—'I was not very good in math'—is just not acceptable for students in today's technological society where jobs increasingly require a sound understanding of math, Lott says" (Allen, 2003, p. 1).

Sherman, Richardson, and Yard (2005) express the same concerns:

> Students should also learn to value learning and the use of mathematics in their daily lives. Too often, people find it socially acceptable to say, 'I am not a math person.' Though they would find it embarrassing to claim that they are not good readers, innumeracy is readily admitted (p. 7).

Young (n.d.) states that "mathematics anxiety is widespread. So rampant is innumeracy that there is little stigma attached to it. Many adults readily confess, 'I was never good at math,' as if displaying a badge of courage for enduring what for them was a painful and useless experience" (p. 3). Paulos (1988) puts it this way:

> Innumeracy, an inability to deal comfortably with the fundamental notions of number and chance, plagues far too many otherwise knowledgeable citizens. The same people who cringe when words such as "imply" and "infer" are confused, react without a trace of embarrassment to even the most egregious of numerical solecisms" (p. 1).

Further testimony comes from Posamentier (2003):

> When I meet someone socially and they discover that my field of interest is mathematics, I am usually confronted with the proud exclamation: "Oh, I was always terrible in math!" For no other subject in the curriculum would an adult be so proud of failure. Having been weak in mathematics is a badge of honor. . . . It is my strong belief that the root of the problem lies in the inherent unpopularity of mathematics. But why is it unpopular? Those who use mathematics are fine with it, but those who do not generally find it an area of study that may have caused them hardship" (p. xiii).

Several studies have explored the problem of cultural attitudes relating to mathematics. Miller, Kelly, and Zhou (2005) have conducted extensive studies in the area of cultural effects on mathematics achievement, comparing attitudes toward mathematics in the United States with attitudes of Chinese parents:

> Mothers of kindergartners in both China and the United States considered literacy skills the most important thing that they themselves learned in first grade (Kelly, 2000). However, when the same mothers were asked to rate how important the mastery of particular math and literacy skills before entry to first grade is for academic success, a different picture emerged. Mothers in China rated literacy and mathematical skills as equally important. U.S. mothers showed a clear bias toward rating the various literacy skills higher than the mathematical skills. Thus, some of the differences in math and reading ability that have been found when comparing young children in China and the United States may be due to the relative importance placed on reading and math skills by mothers in each country (Kelly, 2002) (p. 173).

They explain that "in the same way that different countries emphasize different sports, school mathematics is a national intellectual pursuit in East Asian counties, but not in the United States"

(p. 173). In conclusion, they write that "The picture that emerges from these surveys points to three important facts that have apparent consequences for the nature of early mathematical development" (p. 175): Those facts are as follows:

1. U. S. parents tend to privilege reading over mathematics in preparing their children for school, compared to Chinese parents who show more balance between these two areas.
2. U.S. parents are more likely than Chinese parents to attribute success and failure to innate factors rather than to effort.
3. Finally, a relative lack of communication between home and school may make it difficult for U.S. parents to coordinate with schools any educational efforts they undertake. Stevenson and Stigler (1992) noted that parents of preschoolers in their Minneapolis sample devoted a relatively large amount of effort to teaching their children academic subjects before school started. When school started, that effort switched to nonacademic concerns (such as music lessons, athletics, etc.) at the same time their East Asian parents were becoming heavily involved in helping their children succeed with their schoolwork (p. 175).

Similar conclusions have been drawn by other researchers. For instance, the National Research Council (2001) reports that "Cross cultural research studies have found that U.S. children are more likely to attribute success in school to ability rather than effort when compared with students in East Asian countries" (p. 132). Another research team, Fuson, Kalchman, and Bransford (2005), state it this way:

> In many countries, the ability 'to do math' is assumed to be attributable to the amount of effort people put into learning it. Of course, some people in these countries do progress further than others, and some appear to have an easier time learning mathematics than others. But effort is still considered to be the key variable in success. In contrast, in the United States we are more likely to assume that ability is much more important than effort, and it is socially acceptable, and often even desirable not to put forth effort in learning mathematics (pp. 221-222).

In a review of research about the differences between mathematics achievement of American and Chinese students, Wang and Lin (June/July 2005) conclude the following:

> In general, the studies relevant to family values and processes suggest that Chinese parents set higher expectations for their children's mathematics achievement, engage their children in working more on mathematics at home, and use formal and systematic instructional approaches at home. Exposure to these family values and processes appears to produce children's synergism with parental expectations and may lead to higher general mathematics achievement. Similar family values and processes were also found in Chinese American families (p. 9).

In addition to cultural attitudes, the socio-economic status of parents also determines, in many cases, their attitudes about mathematics. According to Campbell and Silver (1999),

> Social condition, social tradition or culture, and social goals influence student learning. For example, poverty and the educational levels of parents are two social conditions that impact learning. Poverty limits the out-of-school educational experiences and materials that students encounter, affecting both the prior knowledge that students bring to the classroom and access to the tools that students may need to accomplish assigned tasks. Similarly, poverty is often correlated with unstable housing patterns, thereby increasing student mobility and resulting in gaps in learning (p. 30).

Franklin (2003) agrees: "Parents must be encouraged to support learning. . . . Part of the reason so many Latino children are not in college-track courses is because their parents and families do not understand how the school system works and the implications of not taking the right math courses" (p. 5). Phillips, Brooks-Gunn, Duncan, Klebanov, and Crane (1998) report on a study of parenting practices. They conclude that "For parents who want their children to do well on tests (which means almost all parents), middle-class parenting practices seem to work" (p. 127). They also suggest that "it takes at least two generations for changes in parental socioeconomic status to exert their full effect on parenting practices" (p. 127). They continue:

> Even though traditional measures of socioeconomic status account for no more than a third of the test score gap, our results show that a broader index of family environment may explain up to two-thirds of it. Racial differences in grandparents' educational attainment, mothers' household size, mothers' high school quality, mothers' perceived self-efficacy, children's birth weight, and children's household size all seem to be important facts in the gap among young children. Racial differences in parenting practices also appear to be important (p. 128).

Ferguson (1998a) notes the following:

> One common hypothesis is that all children learn more when their home and school environments are well-matched—that is, when there is cultural congruence. Some black children, especially those from low-income households, come from home environments that differ systematically from the typical white mainstream to which schools and teachers are usually oriented (p. 347).

Then he adds, ". . . distinctions of social class may be as important as racial distinctions for understanding the black-white achievement gap and how to reduce it" (p. 350).

There are problems as well in homes of higher economic status. Friedman (2006), for example, quotes a computer science professor:

> I taught at a local university. It was disheartening to see the poor work ethic of many of my students. Of the students I taught over six semesters, I'd only consider hiring two of them. The rest lacked the creativity, problem-solving abilities, and passion for learning (p. 261).

A few pages later Friedman quotes from a consular official in the U.S. embassy in Beijing:

> I do think Americans are oblivious to the huge changes. Every American who comes to visit me in China is just blown away. . . . Your average kid in the U.S. is growing up in a wealthy country with many opportunities, and many are the kids of advantaged educated people and have a sense of entitlement. Well, the hard reality for that kid is that fifteen years from now Wu is going to be his boss and Zhou is going to be the doctor in town. The competition is coming, and many of the kids are going to move into their twenties clueless about these rising forces (p. 264).

Friedman devotes an entire section of his book on the globalization of economies and the competitive flattening of the world to the importance of good parenting. He states that "we need a new generation of parents ready to administer tough love: There comes a time when you've got to put away the Game Boys, turn off the television set, put away the iPod, and get your kids down to work (p. 303).

He continues with this narrative:

> David Baltimore, the Nobel-Prize-winning president of Caltech, knows what it takes to get your child ready to compete against the cream of the global crop. He told me that he is struck by the fact that almost all the students who make it to Caltech, one of the best scientific universities in the world, come from public schools, not from private schools that sometimes nurture a sense that just because you are there, you are special and entitled. "I look at the kids who come to Caltech, and they grew up in families that encouraged them to work hard and to put off a little bit of gratification for the future and to understand that they need to hone their skills to play an important part in the world," Baltimore said. "I give parents enormous credit for this, because these kids are all coming from public schools that people are calling failures. Public education is producing these remarkable students—so it *can* be done. Their parents have nurtured them to make sure that they realize their potential. I think we need a revolution in this country when it comes to parenting around education.
>
> Clearly, foreign-born parents seem to be doing this better. "About one-third of our students have an Asian background or are recent immigrants," he said (pp. 303-304).

To America's great disadvantage then, many homes at both the low and high ends of the economic scale fail to provide the necessary support and expectations to their children to learn mathematics and to learn it well. It is a mistake, then, to believe that only the poor fail to value mathematics and science learning.

The U.S. is not the only country with negative attitudes about the value of mathematics. In their studies, Prenzel and Duit (2000) found problems in Germany, as well:

> It appears that not only the way science and mathematics are taught in German schools is responsible for the deficiencies of German students as revealed by TIMSS *[Third International Mathematics and Science Study]*, but also the image of these school subjects in the broader public. Science and mathematics as well as learning of these subjects are not highly valued in the public and accordingly in the families and the students' peer

> groups. Hence, learning science and mathematics is not sufficiently supported in society as a whole. There is also the common belief that the ability of learning science and mathematics is mainly a matter of being gifted. It, therefore, appears not to be worth the effort when students think they are not gifted (p. 3).

The societal and cultural devaluing of mathematics is not lost on teachers, as several researchers (Allen, 2003) point out:

> One result of the cultural diffidence toward math, some experts say, is that many U.S. teachers follow the path of least resistance and reduce mathematical concepts to a series of 'procedures' to solve a problem. It's largely the American way of teaching math, according to evidence from the Third International Mathematics and Science Study (TIMSS) 1999 Video Study (p. 1).

Fuson, Kalchman, and Bransford (2005) see another cultural effect: "Teachers in some countries believe that it is desirable for students to struggle for a while with problems, whereas teachers in the United States simplify things so that students need not struggle at all" (p. 222). Armington (2002) summarizes as follows: "Coupled with the negative influence of environmental factors is the belief that students who do well in math do so because of native ability, not effort. This misconception, propagated by teachers and society at large, only serves to reinforce negative student behaviors that lead to underperformance in mathematics" (p. 2).

Gardner (1985) summarizes the importance of culture on learning as follows:

> What recent research has shown, virtually incontrovertibly, is that whatever differences may initially appear, early intervention and consistent training can play a decisive role in determining the individual's ultimate level of performance. If a particular behavior is considered important by a culture, if considerable resources are devoted to it, if the individual himself is motivated to achieve in that area, and if proper means of crystallizing and learning are made available, nearly every normal individual can attain impressive competence in an intellectual or a symbolic domain. Conversely, and perhaps more obviously, even the most innately talented individual will flounder without some positive supporting environment. Discovery of an individual's inherent intellectual profile, which I believe may be possible, need not serve, then, as a means of pigeonholing the individual or of consigning him to an intellectual junkheap; rather, such discovery should provide him means for assuring that every individual has available to him as many options as possible as well as the potential to achieve competence in whatever fields he and his society deem important" (p. 316).

Stereotype Threat Effects on Motivation to Learn Mathematics

"Stereotype threat" as it relates to mathematics tends to affect two major groups of learners: girls/women and racial/ethnic minorities. In its influential 2001 report on mathematics, the National Research Council notes such effects:

> Research with older students and adults suggests that a phenomenon termed stereotype threat might account for much of the observed differences in mathematics performance between ethnic groups and between male and female students. . . . So-called wise educational environments can reduce the harmful effect of stereotype threat. These environments emphasize optimistic teacher-student relationships, give challenging work to all students, and stress the expandability of ability, among other factors (p. 133).

Zemelman, Daniels, and Hyde (1998) reflect in their book that "Common myths about mathematics include a variant of the Marine Corps recruiting slogan: math is only for a few good men; most mere mortals (especially women) are not good at it" (p. 83). 'Perseverance in the face of group-based stereotypes about one's limitations poses a daunting challenge,' state Pronin, Steele, and Ross (2003)." They continue:

> Beyond enduring negative expectations and discouragement from others, members of the stereotyped group may respond to inevitable disappointments and difficulties by questioning their own fitness and acceptance in the social environment. And they may be further burdened by the knowledge that individual failures will reinforce the negative views and assumptions held about their group (p. 1).

Ben-Zeev, Duncan, and Forbes (2005) are experts in this discriminatory phenomenon, especially as it pertains to females:

> A useful format for investigating the cause of females' underperformance in the math domain can be found in recent work on stereotype threat—a situational phenomenon that occurs when high-achieving individuals, who are targets of stereotypes alleging intellectual inferiority, are reminded of the possibility of confirming these stereotypes (p. 236).

They point out that not all threats have to be explicit to produce performance deficits. "Stereotype threat," they write, "can be triggered in subtle ways, such as by the gender composition of the individuals in the environment" (p. 237). More explicit triggering of the effects would include "reminding individuals of negative stereotypes about their group." Such reminders "work to hinder these individuals' performance." They further report that "Research has also shown the reverse effect: priming individuals with positive stereotypes can help to facilitate performance" (p. 237).

Spencer, Steele, and Quinn (Jan. 1999) have also contributed to the research on this issue:

> When women perform math, unlike men, they risk being judged by the negative stereotype that women have weaker math ability. We call this predicament stereotype threat and hypothesize that the apprehension it causes may disrupt women's math performance. In Study 1 we demonstrated that the pattern observed in the literature that women underperform on difficult (but not easy) math tests was observed among a highly selected sample of men and women. In Study 2 we demonstrated that this difference in performance could be eliminated when we lowered stereotype threat by describing the tests as not producing gender differences. However, when the test was described as producing gender differences and stereotype threat was high, women performed substantially worse

> than equally qualified men did. A third experiment replicated this finding with a less highly selected population and explored the mediation of the effect. The implication was that stereotype threat may underline gender differences in advanced math performance, even those that have attributed to genetically rooted sex differences (p. 4).

One very interesting study that validates the existence of the negative effects of stereotype threat was conducted by Aronson and his colleagues in 1999: "we induced stereotype threat by invoking a comparison with a minority group stereotyped to excel at math (Asians). As predicted, these stereotype-threatened white males performed worse on a difficult test than a nonstereotype-threatened control group" (p. 29). In other words, even white males, not just minority groups or just women, are susceptible to stereotype threat.

The consequences of stereotype threat have been well studied. Pronin, Steele, and Ross (2003) have summarized the salient studies:

> Previous researchers have explored a number of ego-protective responses to stereotype threat. One well-documented response is "disengagement" (Crocker, Major, & Steele, 1998; Major, Spencer, Schmader, Wolfe, & Crocker, 1998) or "disidentification" (Spencer et al., 1999; Steele, 1997; Steele & Aronson, 1995) with respect to the domain in question. That is, the individual excludes performance in that domain as a basis for self-evaluation, and may reject it as a basis of respect for people in general (Crocker & Major, 1989). Indeed, the individual may even foster an identity "oppositional" to success in that domain (Ogby, 1986) (p. 2).

In other words, once threatened, the individual who feels stigmatized decides not to participate in that domain and "drops out" to save face. Pronin, Steele, and Ross (2003) point out that these protective strategies are "especially costly when the domain in question is relevant to an important avenue for professional or personal advancement," as mathematics certainly is.

All these researchers have also looked for ways to overcome the negative effects. Ben-Zeev, Duncan, and Forbes (2005) ask, "Can we help high-achieving students to develop coping skills for combating the detrimental effects of stereotype threat?" Their answer then follows: "The first wave of proposed interventions, most notably "wise schooling," has been to overcome suspicion and to develop trust or by changing diversity in a student's community (e.g., Steele, 1997)" (p. 244).

Later they add other solutions:

> As research on mediation of stereotype threat progresses, the ways to combat it become even more conceivable. Recent data offer a variety of potential tactics to reverse threat effects, such as by redefining the context with which a test is taken to be less threatening explicitly. . ., having a stigmatized individual engage in self-affirmation thoughts prior to taking a test. . ., or priming women who are taking a difficult math exam with stories of women who have succeeded in male-dominated fields (pp. 245-246).

Finally, they warn: "If test scores are depressed because of stereotype-threat effects, then test scores are not a valid proxy for ability" (p. 246).

Steele and Aronson (1998) conclude their discussion of five studies they conducted on the issue of stereotype threat with this finding: ". . . stereotype threat can impair the test performance of African Americans even if it is created by quite subtle changes of environment. Eliminating stereotype threat can dramatically improve blacks' performance" (p. 423).

Effects of Mathematics Phobia on Motivation to Learn Mathematics

McGuinness (1997) powerfully states a problem that many students have, not only in reading, but in mathematics: "What children want most is to show that they are competent in all areas in which their age mates are competent" (p. 285). Typically, for both children and adults, if we do not feel that we are competent, we do not wish to perform at all, and many of us do not. According to Furner and Duffy (2002),

> Mathematics anxiety has become a concern for our society. Research . . . has shown that only about 7% of Americans have had positive experiences with mathematics from kindergarten through college. Similarly, Burns (1998) has contended that two thirds of U.S. adults fear and loathe math. . . . Many children, including those with disabilities and those without disabilities, as well as adults, do not feel confident in their ability to do math" (p. 67).

There are several related explanations for what is commonly called "math phobia" or "math anxiety." Armington (2002) quotes Sheila Tobias in his:

> Speaking on math anxiety and barriers to student success in mathematics, Sheila Tobias' presentations at NADE 2001 examined both instructional and student issues in learning. According to Tobias, the predominant causes of math anxiety are environmental factors created by math teachers. These include pressures created by timed tests, an overemphasis on one right method and one right answer, humiliation of students at the blackboard, an atmosphere of competition, absence of discussion, and other related dynamics that typify the math classroom. For many students, these factors lead to destructive self-beliefs about the math abilities they possess, avoidance behavior, and an unwillingness to explore mathematical concepts in the classroom environment (p. 2).

Erlauer (2003) likewise connects teacher behavior to math phobia:

> . . . teachers who cause or allow stressful, threatening, or fearful occurrences in the classroom are building memories of those negative issues rather than important academic concepts. Because these students are under stress, their brains are operating in the limbic system rather than the higher-level neocortex, making learning much more difficult (p. 13).

Smith (2002) agrees:

> Teachers and parents who find mathematics boring or incomprehensible easily convey those feelings to a child. Teachers and parents who themselves fear exposure to mathematics easily transfer the fear. Children don't necessarily learn what we hope we are teaching them, but they are most susceptible to learning what we unwittingly demonstrate (p. 127).

Sousa (2001) offers a similar explanation, but distinguishing between math phobia as a difficulty and a true mathematics disability:

> Some children develop a fear (or phobia) of mathematics because of negative experiences in their past or a simple lack of self-confidence with numbers. No doubt, mathematics phobia can be as challenging as any learning disability, but it is important to remember that these students have neurological systems for computation that are normal. They need help primarily in replacing the memory of failure with the possibility of success. Students with mathematics disorders, on the other hand, have a neurological deficit that results in persistent difficulty in processing numbers (p. 140).

Ashcraft and Ridley (2005) add this definition of math phobia: "Math anxiety is defined as a negative reaction to math and to mathematical situations. In Richardson and Suinn's (1972) words, it is '. . . a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations'" (p. 315). These more recent studies agree with the earlier conclusions of Miller and Mercer (1997):

> . . . repeated academic failure frequently results in low self-esteem and emotional passivity in mathematical learning. . . . Confused thinking, disorganization, avoidance behaviors, and math phobia are common results (p. 8).

Ashcraft and Ridley (2005) contribute more than definitions. Their work includes the documentation of behaviors that result from math phobia. Table 4 displays their findings, along with those of Shaley and Gross-Tsur (2001).

Table 4: Math Phobia Manifestations

Researcher(s)	Findings/Conclusions
Ashcraft & Ridley, 2005, 318	". . . highly math-anxious students earn lower grades in their math classes, suggesting strongly that they master less of the math curriculum than their low-anxious counterparts."
Ashcraft & Ridley, 2005, 322	"The basic fact-retrieval portion of keeping-track processes seemed not to be affected by math anxiety, likely because such a substantial portion of that is attributable to relatively automatic long-term memory retrieval. . . . But the procedures of doing arithmetic, including carrying, borrowing, keeping track, and applying rules, seemed likely to depend significantly on working memory processes. On this hypothesis, the disruption in high math-anxious individuals' performance is a disruption in working memory processing."

Researcher(s)	Findings/Conclusions
Ashcraft & Ridley, 2005, 322	"In his [Eysenck, 1992] formulation, anxiety disrupts ongoing cognitive processes to the degree that processing involves working memory. This is because the anxious individual devotes at least some portion of available working memory resources to the anxiety reaction, specifically to worry, intrusive thoughts, concerns over performance evaluation, and so forth. As such, a math-anxious individual's performance in a math task would be expected to deteriorate to the extent that the task arouses the anxiety, but only if the task depends on substantial working memory processing."
Ashcraft & Ridley, 2005, 322	"Ashcraft and Kirk (2001) evaluated exactly that hypothesis in a recent series of studies. Individuals were classified as to their level of math anxiety, were given computation- and language-based assessments of working memory capacity, and then were given arithmetic and counting-based tasks. The results were very straightforward. Higher math anxiety was associated with lower working memory span when the computation-based span task was administered; there was almost no relationship between math anxiety and language-based span. Participants were tested in a standard dual-task paradigm, performing two-column addition while simultaneously holding a string of random letters in working memory for later recall. When the load on working memory was heavy (six random letters), error rates rose dramatically, particularly when the addition problem required carrying. . . . In contrast, when the working memory load was light (two random letters) or when the arithmetic did not involve the procedural component of carrying, error rates were quite low and hardly different across the math anxiety groups."
Ashcraft & Ridley, 2005, 323	"The evidence is entirely consistent with the notion that an affective reaction, whether math-anxiety or induced-stereotype threat, disrupts the functioning of working memory and therefore performance on math problems that rely on working memory."
Shaley & Gross-Tsur, 2001, 339	"Mathematic anxiety may masquerade as or exacerbate dyscalculia because individuals with this problem tend to sacrifice accuracy for speed, and their performance is poor even on the most basic arithmetic exercises."

LeFevre, DeStefano, Coleman, and Shanahan (2005) agree that working memory is affected: "when working memory is required in math tasks, individuals who are high in math anxiety will show degradation of performance in the form of longer reaction times, increased errors, or both" (p. 368).

Understanding, therefore, that math phobia/anxiety can cause problems with processing because of its effect on working memory (see Chapters III-IV for discussion, is important. Otherwise, the student manifesting these symptoms might look very much like a student with mathematics disabilities. Ashcraft and Ridley (2005) conclude:

> The personal and educational consequences of math anxiety have been thoroughly investigated and are well known. The cognitive consequences have only recently come under scrutiny but seem to be lawful and predictable as well; whenever math anxiety is aroused, it will compromise performance—including learning—when working memory is necessary (p. 323).

Furner and Duffy (2002) summarize from the research they have synthesized the following recommendations for preventing mathematics anxiety:

- accommodate for different learning styles
- create a variety of testing environments
- design positive experiences in math classes
- remove the importance of ego from classroom practice
- emphasize that everyone makes mistakes in mathematics
- make math relevant
- let students have some input into their own evaluations
- allow for different social approaches to learning mathematics
- emphasize the importance of original quality thinking rather than rote manipulation of formulas
- characterize math as a human endeavor (p. 68).

In conclusion, the National Research Council (2001), reflects that

> Most U.S. children enter school eager to learn and with positive attitudes toward mathematics. It is critical that they encounter good mathematics teachers in the early grades. Otherwise, those positive attitudes may turn sour as they come to see themselves as poor learners and mathematics as nonsensical, arbitrary, and impossible to learn except by rote memorization. Such views, once adopted, can be extremely difficult to change (p. 132).

Many document the consequences of not changing negative views. The result of math phobia can be, of course, school failure. Kroesbergen (2002) draws these conclusions:

> Students with difficulties learning math obviously have a history of academic failure, which may also result in a lack of confidence with regard to their intellectual abilities and doubts about anything that might help them perform better. This situation can lead to marked passivity in the domain of math and possibly other domains of learning (p. 5).

A related consequence is from McEwan (2000), who connects lack of mathematics proficiency with the nation's dropout problem:

> We have far too many dropouts. Oh, the students I'm talking about don't actually drop out of school; they drop out of mathematics. These students decide very early in their schooling careers that they just don't get it. Maybe they missed some critical learning in an early grade (e.g., place value). Maybe they never mastered the basic facts and algorithms that enable fluent and automatic problem solving. Often, a student's mathematics difficulties are actually reading and writing deficiencies. And sometimes math has just acquired a bad reputation and is mistakenly thought to be a subject for nerds, geeks, and brains (p. 54).

One hears in this analysis an echo of Miller and Mercer (1997), but this time about students with disabilities: ". . . case studies of young adults with learning disabilities who dropped out of school have revealed that a primary reason for leaving school is the feeling that 'further academic efforts would be anxiety provoking and humiliating' . . . ." (p. 8).

Fuson, Kalchman, and Bransford (2005) speak of the same problem:

> Learning about oneself as a learner, thinker, and problem solver is an important aspect of metacognition. In the area of mathematics. . ., many people who take mathematics courses “learn” that “they are not mathematical.” This is an unintended, highly unfortunate, consequence of some approaches to teaching mathematics. It is a consequence that can influence people for a lifetime because they continue to avoid anything mathematical, which in turn ensures that their belief about being “nonmathematical” is true” (p. 236).

Math phobia, then, can create a vicious circle. Fear of not being competent in mathematics leads to anxiety, which leads to inference in learning, which leads to a lack of proficiency, which leads to school failure, which leads to low self-esteem and despair, which leads to becoming a dropout, which results in never becoming competent in mathematics.

Effects of Poor Motivation on Mathematics Achievement

Cultural and social beliefs about the value of mathematics will, of course, affect a student’s motivation to learn. Stereotype threat can convince a student that there is no use in trying to be competent in mathematics, so that impacts motivation to learn. And mathematics phobia or anxiety is so fear-laden that motivation to learn mathematics is practically non-existent. Every teacher can list, however, the names of students who are poorly motivated for reasons not linked to these phenomena. There is a long list in research literature of those reasons. In general, however, they can be understood as unmet psychological needs. Glasser (1965; 1984) categorized the four basic psychological needs as the need (1) to love and be loved; (2) for power or feelings of self-worth; (3) for freedom or choice; and (4) for joy, which includes learning. Tables 5-8 display the findings of various researchers relevant to these unmet needs and their effects on learning in general and on mathematics achievement specifically. The most frequently studied unmet need appears to be the one related to power and self-worth—or self-efficacy (Table 6), as it is sometimes termed. This area also has implications, of course, for those learners who are affected by stereotype threat and mathematics phobia.

Table 5: Need to Belong and Be Loved

Researcher(s)	Findings/Conclusions
Glasser, 1965, 7	“At all times in our lives we must have at least one person who cares about us and whom we care for ourselves. If we do not have this essential person, we will not be able to fulfill our basic needs. Although the person usually is in some direct relationship with us as a mother is to a child or a teacher is to a pupil, he need not be that close as long as we have a strong feeling of his existence and he, no matter how distant, has an equally strong feeling of our existence.”
Glasser, 1965, 9-10	“First is the need to love and be loved. In all its forms, ranging from friendship through mother love, family love, and conjugal love, this need drives us to continuous activity in search of satisfaction. From birth and old age, we need to love and be loved. Throughout our lives, our health and our happiness will depend upon ability to do so.”
Elbaum & Vaughn, 2003, 231	“. . . human beings have a strong drive to maintain significant interpersonal relationships.”

Researcher(s)	Findings/Conclusions
Shaywitz, 2003, 284	"Motivation is critical to learning and can be strengthened by adhering to a few simple principles. First, any child, and particularly one who is dyslexic, needs to know that his teacher cares about him."

Table 6: Need for Power or Self-worth

Researcher(s)	Findings/Conclusions
Glasser, 1965, 10	"Equal in importance to the need for love is the need to feel that we are worthwhile both to ourselves and to others."
Elbaum & Vaughn, 2003, 230	". . . low achievement may be one of the causes of low self-evaluations of competence. At the same time, low self-esteem itself may lead to lowered expectations."
Elbaum & Vaughn, 2003, 231	"Students who lack a positive social self-concept are vulnerable to a host of emotional, social, and learning problems."
Kroesbergen, 2002, 5	"Various aspects of motivation can be distinguished as particularly important for learning. One aspect is the role of attributions or the explanations that students provide for their successes and failures. Students with an internal locus of control tend to explain the outcomes of particular actions on the basis of their own abilities and effort. In contrast, students with an external locus of control tend to think that factors outside their control (such as luck or task difficulty) determine their results. Students with learning difficulties are more likely than normally achieving students to attribute their successes to external factors."
Levin & Long, 1981, 8	". . . students in the mastery group develop higher levels of motivation for later units in the series. Since they have experienced success in the earlier units, they are more confident in their ability to learn well and to succeed in subsequent units."
Smith, 2002, 127	"You can learn inappropriate things about yourself—or about mathematics. The most general inappropriate thing you can learn about yourself is that you can't do mathematics."
Shaywitz, 2003, 284	"Motivation is critical to learning and can be strengthened by adhering to a few simple principles. . . . Second, motivation is increased by a child's having a sense of control, such as a choice about assignments—which book he will read and what topic he will report on. Third, he needs some recognition of how hard he is working as well as tangible evidence that all his effort makes a difference; this can come in the form of improvement on a graph of his fluency rates or receiving a grade on the content of his written work rather than its form."
Wakefield, 1999, 236	"It has always baffled me that some teachers work so hard to promote self-esteem in children yet simultaneously give them inappropriate tasks to perform. Some teachers fail to recognize and acknowledge that a child's response may be 'on the way to being right.' Children develop self-confidence from experiencing success. Teacher praise, sticker awards, and 'all about me' theme projects will not alter the perception children have of themselves if failure dominates their day."
Henderson, 1992, 80	"If one is working with pupils with learning difficulties it is very easy to lower one's aims and objectives; and if a pupil thinks that his teacher does not expect him to achieve very much, then there is a tendency for the pupil not to achieve—the low aspirations of the teacher somehow permeate to the pupil."
Pajares, 1996, 325	"According to Bandura's (1986) social cognitive theory, students' beliefs about their capabilities to successfully perform academic tasks, or self-efficacy beliefs, are strong predictors of their capability to accomplish such tasks. . . . Students' self-efficacy beliefs help determine what students will do with the knowledge and skills they possess. As a consequence, academic performances are highly influenced and predicted by students' perceptions of what they believe they can accomplish."

Researcher(s)	Findings/Conclusions
Pajares, 1996, 325	"Self-efficacy beliefs act as determinants of behavior by influencing the choices that individuals make, the effort they expend, the perseverance they exert in the face of difficulties, and the thought patterns and emotional reactions they experience."
Pajares, 1996, 326	Pajares and Miller (1994) reported that self-efficacy to solve math problems was more predictive of that performance than were prior determinants such as sex or math background or than variables such as math anxiety, math self-concept, or perceived usefulness of mathematics."
Pajares, 1996, 340	"Some self-efficacy researchers have suggested that teachers should pay as much attention to students' perceptions of capability as to actual capability, for it is the perceptions that may more accurately predict students' motivation and future academic choices (see Hackett & Betz, 1989)."
Karp & Howell, Oct. 2004, 122	"The desire to provide clarity can lead us to overcompensating for students who are struggling, however, and never challenging them to take risks and grapple with the unknown."
Zeldin & Pajares, 2000, 2	"Bandura (1986, 1997) has argued that the most important source of information comes from the interpreted results of one's past performance, which he called mastery experiences. Authentic mastery of a given task can create a strong sense of efficacy to accomplish similar tasks in the future. Alternatively, repeated failure can lower efficacy perceptions, especially when such failures occur early in the course of events and cannot be attributed to lack of effort or external circumstances. Continued success, on the other hand, can create hardy efficacy beliefs that occasional failures are unlikely to undermine."
Zeldin & Pajares, 2000, 2	"Beliefs of personal competence are also influenced by the verbal persuasions one receives. Verbal messages and social encouragement help individuals to exert extra effort and maintain the persistence required to succeed, resulting in the continued development of skills and of personal efficacy."
Zeldin & Pajares, 2000, 3	"Individuals with strong self-efficacy beliefs work harder and persist longer when they encounter difficulties than those who doubt their capacities."
Pajares, 2004, 396	"People also form their self-efficacy beliefs through the vicarious experience of observing others perform tasks. This source of information is weaker than mastery experience in helping create self-efficacy beliefs, but, when people are uncertain about their own abilities, they become more sensitive to it. The effects of modeling are particularly relevant in this context, especially when the individual has little prior experience with the task."
Pajares, 2004, 397	"And, just as positive persuasions may work to encourage and empower, negative persuasions can work to defeat and weaken self-efficacy beliefs. In fact, it is usually easier to weaken self-efficacy beliefs through negative appraisals than to strengthen such beliefs through positive encouragement."
Ferguson, 1998b, 313	"My bottom line conclusion is that teachers' perceptions, expectations, and behaviors probably do help to sustain, and perhaps even to expand, the black-white test score gap. The magnitude of the effect is uncertain, but it may be quite substantial if effects accumulate from kindergarten through high school. The full story is quite complicated and parts of it currently hang by thin threads of evidence. Much remains on this research agenda."
Marzano, 1992, 27	"Learners who believe they have the inner resources to successfully complete a task attribute their success to effort; there is no task they consider absolutely beyond their reach."
Marzano, Pickering, & Pollock, 2001, 50	"Not all students realize the importance of believing in effort. . . . The implication here is that teachers should explain and exemplify the 'effort belief' to students."
Marzano, Pickering, & Pollock, 2001, 52	"A powerful way to help [students] make this connection [between effort and achievement] is to ask students to periodically keep track of their efforts and its relationship to achievement."

Researcher(s)	Findings/Conclusions
Sousa, 2001, 209	"Look for abilities, not just disabilities. Sometimes we get so concerned about the students' problems that we miss the opportunity to capitalize on their strengths. Many studies indicate that using an individual's strengths to mitigate areas of weakness often results in improved performance and a well-needed boost to that person's self-esteem."
Smey-Richman, 1988, 24-25	"Success at novel and challenging tasks is important to low achieving students . . ., but overly difficult tasks produce confusion and discouragement. According to Brophy, the degree of cognitive strain produced by tasks that allow students a 50 percent or less success rate is so great that it exceeds the tolerance level of the slow learner. In this regard, Harter has shown that students feel motivated when they experience success with what they perceive as reasonable effort, but are discouraged when they achieve success only with sustained effort."
Smey-Richman, 1988, 25	". . . the combination of high effort and failure is especially damaging, as it leads to suspicion of low ability. It is this self-realization of incompetency that triggers humiliation and shame."
Smey-Richman, 1988, 35	". . . continued success on easy tasks is ineffective in producing challenge-seeking and persistent behavior. . .; consistently easy tasks lower self-confidence."
Tileston, 2000, 5	"Jenson believes that enrichment in the classroom comes primarily from challenge and feedback. He warns that too little challenge in the classroom breeds boredom and that too much can intimidate. Challenge should be filtered so that it provides stimulating and fun experiences that match the ability level of the student without causing frustration."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 95	"Research shows that students may not realize the influence effort has on their success in school, but they can learn that effort helps them succeed. Simply teaching students that added effort pays off in terms of enhanced achievement actually increases student achievement. In fact, one study (Van Overwalle & De Metsenaere, 1990) found that students who were taught about the relationship between effort and achievement achieved more than students who were taught techniques for time management and comprehension of new material."
Rose & Meyer, 2002, 127	"We know that students learn best in their 'zone of proximal development' (Vygotsky, 1962), where challenge is just beyond their current capacity but not out of reach. Students' comfort zones—the level of difficulty, challenge, and frustration optimal for them—vary considerably. Teachers who hope to sustain students' engagement must be able to continually adjust the challenge for and among different learners."
Rose & Meyer, 2002, 127	"Adjustable levels of challenge have advantages beyond the immediate power to engage. Providing such choices for students makes the process of goal-setting explicit and provides a structured opportunity for students to practice setting realistic goals and optimal challenges for themselves."
Mercer & Mercer, 2005, 45	"The need for students to experience high levels of success has substantial research support. In this research, success refers to the rate at which the student understands and correctly completes exercises (Borich, 1992). . . . Apparently, during high success more content is covered at the learner's appropriate instructional level. . . . Borish (1992) claims that research suggests that students need to spend about 60 to 70 percent of their time on tasks that allow almost complete understanding with occasional careless errors. Instruction that promotes high success not only contributes to improved achievement but also fosters increased levels of self-esteem and positive attitudes toward academic learning and school. . . . Lack of success can lead to anxiety, frustration, inappropriate behavior, and poor motivation. In contrast, success can improve motivation, attitudes, academic progress, and classroom behavior."
Mercer & Mercer, 2005, 139	"Once students learn that successes are the result of their own efforts, they are more likely to feel in control of their learning and develop more independent learning behaviors."

Researcher(s)	Findings/Conclusions
Marzano, Pickering, & Pollock, 2001, 50	". . . this body of research demonstrates that people generally attribute success at any given task to one of four causes: • Ability • Effort • Other people • Luck Three of these four beliefs ultimately inhibit achievement. . . . Belief in effort is clearly the most useful attribution."
Marzano, Pickering, & Pollock, 2001, 51	"An interesting set of studies has shown that simply demonstrating that added effort will pay off in terms of enhanced achievement actually increases student achievement."
Jones, Wilson, & Bhojwani, 1997, 152	"Chapman concluded that students who come to doubt their abilities (a) tend to blame their academic failures on those deficits, (b) generally consider their low abilities to be unchangeable, (c) generally expect to fail in the future, and (d) give up readily when confronted with difficult tasks. Unless interrupted by successful experiences, continued failure tends to confirm low expectations of achievement, which in turn sets the occasion for additional failure."
Jones, Wilson, & Bhojwani, 1997, 152	"Specific student estimates of self-efficacy were more accurate predictors of performance than prior experience in mathematics."
Jones, Wilson, & Bhojwani, 1997, 152	". . . negative expectations and motivational problems may be reduced by interventions to eliminate deficits in specific mathematics skills."
Balfanz, McPartland, & Shaw, Apr. 2002, 18	". . . learning activities need to be structured so that students can experience success, receive positive reinforcement, and exercise some control over their learning process."
Sherman, Richardson, & Yard, 2005, 3	"Some students believe that their mathematical achievement is mainly attributable to factors beyond their control, such as luck. These students think that, if they scored well on a mathematics assignment, they did so only because the content happened to be easy. These students do not attribute their success to understanding or hard work. Their locus is external because they believe achievement is due to factors beyond their control and do not acknowledge that diligence and a positive attitude play a significant role in accomplishment. Students might also believe that failure is related to either the lack of innate mathematical ability or level of intelligence. They view their achievement as accidental and poor progress as inevitable. In doing so, they limit their capacity to study and move ahead (Beck, 2000; Phillips & Gully, 1997).
National Research Council, 2001, 131	"Productive disposition refers to the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics."
Vaughn, Gersten, & Chard, 2000, 8	"Critical variables that influence intervention effectiveness are the use of strategies used to enhance task persistence and the moderation of task difficulty. . . . Controlling for task difficulty to ensure that students experience success and persist in learning activities has long been recognized as a critical feature of effective instruction for students with LD (Gersten, Carnine, & White, 1984). Furthermore, while academic engagement (Anderson, Evertson, & Brophy, 1979; Greenwood, 1999) has been established as an essential factor linked to enhanced academic outcomes, time on task and persistence with tasks is affected by students' motivation to learn. Students' working on tasks that are challenging and meaningful but not beyond their reach influence all of these. Students who experience some successes in school are much more likely to participate actively in educational or work experiences following school (Blackorby & Wagner, 1996). Conscious attention to task difficulty is likely to be linked to higher levels of student achievement."
Vaughn, Gersten, & Chard, 2000, 9	". . . a recent synthesis examining the effects of intervention research on the self-concept of students with LD indicates at the elementary level that academic interventions are the most effective means to improved self-concept (Elbaum & Vaughn, 1999)."

Researcher(s)	Findings/Conclusions
National Research Council, 2001, 339	"Students' motivation depends on both expectation and value. That is, students are motivated to perform the task successfully if they apply themselves and the degree to which they value the task or the rewards that performing it successfully will bring. Therefore, teachers can motivate students to strive for mathematical proficiency both by supporting their expectations for achieving success through a reasonable investment of effort and by helping them appreciate the value of what they are learning."
Bruer, 1993, 258	"If we want more students to thrive, we will have to restructure classrooms and schools to create environments where children believe that, if they try, they can learn."

Table 7: Need for Freedom or Choice

Researcher(s)	Findings/Conclusions
Glasser, 1984, 12	"What we want is the freedom to choose how we are to live our lives, to express ourselves freely, associate with whom we choose, read and write what satisfies us, and worship or not worship as we believe."
Levine, n.d.,3	"So a student can lose motivation because he doesn't like a goal, because he feels he could never get that goal, or because the goal would be much too hard to get. You can see how a student with learning disorders might lose motivation when it comes to getting a good report card."
Shaywitz, 2003, 284	"Motivation is critical to learning and can be strengthened by adhering to a few simple principles. First, any child, and particularly one who is dyslexic, needs to know that his teacher cares about him. Second, motivation is increased by a child's having a sense of control, such as a choice about assignments—which book he will read and what topic he will report on. Third, he needs some recognition of how hard he is working as well as tangible evidence that all his effort makes a difference; this can come in the form of improvement on a graph of his fluency rates or receiving a grade on the content of his written work rather than its form."
Marzano, 1992, 25	"Current research and theory on motivation . . . indicate that learners are most motivated when they believe the tasks they're involved in are relevant to their personal goals."

Table 8: Need for Joy and Learning

Researcher(s)	Findings/Conclusions
Glasser, 1984, 13-15	"While most of us do not feel as driven by fun as we are by power, freedom, or belonging, I believe it is as much a basic need as any other. . . . I believe that fun is a basic genetic instruction for all higher animals because it is the way they learn. . . . When we are both learning and having fun, we often look forward to hard work and long hours; without fun, these become drudgery."
Smith, 2002, 126	"The most general inappropriate things you can learn about mathematics are that it is boring, alien, bewildering, and a cause of anxiety. . . ."
Rose & Meyer, 2002, 35	"Understanding affective issues can help teachers support all learners more appropriately. Of the three learning networks, affective networks are perhaps intuitively the most essential for learning, yet they are given the least formal emphasis in the curriculum. All teachers know how important it is to engage students in the learning process, to help them to love learning, to enjoy challenges, to connect with subject matter, and to persist when things get tough. When students withdraw their effort and engagement, it is tempting to consider this a problem outside the core enterprise of teaching. We believe this is a mistake. Attending to affective issues when considering students' needs is an integral component of instruction, and it can increase teaching effectiveness significantly."

Researcher(s)	Findings/Conclusions
Kujala, Karma, Ceponiene, Belitz, Turkkila, Tervaniemi, & Naatanen, 2001, 7	"As previous studies have shown, attention and motivation are important facts in causing plastic neural changes in the brain."
Erlauer, 2003, 73	"Success breeds success. If we can make a student feel successful in learning and satisfied with life within the classroom and school, he or she will be motivated to continue striving to achieve. Part of making students feel successful is meeting their personal learning needs. When students find school and learning interesting, they want to learn. Making lessons interesting and the content and skills being taught meaningful and relevant to the students is one way of meeting students' needs. Another way to meet the needs of students is through recognizing their individual abilities and learning styles and implementing practices related to those individual differences."
Center for Development and Learning, 2005, 1	"Using diverse instructional strategies and tactics for diverse learners connects with, engages, and motivates students. That may sound simple, but the underlying knowledge a teacher must have to pull that off day after day, hour after hour—assessing his/her students and adjusting strategies and tactics moment by moment—requires a sophisticated skill level."
Providing Appropriate Levels of Challenge, 2000, 1	"The right level of challenge is always a moving target. As skill improves, the next challenge tests new mastery to just the right extent. The same kind of incremental, responsive challenge can foster engagement in the classroom. Without new challenges, students become bored; impossible challenges frustrate and dishearten them. The right level of challenge at the right time can 'pull in' students the way video games do, building mastery a step at a time."
Csikesentmihalyi, 1991, 49	"When people reflect on how it feels when their experience is most positive, they mention at least one, and often all of the following: First, the experience usually occurs when we confront tasks we have a chance of completing. Second, we must be able to concentrate on what we are doing. Third and fourth, the concentration is usually possible because the task undertaken has clear goals and provides immediate feedback. Fifth, one acts with a deep but effortless involvement that removes from awareness the worries and frustrations of everyday life. Sixth, enjoyable experiences allow people to exercise a sense of control over their actions. Seventh, concern for the self disappears, yet paradoxically the sense of self emerges stronger after the flow experience is over. Finally, the sense of the duration of time is altered; hours pass by in minutes, and minutes can stretch out to seem like hours. The combination of all these elements causes a sense of deep enjoyment that is so rewarding people feel that expending a great deal of energy is worthwhile simply to be able to feel it."
Rose & Meyer, 2002, 125	"Affect is the fuel that students bring to the classroom, connecting them to the 'why' of learning. The work of Goleman (1995) shares the UDL *[Universal Design for Learning]* prospective that motivation is at least as important for school success as the capacity to recognize and generate patterns. Affect goes beyond simple enjoyment, and among other things, it plays a part in the development of persistence and deep interest in a subject. If we emphasize skills and knowledge to the exclusion of emotion, we may breed negative feelings toward learning, especially in students having difficulties."

Given the plethora of references to the importance of self-worth or self-efficacy, it is not surprising that there have been many studies conducted about the role of rewards and recognition as strategies to improve motivation for learning. Table 9 includes the findings from that work, much of it a synthesis of studies reviewed by Marzano, Pickering, and Pollock (2001).

Table 9: Efficacy of Rewards and Recognition in Improving Motivation

Researchers(s)	Findings/Conclusions
Marzano, Pickering, & Pollock, 2001, 55	"Rewards do not necessarily have a negative effect on intrinsic motivation."
Marzano, Pickering, & Pollock, 2001, 55-56	"Reward is most effective when it is contingent on the attainment of some standard of performance."
Marzano, Pickering, & Pollock, 2001, 57-58	". . . it appears obvious that abstract rewards—particularly praise—when given for accomplishing specific performance goals can be a powerful motivator for students."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 95	"Research . . . shows that rewards do not necessarily decrease student motivation and that reward is most effective when contingent on successfully completing a specific level of performance. We also know symbolic recognition is more powerful than tangible rewards."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 109	"When used properly, praise is highly effective. Generally, it is best to provide recognition for specific elements of an accomplishment."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 109	"Symbolic tokens, such as stickers or certificates, can be effective tools to recognize the successful completion of special learning goals. However, to keep students from losing their intrinsic motivation, teachers should avoid rewarding students for simply completing an activity."
Marzano, Pickering, & Pollock, 2001, 55	"Those who have carefully analyzed all the research on rewards, commonly came to the conclusion that they do not necessarily decrease intrinsic motivation."
Marzano, Pickering, & Pollock, 2001, 57	"Abstract symbolic recognition is more effective than tangible awards. . . . the more abstract and symbolic forms of reward are, the more powerful they are. . . . verbal reward seems to work no matter how one measures intrinsic motivation. Tangible rewards, on the other hand, do not seem to work well as motivators, regardless of how motivation is measured."
Marzano, Pickering, & Pollock, 2001, 58	". . . it is best to make this recognition as personal to the students as possible."
Marzano, Pickering, & Pollock, 2001, 59	"Reinforcing effort can help teach students one of the most valuable lessons they can learn—the harder you try, the more successful you are. In addition, providing recognition for attainment of specific goals not only enhances achievement, but it stimulates motivation."
Ontario Ministry of Education, 2005, 116	"Positive reinforcements should outweigh negative reinforcements by a ratio of four to one (Gottfredson, 1997; Lipsky, 1996). Rules should be stated in terms of what students will do, rather than what not to do. . . ."

The Center for Development and Learning (2005) summarizes the important role of schools in motivating students to learn, regardless of the cause.

> So whose job is it to motivate students? It is every teacher's job to motivate every student. Learning more about the brain and the development of the mind, studying the new information on learning, making learning meaningful and learning about learning, watching the learning process, monitoring closely for breakdowns, and applying specific strategies that directly address the breakdowns—these are teachers' challenges as they work to create classrooms that honor diversity (p. 4).

Gardner (1985) offers a broad view that includes the importance of culture, of eliminating such environmental practices as those that produce stereotype threat, of eliminating the fear that causes mathematics phobia, and of motivation in general in improving learning:

> Most contemporary psychological analyses assume an individual eager to learn; but, in fact, such factors as proper motivation, an affective state conducive to learning, a set of values that favors a particular kind of learning, and a supporting cultural context are indispensable (though often elusive) facts in the educational process (p. 373).

Effects of Language Differences on Mathematics Achievement

In addition to the effects of cultural beliefs and values, stereotype threat, mathematics phobia, and other kinds of negative motivation on mathematics achievement, teachers may encounter students with difficulties in mathematics that are due simply to the structure of their home language. Research in this area makes it clear that the cultural support for mathematics in Asia (in contrast to the apathy or even overt distaste in the United States) is not the only advantage that Asian children have in learning mathematics. One particular study's findings are cited in Table 10. What the research shows is that Chinese is far superior to English and to other western languages in its incorporation of the base-10 system in letter names and in its names for what English-speakers call "numerators" and "denominators." Consequently, Chinese children learn to count much more quickly than English-speaking children, especially in the numbers from 11 through 19; and Chinese children learn fractions much quicker than English-speaking children.

Table 10: Advantages of Asian Languages on Mathematics Achievement

Researcher(s)	Findings/Conclusions
Miller, Kelly, & Zhou, 2005, 167	"A claim that language influences acquisition of mathematical competence is strengthened if the observed differences in development between speakers of different languages (a) correspond in a sensible way to specific linguistic differences, (b) appear developmentally when children are acquiring concepts or skills related to those linguistic differences, and (c) are limited to those areas where languages differ."
Miller, Kelly, & Zhou, 2005, 167	"Our review of differences in the morphology of cardinal number names in Chinese and English leads to clear predictors about the nature, timing, and limits of differences in counting acquisition that might be expected if the structure of number names is a key source of difficulty in acquisition. Differences should (a) favor Chinese-speaking children, (b) appear when children are learning the number names termed 'teens' in English, and (c) be limited to the acquisition of number names and the base ten concept."
Miller, Kelly, & Zhou, 2005, 169	". . . these data confirm what is evident from inspection, that there are no significant differences between the percentage of Chinese and U.S. preschoolers who could count to ten, but that counting from 10 to 20 is significantly easier for Chinese children. Of the children who could count to 20, there was no significant difference in the percentage of American or Chinese preschoolers who could count to 100."
Miller, Kelly, & Zhou, 2005, 170	"The structure of number names is associated with a specific, limited difference in the course of counting acquisition between English-speaking and Chinese-speaking children. One area where there may be conceptual consequences of these linguistic differences is in children's understanding of the base-ten principle that underlies the structure of Arabic numerals. This base-ten structure is a feature of a particular representational system rather than a fundamental mathematical fact, but it is a feature that is incorporated into many of the algorithms children learn for performing arithmetic and, thus, it is a powerful concept in early mathematical development. Because English number names do not show a base-ten structure as consistently or as early as do Chinese number names, English-speaking children's conceptual understanding of this base-ten structure may be delayed compared to their Chinese-speaking peers."

Researchers(s)	Findings/Conclusions
Miller, Kelly, & Zhou, 2005, 170	"Fuson and her colleagues report success with explicitly tutoring low-SES urban first graders about the base ten structure of numbers, with the result that their end-of-year arithmetic performance approximated that reported of East Asian children."
Miller, Kelly, & Zhou, 2005, 171	"Towse and Saxton concluded that the difficulty English-speaking children have may be largely limited to difficulty understanding the irregular 'teens' of English and may also in part be due to social factors. . . ."
Miller, Kelly, & Zhou, 2005, 171	"Chinese forms ordinal numbers by adding an ordinal prefix to the cardinal number name, in contrast to the more complex English ordinals (first, second, third, etc.) that often bear no clear relation to the corresponding cardinal number name. Because of this complexity, it is possible to study in older English-speaking children some of the counting acquisition processes previously described for preschoolers."
Miller, Kelly, & Zhou, 2005, 171	"The linguistic representation of fractions in Chinese differs from that used in English in three ways. . . . The difficulty English-speaking children have with ordinal names has been described, so the use of ordinal number names in generating the names for rational numbers may be a stumbling block for English speakers. Finally, the names for the components of a fraction may also be more transparent than those in English; the words for numerator and denominator correspond to 'fraction child' and 'fraction mother,' respectively."
Miller, Kelly, & Zhou, 2005, 172	"Simple, consistent, and transparent language may make concepts more accessible than they would otherwise be, but some concepts will certainly remain difficult for children to grasp. In our lab, we have been videotaping classes on rational number in China and the United States at grades 4 and 5. We have been struck by how much time U.S. teachers need to spend on teaching children the terms 'numerator' and 'denominator,' often coming up with non-mathematical mneumonics such as 'Notre Dame' to help children remember these terms. Chinese teachers do not face these difficulties, but rational number remains a difficult and somewhat counterintuitive concept, nonetheless."
Miller, Kelly, & Zhou, 2005, 173	"The linguistic representation of mathematical concepts in particular languages can present stumbling blocks for children, but ones that can be overcome with instruction aimed at making clear what language obscures. Cross-linguistic research of the kind described here is useful in distinguishing the problems in mathematical development that reflect features of particular languages from those that stem from more general limitations of children's cognitive development."
Wang & Lin, 2005, 8	". . . the research in this area points to several possible advantages of Chinese language for mathematics performance. For instance, the fact that the Chinese number naming is consistent with a base-ten numbering system may help students do well on tasks relevant to base-ten values, such as counting skills and place-value competence. The clarity of the Chinese language in representing mathematics concepts may also contribute to better conceptual understanding, and there may be a close connection between Chinese writing and spatial abilities. These findings seem to confirm the weaker form of the Sapir-Whorf hypothesis that language and culture can influence each other mutually (Sapir, 1949; Whorf, 1956)."

The National Research Council's 2001 report on mathematics teaching and learning reflected an awareness of these research findings. Their conclusions follow:

> Several studies comparing English- and Chinese-speaking children demonstrate that the organization of number names does indeed play a significant role in mediating children's mastery of the symbolic system. These studies have reported that (a) differences in performance on counting-related tasks do not emerge until children in both the United States and China begin learning the second decade of number names, sometime between 3 and 4 years of age; (b) those differences are generally limited to the verbal aspects of

> counting, rather than affecting children's ability to use counting in problem solving or their understanding of basic counting principles; and (c) differences in the patterns of mistakes that children make in learning to count reflect the structure of the systems they are learning (pp. 166-167).

They add:

> Speakers of language whose number names are patterned after Chinese (including Korean and Japanese) are better able than speakers of English and other European languages to represent numbers using base-10 blocks and to perform other place-value tasks. Because school algorithms are largely structured around place value, the finding of a relationship between the complexity of number names and the ease with which children learn to count has important educational implications (p. 167).

Again, this kind of information is extremely valuable for educators. Otherwise, it would be difficult to know how to make clear "what language obscures," (p. 173) as Miller, Kelly, and Zhou (2005) advise. Further, as they state, these kinds of studies make it possible, again, to distinguish between students who have difficulties due to language features from those who may be cognitively limited due to learning disabilities.

English-language learners face an even more diverse set of difficulties in learning mathematics than do many native speakers. As Short and Echevarria (Dec. 2004/Jan. 2005) state, "We do English language learners a disservice if we think of them as one-dimensional on the basis of their limited English proficiency" (p. 9). These children come in increasing numbers from all over the world, so, first of all, there are linguistic differences between English and the multitude of languages spoken by children in American schools. The depth of research on these differences and their effects on mathematics learning has not been done to the extent it has for Chinese and its related languages. Also, many English-language learners come to schools in the United States without any prior schooling and with little or no informal home education leading to readiness to learn mathematics. They are, where bilingual education is not offered, therefore, confronted with the onerous tasks of learning mathematics and learning mathematics in English at the same time. Indeed, as Ortiz (2001) points out, ". . . students with limited English may fail because they do not have access to effective bilingual or English as a second language (ESL) instruction" (p. 1). A recent article in *Education Week* (Cavanagh, 2005) describes the problem:

> Malinda Evans spends about an hour and a half each day teaching mathematics to her 5th graders at Navajo Elementary School in the working-class South Valley neighborhood of Albuquerque, N. M. Whether the topic is basic division, geometry, or word problems, it is invariably also a lesson in the English language, which vexes many of her pupils more than any single equation ever could (p. 1).

Cavanagh goes on to point out that "While math has long been regarded a universal language because of its foundation in numbers, the subject poses nearly as many hurdles for students with limited English as lessons that rely more heavily on reading. . . ." (p. 22). She continues, as follows:

> Whether students' first language is Spanish or another, they face several challenges in math. The academic language of the subject presents terms that almost never come up in everyday conversation, such as "quotient" and "exponent." It also presents them with words that have double meanings, like "table," and idiosyncratic English expressions, such as questions asking for the "difference" between two numbers. Many students mistakenly take that as a cue to describe numbers' different characteristics, rather than a call to perform subtraction (p. 22).

The end of the article quotes again the teacher, Ms. Evans:

> Students who began their formal math studies in another country may find that familiar symbols, expressions, and methods differ from those they encounter in U. S. classrooms. Those barriers become more pronounced as students delve into word and story problems that can be worded a thousand different ways. . . . (p. 22).

Raborn (1995) would concur:

> Linguistic factors must be considered during planning and instruction of mathematics. Math vocabulary is precise but not always familiar. It may be difficult, even for students who are not bilingual, to determine which meaning of "odd" is intended in a problem (odd as in something peculiar or odd as in numbers that are not divisible by two). Special problems may exist for students with learning disabilities who are concurrently learning the English language. Cuevas and Beech (1983) noted the importance of considering issues of language comprehension, knowledge of syntax and vocabulary, and understanding of relational terms as they apply in mathematics. Students may experience difficulty distinguishing differences and making comparisons in relationships that pertain to size, speed, space, and time. Students with learning difficulties from language-minority backgrounds are likely to encounter difficulty with language concepts and structures even in their native language (p. 2).

In Table 11, the research on some of the difficulties experienced by ELL's in learning mathematics is presented.

Table 11: ELL's Difficulties in Learning Mathematics in English

Researcher(s)	Findings/Conclusions
Marzano, 1998, 15	". . . the fact that language shapes perceptions to some extent is widely accepted among psychologists."
Marzano, 1998, 17	". . . learning new words is tantamount to learning new distinctions within a society, which is tantamount to learning new abstract concepts important to a society. This might explain why word knowledge has been shown to be highly correlated with achievement, aptitude, and intelligence (see Nagy, 1988; Nagy and Anderson, 1984; Sternberg, 1987; Sternberg and Powell, 1983; Marzano and Marzano, 1988)."

Researcher(s)	Findings/Conclusions
McEwan, 2000, 33	"Durkin and Shire (1991) describe just one area in which the lack of coordination and articulation can cause poor performance and student anxiety in mathematics—lexical ambiguity (72). Mathematics has more than its share of ambiguous words. For students whose language development and reading skills are below grade level, words that are spelled the same but have different meanings (leaves as outgrowths of trees or leaves as used in the process of subtraction) and words that have different but related meanings (product as something that is made and product as a quantity obtained by multiplication) pose serious problems."
McEwan, 2000, 33	"When the teacher begins to attach symbols to different words in different contexts, confusion reigns supreme. Consider the different words used for the equal sign (=): equals, means, makes, leaves, the same as, gives, and results in. And any one of these words in itself has multiple meanings."
Smith, 2002, 134	"Children . . . may have particular trouble with the equals (=) sign. They think it is an instruction to do something."
Gersten & Baker, 2006, 103	"The connection between language development and acquisition of academic content and strategies for reading and problem solving is fundamental to virtually all instructional research for this population."
Gersten & Baker, 2006, 103	"It is still common for teachers to make the erroneous assumption that possessing a command of conversational English means a child can follow abstract discussions of concepts. . . ."

Sousa (2001) points out similar issues and how they are magnified in students who also have disabilities:

> Although the language of mathematics is precise, it is not always translated by ESL students. Those who also have learning disabilities already have problems understanding mathematical concepts in their native language. When faced with mathematical statements in English, these difficulties are compounded. The students have to cope with applying the rules of vocabulary, syntax, and grammar to both the English language and to mathematics. Consequently, they may have problems distinguishing in mathematical relationships, such as size, time, speed, and space (p. 146).

In addition to the problems of language, English-language learners with prior schooling in their native countries are frequently confused by the algorithms taught by American teachers, which may involve different steps from those they previously learned. Raborn (1995) makes the following observation:

> Another difficulty that may arise with language-minority students is that of differences in algorithms. An algorithm is the procedure used for finding the solution to a mathematical problem. Most Americans learn to calculate using set algorithms taught in the schools. Students from South America or Asian countries learn algorithms that are different in sequence. The position of numbers on the written page often does not match the algorithms typically used in American schools. Therefore, when students are asked to calculate, a difference in the algorithm may be misinterpreted as lack of math ability. For language-minority students, it is more important to determine if the student knows how to obtain the correct answer and if they can explain the procedure, rather than how well the student's algorithms match those used in our schools (p. 2).

Later, Sousa (2001) draws a strikingly similar conclusion, obviously drawing from the same research:

> Differences in algorithms—the procedures used to find the solution to a mathematical problem—also poses difficulties for ESL students. Asian and South American students, for example, learn algorithms that are different in sequence from those taught in American schools. The algorithms that an ESL student uses to make calculations may be misinterpreted as a mathematics disorder. Rather than being concerned about differing algorithms, teachers should determine whether the student can explain the procedure and arrive at the correct answer (p. 147).

Interestingly, it is frequently the bilingual teachers, many of whom are themselves immigrants, who point out this source of difficulty to native-born teachers. They recognize that the algorithms that they learned in their native country differ in many ways from those traditionally taught in the United States, so they are quicker to recognize the source of confusion that occurs when an English-language learner resorts to his or her prior learning to solve a problem.

It is easy to gloss over the complexity of teaching English-language learners well. Educators may see the job as chiefly one of teaching them English. ELLs come to school, however, with the same diversity as native-born Americans. It is important to remember that all the motivational issues that may affect achievement of native-born American children may also affect English-language learners; plus they may also be victims of inadequate or inappropriate instruction.

Effects of Inadequate or Inappropriate Instruction on Mathematics Achievement

A review of the research on reading achievement will certainly lead to many references to the existence of inadequate or inappropriate instruction being the cause for most student failure in learning to read. In fact, Lyon (1996), one of the major advocates for scientifically-based evidence in education (now mandated in *NCLB*), for the Reading First program institutionalized in *NCLB*, and for the Response-to-Intervention model advocated in *IDEA*, maintains that inadequate and inappropriate instruction is the cause for massive over-identification of children for special education. The abundancy of these references in reading research, however, almost pale in comparison to those found in mathematics research. It is difficult to find a study that does not at least allude to the need for preschool education in mathematics, for better teacher preparation to teach mathematics, for better curriculum, for more effective teaching strategies, for better assessments to measure progress, and on and on. Clearly, many prominent mathematicians and mathematics educators believe that the vast majority of student failure to learn mathematics is the result of inadequate or poor instruction, especially given that most people are more dependent on the school to teach them mathematics than they are for literacy development.

Table 12 includes the findings/conclusions of researchers who identify inadequate instruction (not necessarily "inappropriate" instruction) as a major problem:

Table 12: Effects of Inadequate Instruction on Mathematics Achievement

Researcher(s)	Findings/Conclusions
Siegler, 2003, 225	"Why do some children encounter such large problems with arithmetic? One reason is limited exposure to numbers before entering school. Many children labeled 'mathematically disabled' come from poor families with little formal education. By the time children from such backgrounds enter school, they are already far behind other children in counting skill, knowledge of numerical magnitudes, and knowledge of arithmetic facts."
Butterworth, 2005, 456	". . . there are many reasons for being bad at mathematics, including inappropriate teaching, behavioral problems, anxiety, and missing lessons."
Garnett, 1992, 2	"Clearly, some difficulties in math learning constitute a substantial disability stemming from factors within the learner. On the other hand, large numbers of children in American elementary schools do poorly in mathematics as a consequence of inadequate math teaching (Dossey, Mullis, Lindquist, & Chambers, 1988). The poor math achievement of American students in general has been attributed largely to classroom factors. These include: too little time spent on arithmetic, insufficient interaction during math practice, and inadequate connecting of concepts with language, with written symbols, and with practical applications. Thus, although the poor math performance of many students with learning disabilities may indeed reflect intrinsic weaknesses, these weaknesses are likely to be seriously exacerbated by poor math instruction (Cawley & Miller, 1989)."
Cawelti, 1995, 102	"International comparisons at the elementary school level show that American students in general receive much less instruction in mathematics than do their age cohorts in other countries."
Dowker, 2004, 16	"There is much research that indicates that the school environment and teaching methods are important influences on the mathematical performance of children throughout the ability range. Appropriate teaching may prevent some mathematical difficulties from ever becoming apparent; and many mathematical difficulties are undoubtedly mainly the result of limited or inappropriate teaching (or, worldwide, to a complete or near-complete lack of schooling)."

Tables 13 and 14 include the findings of researchers on the effects of inappropriate curriculum and curriculum materials on student learning in mathematics:

Table 13: Effects of Inappropriate Curriculum on Mathematics Achievement

Researcher(s)	Findings/Conclusions
Miller & Mercer, 1997, 7	"Another factor that undoubtedly contributes to poor math performance among students with disabilities is poor curricula and instruction."
Miller & Mercer, 1997, 9	"Forcing all students to follow one designated curriculum is a vivid example of fitting students to the curriculum rather than fitting the curriculum to students. Plainly stated, such an approach violates the basic principles of special education. Students with disabilities often need 'things' that differ from what schools typically provide."
Battista, 1999, 431	"One of the major consequences of the blatant disregard of modern scientific research on mathematics learning is the almost universal belief in what I call the 'myth of coverage.' According to this myth, 'If mathematics is covered, students will learn it.'"
Caine & Caine, 1991, 13	"The school based on the factory approach fails to prepare students for two reasons. First, the relevant skills and attributes students need for this century and the next tend not to be addressed. Second, the organization and methods of teaching content and skills are inadequate because they fail to take advantage of the brain's capacity to learn."

Researcher(s)	Findings/Conclusions
Reys, 2001, 258	"People who demand research to document the effectiveness of reform curricula are either unaware of the history of student performance using the traditional curricula or choose to ignore more than 30 years of widely reported results. In fact, to assume that traditional mathematics programs have shown themselves to be successful is, according to James Hiebert, 'ignoring the largest database we have.' Hiebert goes on to say, 'The evidence indicates that the traditional methods in the United States are not serving our students well.'"
Mercer & Mercer, 2005, 408	"Kelly et al. conclude that a strong relationship exists between the number and types of errors and the curriculum."
Dowker, 2004, 17	"When considering the diversity among all students with and without disabilities, it is unrealistic to assume that one curriculum or one set of standards will suit the math needs of everyone."

Table 14: Effects of Inappropriate Curriculum Materials on Mathematics Achievement

Researcher(s)	Findings/Conclusions
Moss, 2005, 319	"Why does instruction so often fail to change students' whole-number conceptions? Analyses of commonly used textbooks suggest that the principles of *How People Learn* are routinely violated. First, it has been noted that—in contrast to units on whole-number learning—topics in rational number are typically covered quickly and superficially. Yet, the major conceptual shift required will take time for students to master thoroughly. Within the allotted time, too little is devoted to teaching the conceptual meaning of rational number, while procedures for manipulating rational numbers receive greater emphasis. While procedural competence is certainly important, it must be anchored by conceptual understanding. For a great many students, it is not."
Miller & Mercer, 1997, 8	"The lack of appropriate math materials for teachers to use compounds the problem of poor curricula and instruction."
Miller & Mercer, 1997, 7	"The primary concerns regarding basal programs are the lack of adequate practice and review, inadequate sequencing of problems, and an absence of strategy teaching and step-by-step procedures for teaching problem solving. . . . Research has demonstrated that the basal approach to teaching mathematics is particularly detrimental to students who have learning difficulties. . . ."
Committee on *How People Learn,* 2005, 370	"The central problem with most textbook instruction, many researchers agree, is the failure of textbooks to provide a grounding for the major conceptual shift to multiplicative reasoning that is essential to mastering rational numbers."
Moss, 2005, 319	"Textbooks typically treat the notation system as something that is obvious and transparent and can simply be given by definition at a lesson's outset. Further, operations tend to be taught in isolation and divorced from meaning. Virtually no time is spent in relating the various representations—decimals, fractions, percents—to each other."
Moss, 2005, 320	"The central problem with most textbook instruction, many researchers agree, is the failure of textbooks to provide a grounding for the major conceptual shift to multiplicative reasoning that is essential to mastering rational number."
Jones, Wilson, & Bhojwani, 1997, 153	"Two deficiencies that contribute to inefficient instruction and chronic error patterns in the management of instructional examples are common to commercial math curricula. First, the number of instructional examples and the organization of practice activities are frequently insufficient for students to achieve mastery. . . . A second deficiency is an inadequate sampling of the range of examples that define a given concept. If some instances of a concept are under represented in instruction or simply not included in instruction, students with LD will predictably fail to learn that concept adequately."

Researcher(s)	Findings/Conclusions
Fuchs & Fuchs, 2001, 85	". . . mathematics textbooks, which do a poor job of adhering to important instructional principles for students with and without disabilities (Jitendra, Salmento, & Haydt, 1999), account for approximately 75% of what occurs in mathematics instruction in general education (Porter, 1989)."

The vast majority of references to poor education being the cause of low achievement in mathematics centers on inappropriate (in contrast to "inadequate") instruction—the methods or strategies used to teach students. Table 15 includes an overview of those concerns:

Table 15: Effects of Inappropriate Instruction on Mathematics Achievement

Researcher(s)	Findings/Conclusions
Mercer & Mercer, 2005, 3	". . . the knowledge gap between what is known about effective teaching and what is routinely practiced in classrooms is enormous."
Miller & Mercer, 1997, 7	"Another factor that undoubtedly contributes to poor math performance among students with disabilities is poor curricula and instruction."
Dowker, 2004, 16	"There is much research that indicates that the school environment and teaching methods are important influences on the mathematical performance of children throughout the ability range. Appropriate teaching may prevent some mathematical difficulties from ever becoming apparent; and many mathematical difficulties are undoubtedly mainly the result of limited or inappropriate teaching (or, worldwide, to a complete or near-complete lack of schooling)."
Mercer & Mercer, 2005, 428	"Although many students with math deficiencies exhibit characteristics (such as problems in memory, language, reading, reasoning, and metacognition) that predispose them to math disabilities, their learning difficulties often are compounded by ineffective instruction. Many authorities . . . believe that poor or traditional instruction is the primary cause of the math difficulties of many students with learning problems. Numerous studies support the position that students with math disabilities can be taught to improve their mathematical performance."
Shaley & Gross-Tsur, 2001, 339	"It is important to realize that although children learn some arithmetic on their own, for the most part this skill is taught in school. Within this formal setting, inadequate teaching methods may be one of the reasons why children have trouble learning arithmetic."
Bisanz, Sherman, Rasmussen, & Ho, 2005, 144	". . . some children fail to receive appropriate instruction until long after they have entered school."
Battista, 1999, 426	"For most students, school mathematics is an endless sequence of memorizing and forgetting facts and procedures that make little sense to them. Though the same topics are taught and retaught year after year, the students do not learn them. Numerous scientific studies have shown that traditional methods of teaching mathematics not only are ineffective but also seriously stunt the growth of students' mathematical reasoning and problem-solving skills."
Butterworth, 2005, 456	". . . there are many reasons for being bad at mathematics, including inappropriate teaching, behavioral problems, anxiety, and missing lessons."
Battista, 1999, 427	"In traditional mathematics instruction, every day is the same: the teacher shows students several examples of how to solve a certain type of problem and then has them practice this method in class and in homework. The National Research Council has dubbed the 'learning' produced by such instruction as 'mindless mimicry mathematics.' Instead of understanding what they are doing, students parrot what they have seen and heard."
Battista, 1999, 430	"Because traditional instruction ignores students' personal construction of mathematical meaning, the development of their mathematical thought is not properly nurtured, resulting in stunted growth."

Researcher(s)	Findings/Conclusions
Smith, 2002, 97	"Small wonder that many people never fully understand 'operations' involved in the multiplication and division of fractions. They may learn the rituals for long enough to get through examinations, but what doesn't make sense to them is rarely retained for any length of time. What is the point of remembering something that is seemingly nonsense? "It is not that such people have learned nothing in school, but they have learned the wrong things. They have learned to expect that multiplication makes something bigger and that division makes it smaller. They have learned this from innumerable classroom explanations and demonstrations. And it all seems intuitively obvious—until they have a need or desire to think mathematically."
Bisanz, Sherman, Rasmussen, & Ho, 2005, 144	"When instructional practices do not match the cognitive skills and inclinations children bring with them to school, learning can be hampered severely. If instruction and early assessment are to be optimized for the benefit of children, they must be based on a thorough understanding of what children do and do not know about arithmetic prior to schooling."
Fuson, Kalchman, & Bransford, 2005, 242-243	". . . some suggest that students must invent all their mathematical ideas and that we should wait until they do so rather than teach ideas. This view, of course, ignores the fact that all inventions are made within a supportive culture and that providing appropriate supports can speed such inventions. Too much focus on student-invented methods per se can hold students back; those who use time-consuming methods that are not easily generalized need to be helped to move on to more rapid and generalizable 'good enough' methods. A focus on sense making and understanding of the methods that are used is the balanced focus, rather than an emphasis on whether the method was invented by the student using it."
Caine & Caine, 1991, 13	"The school based on the factory approach fails to prepare students for two reasons. First, the relevant skills and attributes students need for this century and the next tend not to be addressed. Second, the organization and methods of teaching content and skills are inadequate because they fail to take advantage of the brain's capacity to learn."
Furner & Duffy, 2002, 67	"Oberlin (1982) found that several common teaching techniques cause math anxiety, such as assigning the same work for everyone, teaching the textbook problem by problem, and insisting on only one correct way to complete a problem. Also, a student's lack of success with math may be caused by any one of several factors, such as poor math instruction, insufficient number of math courses in high school, or misinformation about what math is and what it is not."
Jones, Wilson, & Bhojwani, 1997, 152	"Individual differences in cognitive development certainly affect the achievement of academic skills. In earlier years, many professionals readily accepted that individual psychological differences accounted for failure to learn in school. Currently, a more parsimonious explanation is that many students fail as a result of ineffective instruction. . . . Students' expectations for failure frequently develop as a result of prolonged experiences with instruction that fails to result in successful performance."
Jones, Wilson, & Bhojwani, 1997, 153	"Generally, students are taught that fractions represent equal divisions of one whole. . . . This representation of the concept of a fraction is inadequate, however, because not all fractions are less than one whole unit. Some fractions are equal to or greater than 1. . . . Representing the concept of a fraction as a quantity less than one whole limits student understanding of the wider range of possible fraction concepts. An inadequate conceptualization of fractions contributes to inadequate understanding of computation with fractions and, thus, severely, limits problemsolving skills."
Jones, Wilson, & Bhojwani, 1997, 153	"To learn correct conceptualizations, students must be taught which attributes are relevant and which are irrelevant. If sets of instructional examples consistently contain attributes that are irrelevant to a concept, then students will predictably learn misconceptualizations that may seriously hinder achievement. It is not uncommon to find that presentations of misleading variables have inhibited mathematics achievement."

Researcher(s)	Findings/Conclusions
Kroesbergen, 2002, 2.1	"Another important cause of such difficulties (difficulties in learning math) may be a poor fit between the learning characteristics of individual students and the instruction they receive (Carnine, 1997). In the case of a poor fit, the instruction must be adapted to the students' needs. In other words, all students with mathematics difficulties require special attention (Geary, 1994). These students have special educational needs, need extra help, and typically require some type of specific mathematics intervention. . . ."
D'Arcangelo, 2002, 84	Interview with Brian Butterworth: "Not being good at mathematics may have two main causes. The first is genetic. A minority of people may be born with a condition that makes it difficult for them to learn mathematics; that is, they are born with dyscalculia or are born with dyslexia, which also can have a consequence for mathematics learning. A far more likely cause is that they were taught badly. That means taught in a way that left them failing to understand what they were doing. Thus, everything else that they learned that was based upon what they didn't understand was going to be very fragile. So, they avoided mathematics."
Stigler & Hiebert, 1999, 10	"What we see clearly is that American mathematics teaching is extremely limited, focused for the most part on a very narrow band of procedural skills. Whether students are in rows working individually or sitting in groups, whether they have access to the latest technology or are working only with paper and pencil, they spend most of their time acquiring isolated skills through repeated practice. Japanese teaching is distinguished not so much by the competence of the teachers as by the images it provides of what it can look like to teach mathematics in a deeper way, teaching for conceptual understanding."
Paulos, 1988, 72-73	"Why is innumeracy so widespread even among otherwise educated people? The reasons, to be a little simplistic, are poor education, psychological blocks, and romantic misconceptions about the nature of mathematics. . . early mathematics education is generally poor. Elementary schools by and large do manage to teach the basic algorithms for multiplication and division, addition and subtraction, as well as methods for handling fractions, decimals, and percentages. Unfortunately, they don't do as effective a job in teaching when to add or subtract, when to multiply or divide, or how to convert from fractions to decimals or percentages. Seldom are arithmetic problems integrated into other schoolwork—how much, how far, how old, how many. Older students fear word problems in part because they have not been asked to find solutions to such quantitative questions at the elementary level."
Sousa, 2001, 140	"One critical factor in how well students learn mathematics is the quality of the teaching. Recent studies show that student achievement in mathematics is strongly linked to the teacher's expertise in mathematics."
Ma, 1999, 36	"Limited subject matter knowledge restricts a teacher's capacity to promote conceptual learning among students. Even a strong belief of 'teaching mathematics for understanding' cannot remedy or supplement a teacher's disadvantage in subject matter knowledge. A few beginning teachers in the procedurally directed group wanted to 'teach for understanding.' They intended to involve students in the learning process, and to promote conceptual learning that explained the rationale underlying the procedure. However, because of their own deficiency in subject matter knowledge, their conception of teaching could not be realized."
Ma, 1999, 64	"Although 43% of the U.S. teachers successfully calculated 1 ¾ divided by ½, almost all failed to come up with a representation of division by fractions. Among the 23 teachers, 6 could not create a story and 16 made up stories with misconceptions. Only one teacher provided a conceptually correct but pedagogically problematic representation. The teachers displayed various misconceptions about the meaning of division by fractions."

Researcher(s)	Findings/Conclusions
Jerald, 2006, 3	"After all, NCLB places a strong emphasis on mathematics for some very good reasons. Students who fall behind educationally seldom catch up later on, and this is especially true in math, a 'cumulative' subject area that builds new knowledge upon foundational skills mastered previously. A weak math foundation can have profoundly negative consequences for young people later in life, including the following: • Many American teenagers struggle with algebra when they reach high school, and researchers have found that failing ninth-grade algebra is a strong predictor of dropping out. • Multiple large-scale federal studies have revealed taking and passing high school math courses beyond algebra II has a strong impact on whether students complete college, regardless of family background. • Whether they graduate from college or not, young people with low math skills now struggle to find decent jobs in an economy where skill demands have increased dramatically during the last 20 years—even in so-called 'blue collar' jobs that require little or no postsecondary education."

The Math Wars

Even though many mathematicians and mathematics educators agree that inappropriate curriculum and instruction result in low achievement among American students, there is not a consensus on what constitutes inappropriate curriculum or instruction. The ensuing debate is referred to as "the math wars," and they continue, unfortunately, to rage.

Table 16: Math Wars

Researcher(s)	Findings/Conclusions
Martin, n.d.,1	"Who would have ever thought that mathematics education would be the subject of heated editorials in the newspaper, of morning talk show head-to-heads, and of political intrigue at all levels?"
Goya, 2006, 370	"When you think about it, the math wars are pointless. Debates about this reform or that reform always miss the mark."
Bass, 2005, 417	"There has been much attention to the so-called 'math wars,' an unfortunate term coined in the U.S. to describe the conflicts between mathematicians and educators over the content, goals, and pedagogy of the curriculum."
Stumbo & Lusi, 2005, 2	"The coming together of divergent paths has been a critical step forward in the improvement of mathematics education."
Reys, 2001, 258	"All interested parties should stop trying to defend the past and work together to improve children's mathematics education for the future."
Martin, n.d.,7-8	"On the final examination for a graduate course I was teaching last semester, I asked students to 'briefly discuss ideas for how the 'math wars' could be brought to a peaceful end.' One student wrote, 'The wars shouldn't end. The two sides need each other.' I think there is at least some truth in that response, although one might wish for some changes in the rules of engagement. We need to learn from the critics, rather than treating them only as adversaries to be defeated."

The mathematics curriculum wars whirl around the issue of whether to emphasize concept development, procedures (algorithms), fact fluency, or problem solving in the curriculum, but there is hope in at least détente since much recent research verifies that all are necessary in a balanced mathematics curriculum. Table 17 includes the research and analysis relating to the curriculum debate in the math wars.

Table 17: Math Wars—Curriculum Issues

Researcher(s)	Findings/Conclusions
Klein, Askey, Milgram, Wu, et al., 1999, 1	"In early October of 1999, the United States Department of Education endorsed ten K-12 mathematics programs by describing them as 'exemplary' or 'promising.' . . . It is not likely that the mainstream views of practicing mathematicians and scientists were shared by those who designed the criteria for selection of 'exemplary' and 'promising' mathematics curricula. . . . Even before the endorsements by the Department of Education were announced, mathematicians and scientists from leading universities had already expressed opposition to several of the programs listed and had pointed out serious mathematical shortcomings in them. . . . we believe that it is premature for the United States government to recommend these ten mathematics programs to schools throughout the nation. We respectfully urge you to withdraw the entire list of 'exemplary' and 'promising' mathematics curricula, for further consideration, and to announce the withdrawal to the public."
Miller & Mercer, 1997, 9	". . . if a particular instructional approach results in student success, it should be valued regardless of its paradigm affiliation."
RAND, 2002, 3	"Further complicating the process of improving school mathematics are disputes about what content should be taught and how it should be taught. Some argue that mathematics be taught primarily by teachers giving clear, organized expositions of concepts and procedures and then giving students opportunities to practice and apply. Others contend that teachers should design ways to engage students firsthand in exploring the meaning of mathematical procedures, rather than simply showing them how to carry them out. Some want students to memorize procedures and develop skill so that understanding can follow; others want to put understanding first and foremost, contending that in the computer age a heavy emphasis on procedural skill is no longer relevant. Arguments also rage over the nature of school mathematics: Should it be mostly abstract and formal or mostly concrete and practical? With these basic issues in play, battles have been waged over curriculum materials. The intense debates that filled the past decade have often impeded much-needed collective work on improvement. Moreover, they have been based more often on ideology than on evidence."
Stumbo & Lusi, 2005, 2	"Today, most mathematics educators agree that a balanced approach to teaching mathematics that honors the need for students to attain proficiency in performing both simple and complex computational skills, as well as knowing how and when to apply those skills when presented with a problem, is called for."
Siegler, 2003, 226	"Throughout this century, instructional reform has oscillated between emphasizing mastering of facts and procedures on the one hand and emphasizing understanding of concepts on the other (Hiebert & LeFevre, 1986). Few today would argue that either type of mathematical knowledge should be taught to the exclusion of the other. Much less agreement exists, however, concerning the balance between the two that should be pursued or concerning how to design instruction that will inculcate both types of knowledge."
RAND, 2002, xii	"Complicating the process of improving school mathematics are disputes about what content should be taught and how it should be taught. Arguments rage over curriculum materials, instructional approaches, and what aspects of the content to emphasize. Should students be taught the conventional computational algorithms or is there merit in exploring alternative procedures? Should calculators be used in instruction? What degree of fluency is necessary and how much depth of conceptual understanding? What is the most appropriate view of algebra? These questions unhelpfully dichotomize important instructional issues. The intense debates that filled the past decade, often based more on ideology than on evidence, have hindered improvement."

Researcher(s)	Findings/Conclusions
RAND, 2002, 8	"These strands of proficiency [conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition] are interconnected and coordinated in skilled mathematical reasoning and problem solving. Pitting one against another—e.g., conceptual understanding versus procedural fluency—misconstrues the nature of mathematical proficiency. Because the five strands are interdependent, the question is not which ones are critical, but rather when and how they are interactively engaged. The core issue is one of balance and completeness, which suggests that school mathematics requires approaches that address all of the strands. Mathematical proficiency is more complex than extreme or simplistic positions allow."
Bohan, 2002, 36	"The issue of computation continues to be problematic. The 1989 NCTM *Curriculum and Evaluation Standards for School Mathematics* stated that there should be 'diminished' emphasis on rote memorization of facts and pencil-and-paper computation, but many interpreted this as a call for 'elimination' of these activities. It was not the intention of the 1989 NCTM *Standards* that these critical components of mathematics competency be eliminated; rather it was suggested that they be taught in greater balance with conceptual and application-oriented activities."
Geary, 2003a, 456	"These arguments are not to say that students should be taught procedures without understanding the associated concepts; students should be taught effective computational procedures—and practice them to the point of automaticity—and should know the associated concepts (see Geary, 1995, 2001). It is now clear that the learning of conceptual and procedural skills are interrelated. A solid conceptual understanding of the domain (e.g., base-ten system) is important for avoiding and correcting procedural errors (e.g., Sophian, 1997), and the practice of procedures provides a context for children to learn associated concepts and problem-solving strategies (Siegler & Stern, 1998). Nor does this mean that constructivist techniques cannot sometimes be useful in mathematics education."
Wu, n.d., 3	"The overriding characteristic of the traditional curriculum is its emphasis on learning algorithms by rote: mathematics becomes a set of algorithms to be memorized and regurgitated at exam time. . . . Anyone who teaches freshman calculus regularly knows only too well the ill effects of this kind of mathematics education."
Wu, n.d., 4	"From a mathematical point of view, the main problem with the traditional curriculum is that it deals with the *how* of mathematics, but not with *why*. The basic questions of why something is true and why something is important are allowed to remain unanswered. What we need is a curriculum that provides answers to these questions."
Wu, n.d., 6	"The traditional curriculum is driven by algorithms-without-explanations. By oversimplifying mathematics in this fashion, this curriculum acquires several virtues: it has built-in precision; it brings computational skills to the forefront; it sets a clear goal for students—always strive to produce a correct answer; and finally, it lets teachers know unambiguously what to teach. Its weaknesses are that, especially in unskilled hands, it can easily degenerate into mindless number crunching and symbol-pushing, so that students end up not learning even the computational skills. These weaknesses are correctable: supply the motivation and reasoning lying behind the algorithms, and replace some of the routine drills with exercises that make a greater demand on students' conceptual understanding."
National Research Council, 1997, 127	"The assumption that mastery of basic skills is not a prerequisite for advanced learning appears tenuous for many students with cognitive disabilities."
Stotsky, S. (2005), 2-3	"Much understanding of math comes from mastery of basic skills, an idea backed by most professors of mathematics. The idea of having to make a choice between conceptual understanding and skills is essentially false, a bogus dichotomy. That students will only remember what they have extensively practiced and that they will only remember for the long term that which they have practiced in a sustained way over many years are realities that can't be bypassed."

Researcher(s)	Findings/Conclusions
Berliner & Casanova, 1993, 16	"We need to remind ourselves that knowing how to learn and memorize are extremely important skills, and that not all children come to school with them."
Cavanagh, Feb. 15, 2006, 2	"Mathematicians, scholars, and teachers have long argued over what approaches to math instruction work best in the classroom. At issue, in broad terms, is whether teachers should focus more on building students' conceptual understanding of the subject, or emphasize nurturing their mastery of basic math skills."
Ocken, 2001, 1	". . . prominent educators. . . have proposed that the standard algorithms be banished from American classrooms because they undermine children's understanding, performance, and even emotional well-being. None of these effects has ever been documented by a properly controlled experimental study using a statistically valid sample population."
Ocken, 2001, 4	". . . it is naïve and counterproductive to suggest that calculators, or even symbolic algebra software, eliminate or reduce students' need for formal algebraic skills. . . . In my own experience with technology-based courses, it is precisely those students with the weakest symbolic and algebraic skills who have the most trouble correctly entering formulas and expressions into their calculator or computer."
Bruer, 1993, 99	"The debate on how to improve elementary math instruction often pits those who argue for more number facts and computational fluency (primacy of procedural knowledge) against those who argue for more knowledge about the number system (primacy of conceptual knowledge). The NAEP results, and the data on Resnick's subject . . ., show that computational skill doesn't ensure conceptual understanding. However, conceptually oriented instruction, although it recognizes the importance of relating underlying concepts to computational rules, sometimes leaves too much to children's inventive minds. Resnick's subjects . . . demonstrate that we can't assume, as some conceptual approaches do, that with conceptual understanding children can invent or master the computational procedures. The power of a technique such as mapping instruction is that it explicitly combines the two kinds of knowledge that children need to build their mathematical expertise. "Most school math does not do this. Instruction either doesn't teach the underlying representations (such as the mental number line and the part-whole schema) or doesn't make the link between concept and procedure explicit. Some children make the connection on their own, but many do not. Without the link, mathematics is meaningless. "All children would benefit from understanding how number concepts support and give meaning to procedural skills. Some educators and critics may find this a painfully obvious conclusion. If so, it is an obvious conclusion that many schools apparently find hard to implement."
Ocken, 2001, 1	"Instead of being forced into programs that de-emphasize, denigrate, or discard the traditional algorithms of arithmetic, all American children should receive balanced K-8 mathematics instruction that includes appropriate emphasis on the development of formal and algorithmic skills. Only then will they have a meaningful chance to develop their own competencies and thereby obtain access to careers that are both personally rewarding and crucial to the well-being of the larger society."
Wu, 1999, 6	"If there is any so-called harmful effect in learning the algorithms, it could only be because they are not taught properly."
National Research Council, 2001, 122	"Procedural fluency and conceptual understanding are often seen as competing for attention in school mathematics. But pitting skill against understanding creates a false dichotomy. As we noted earlier, the two are interwoven. Understanding makes learning skills easier, less susceptible to common errors, and less prone to forgetting. By the same token, a certain level of skill is required to learn many mathematical concepts with understanding, and using procedures can help strengthen and develop that understanding."

Table 18 includes the research on the math wars as they relate to instructional strategies—chiefly the debate on constructivism versus direct instruction.

Table 18: Math Wars—Instructional Strategies

Researcher(s)	Findings/Conclusions
National Research Council, 2001, xiv	"Mathematics is invented, and it is discovered as well. Students learn it on their own, and they learn it from others, most especially their teachers. If students are to become proficient in mathematics, teachers must create learning opportunities both constrained and open."
National Research Council, 2001, xiv	"A claim used to advocate movement in one direction is that mathematics is bound by history and culture, that students learn by creating mathematics through their own investigations of problematic situations, and that teachers should set up situations and then step aside so that students can learn. A countervailing claim is that mathematics is universal and eternal, that students learn by absorbing clearly presented ideas and remembering them, and that teachers should offer careful explanations followed by organized opportunities for students to connect, rehearse, and review what they have learned. The trouble with these claims is not that one is true and the other false; it is that both are incomplete. They fail to capture the complexity of mathematics, of learning, and of teaching."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 326	"A common misconception in education is that allowing students to discover how to perform a skill or process is always better than directly teaching the skill or process. This idea probably gained favor in reaction to the long-held misconception that drill and practice in specific steps is always the best way to teach skills. The truth about how to teach skills and processes lies somewhere between the 'discovery' and 'drill-and-practice' techniques. Students learn some skills better through discovery, but they learn other skills better through direct instruction. For example, consider the skills of addition, subtraction, multiplication, and division. Asking students to discover the steps involved in these computation processes makes little sense. Students would probably understand these skills well if they discovered the steps to addition, but this process would take an excessive amount of time."
Mercer & Mercer, 2005, 129	"Students with histories of problems in automaticity, metacognitive strategies, memory, attention, generalization, proactive learning, and motivation cannot plausibly engage in efficient self-discovery learning (i.e., implicit teaching)."
Mercer & Mercer, 2005, 130	"Howell and Nolet (2000) provide guidelines regarding student factors when selecting an instructional approach. Howell and Nolet suggest using a more implicit approach when the student has an adaptive or flexible motivational system, has significant prior knowledge of the task or concept, and encounters consistent success with the content. They recommend a more explicit approach when the student has a rigid motivational pattern, lacks significant prior knowledge, or encounters repeated failure on the task."
Mercer & Mercer, 2005, 130	". . . an explicit approach is more appropriate when the task is complex, is poorly defined, has missing information, or requires a task-specific strategy. Explicit instruction is warranted if the content is critical to subsequent learning or requires a high level of proficiency. If time is limited or if a priority on mastery exists, an explicit approach works best."
Mercer & Mercer, 2005, 131	"Examination of the setting [implicit learning] demands and prevalent characteristics of students with learning problems demonstrates that many such students lack the attributes, skills, and knowledge needed to succeed in an implicit setting. These and other students at risk for school failure require much teacher support and direction to begin moving toward becoming self-regulated lifelong learners."
Jones, Wilson, & Bhojwani, 1997, 155	"Explicitness of curriculum design refers to the unambiguous presentation of important concepts and skills and the relationships among them. . . . a highly explicit math curriculum produced greater student achievement than a less explicit curriculum."

Researcher(s)	Findings/Conclusions
Jones, Wilson, & Bhojwani, 1997, 155	". . . if hierarchical sequences are not developed around explicit instructional priorities, it is unlikely that students with learning difficulties will progress efficiently."
Jones, Wilson, & Bhojwani, 1997, 158	". . . in constructivist approaches, students often persevere through trial-and-error learning. Under such learning conditions, students with LD are apt to make many more errors than their more capable peers. If they expect to fail, they are prone to give up or to withdraw from instruction."
Kroesbergen, 2002, 3.1	"Research suggests that instruction based on constructivist principles leads to better results than more direct, traditional mathematics education (Cobb et al., 1991; Gravemeijer et al., 1993; Klein, 1998). And many researchers have observed that learning in such a manner is more motivating, exciting, and challenging (Ginsburg-Block & Fantuzzo, 1998). Students who learn to apply active learning strategies are also expected to acquire more useful and transferable knowledge because, for example, problem solving requires active participation on the part of the learner (Gabrys, Weiner, & Lesgold, 1993). "The question that remains is whether constructivist approaches and RME (realistic mathematics education or discovery learning] are also beneficial for *low* performing students (Klein et al., 1998). Van Zoelen, Houtveen, and Booij (1997) conclude that although the average and good students profit from RME, the weaker students appear to benefit much less from this method. Woodward and Baxter (1997) also state that special educators have raised objections to the instructional methods and materials put forth in the NCTM standards because they are too discovery-oriented, and not very sensitive to teaching students with math difficulties. In research, they also show this kind of instruction to benefit most students but those with learning difficulties and low-achievers to a much lesser extent."
Kroesbergen, 2002, 3.1	"The recommendations mentioned in the literature for teaching students with learning disabilities or low-performing students appear to be in clear opposition to the constructivist principle of guided re-invention. The question is whether teachers can ask low-performing children to actively contribute to lessons by inventing new strategies. Asking for such a contribution actually appears to deny the special status of these children, who clearly have more difficulties with knowledge generalization, connecting new information to old, and the automatization of basic facts."
Kroesbergen, 2002, 3.4	". . . guided instruction appears to be particularly well-suited to students in regular education and structured instruction to students in special education."
Kroesbergen, 2002, 4.4	"To conclude, directed math instruction was found to be more effective than guided math instruction although the guided instruction was still more effective than regular instruction for the low-achieving students examined in the present study. This means that the current reforms in the mathematics curricula are not based on the most adequate instruction principles for low-achieving students. Low-achieving students have special needs, and their instruction should clearly be adapted to these special needs."
McEwan, 2000, 45	"Three studies examined whether initial instruction should be given through a discovery mode, a guided discovery mode, or a direct (didactic) mode (Anastasiow, Sibley, Leonhardt, & Borich, 1970; Lackner, 1972; Olander & Robertson, 1973). A slight advantage appears to go to the guided discovery mode for some content and with higher-performing students, and to the direct mode for some content and with lower-performing students. There are no advantages indicated in these studies for a strict discovery mode (constructivist learning). Several additional studies in the review investigated some other aspect of mathematics instruction, but as part of their study found positive effects for explicit instruction as well (Dixon, Carnine, Lee, Wallin, & Chard, 1998b, 2)."

Researcher(s)	Findings/Conclusions
Jones, Wilson, & Bhojwani, 1997, 159	"In summary, the constructivist perspective, though intuitively appealing, is currently unsupported by empirical research and is logically inadequate for the task of teaching adolescents with LD. . . . The premise that secondary students with LD will construct their own knowledge about important mathematical concepts, skills, and relationships, or that in the absence of specific instruction or prompting they will learn how or when to apply what they have learned, is indefensible, illogical, and unsupported by empirical investigations."
Smith, 2002, 128	"The constructivist stance is that mathematical understanding is not something that can be explained to children, nor is it a property of objects or other aspects of the physical world. Instead, children must 'reinvent' mathematics in situations analogous to those in which relevant aspects of mathematics were invented or discovered in the first place. They must construct mathematics for themselves, using the same mental tools and attitudes they employ to construct understanding of the language they hear around them. . . . "None of this means that children should be left to their own resources to recapitulate 5000 years of mathematical history, including all of the false turns and blind alleys. But it does assert that children can and must invent mathematics for themselves if given opportunities for relevant experiences and reflection."
Kroesbergen, Van Luit, & Maas, 2004, 1	"In this study we compared the effects of small-group constructivist and explicit mathematics instruction in basic multiplication on low-achieving students' performance and motivation. . . . Results showed that the math performance of students in the explicit instruction condition improved significantly more than that of students in the constructivist condition, and the performance of students in both experimental conditions improved significantly more than that of students in the control condition. Only a few effects on motivation were found. We therefore concluded that recent reforms in mathematics instruction requiring students to construct their own knowledge may not be effective for low-achieving students."
Jones, Wilson, & Bhojwani, 1997, 158	"Compelling research on effective instructional practices is frequently ignored (Carnine, 1992). Instead, appealing but unvalidated trends, such as constructivism and discovery learning, have caught educators' attention. These ideologies tend to be vague and allow support for haphazard and poorly designed instruction. They are logically antithetical to the existing empirical evidence on best practices for students with LD. Educational practices that are derived from ideologies must be critically evaluated—and not merely for their fit with the political sensibilities or any particular ideology, but for their effect on the achievement of children and youth."
Whitehurst, n.d., 4	"A second finding that complicates the basic constructivist view is that discovery activities may substantially compromise learning unless the child already has mastered the background knowledge that is relevant to the problem to be explored."
Whitehurst, n.d., 5	". . . it may take a very long time for some children to discover that they have to pay attention to the first digit in solving decimal fractions. Why not tell them? As the famous psychologist Jerome Bruner said about discovery learning back in 1966, 'it is the most inefficient technique possible for regaining what has been gathered over a long period of time.' The algorithms, procedures, and facts of mathematics are powerful cultural inventions that have accumulated over thousands of years of human history. We simply cannot expect every child to discover the Pythagorean theorem."
Geary, 2003a, 453	"At one extreme, Cobb et al. (1992) have argued that, with appropriate social-mathematical contexts, 'it is possible for students to construct for themselves the mathematical practices that, historically, took several thousand years to evolve' (28). Claims such as these are highly speculative and almost certainly untrue. It is very unlikely that most, or even a handful of, students will be capable of discovering all of the mathematical concepts and procedures that will enable them to succeed, for instance, in college-level mathematics courses (Geary, 1995)."

Researcher(s)	Findings/Conclusions
Geary, 2003a, 455	". . . Fuson and Burghardt suggest that some balance between constructivist and direct instruction will be needed. I agree with this suggestion, with the caveat that the determination of this balance must be based on empirical research, that is, controlled studies that assess the most effective approach to teaching concepts and procedures. However, I disagree with Ambrose et al.'s suggestion that basic procedural and computational skills do not need to be taught in school and that the use of constructivist or direct instruction is a matter of values, not just empirical research."
Geary, 2003a, 456	"Even if the constructivist approach was fully effective with all children (which has not been proven), the time required to construct invented algorithms is time that cannot be spent on other mathematical topics. Of course, this is why early mathematicians invested so much time in developing procedures and representational systems (e.g., the base-ten system) for number and arithmetic. . . . The goal of teaching these algorithms is to circumvent the centuries it took to develop these procedures (e.g., Al-Uqlidisi, 952/1978) in favor of more important aspects of arithmetic, such as conceptually understanding the base-ten system."
Kroesbergen, 2002, 6	"From a constructivist perspective, a repertoire of strategies can be built via exposure to and practice with different problems; the students are not told how to solve the problems and must therefore discover which strategies to use in discussion with other students. In such a manner, students learn from their own experiences. Such instruction is rarely recommended for students with difficulties learning math, however (Kroesbergen & Van Luit)."
Kroesbergen, 2002, 7	"Although instruction based on the principles of realistic mathematics education ["discovery" mathematics based in real-world problems] has shown promising results (Cobb, et al., 1991; Gravemeijer et al., 1993; Kelin et al., 1998), its beneficial value for students with learning difficulties is highly doubted (Klein et al., 1998; Van Zoelen, Houtveen & Booij, 1997; Woodward & Baxter, 1997). . . . children with difficulties learning math appear to need more directed instruction than provided within the framework of realistic mathematics education. Special educators thus tend to employ instructional methods based on cognitive behavior modification principles or direct instruction principles."
Committee on *How People Learn,* 2005, 242	". . . some suggest that students must invent all their mathematical ideas and that we should wait until they do so rather than teach ideas. This view, of course, ignores the fact that all inventions are made within a supportive culture and that providing appropriate supports can speed such inventions. Too much focus on student-invented methods per se can hold students back; those who use time-consuming methods that are not easily generalized need to be helped to move on to more rapid and generalizable 'good-enough' methods. A focus on sense making and understanding of the methods that are used is the balanced focus, rather than an emphasis on whether the method was invented by the student using it."
Klahr & Nigam, 2004, 1	". . . the theoretical basis for the predicted superiority of discovery over direct instruction is vague and, in many cases, inconsistent with much of the literature on learning and memory. For example, in most cases children in discovery situations are more likely than those receiving direct instruction to encounter inconsistent or misleading feedback, encoding errors, causal misattributions, and inadequate practice and elaborations. These severe cognitive disadvantages may overwhelm the increased motivational aspects that are commonly attributed to discovery learning. In sum, we question the widely accepted view that discovery learning usually trumps direct instruction."

Researcher(s)	Findings/Conclusions
Anderson, Reder, & Simon, 2000, 12	"When, for whatever reason, students cannot construct the knowledge for themselves, they need some instruction. The argument that knowledge must be constructed is very similar to the earlier arguments that discovery learning is superior to direct instruction. In point of fact, there is very little positive evidence for discovery learning and it is often inferior (e.g., Charney, Reder, & Kusbit, 1990). Discovery learning even when successful in acquiring the desired construct, may take a great deal of valuable time that could have been spent practicing this construct if it had been instructed. Because most of the learning in discovery learning only takes place after the construct has been found, when the search is lengthy or unsuccessful, motivation commonly flags."
Marzano, Pickering, & Pollock, 2001, 137	"Although the discovery approach has captured the fancy of many educators, there is not much research to indicate its superiority to other methods. Indeed, some researchers have made strong assertions about the lack of effectiveness of discovery learning, particularly as it relates to skills. For example, researchers McDaniel and Schlager (1990) note: 'In our view, discovery learning does not produce better skill' (p. 153)."
Ball, Ferrini-Mundy, Kilpatrick, Milgram, Schmid, & Schaar, 2005, 3	"Some have suggested the exclusive use of small groups or discovery learning at the expense of direct instruction in teaching mathematics. Students can learn effectively via a mixture of direct instruction, structured investigation, and open exploration. Decisions about what is better taught through direct instruction and what might be better taught by structuring explorations for students should be made on the basis of the particular mathematics, the goals for learning, and the students' present skills and knowledge. For example, mathematical conventions and definitions should not be taught by pure discovery."
Anderson, Reder, & Simon, 13	"This criticism of practice (called 'drill and kill' as if this phrase constituted empirical evaluation) is prominent in constructivist writings. Nothing flies more in the face of the last 20 years of research than the assertion that practice is bad. All evidence, from the laboratory and from extensive case studies of professionals, indicates that real competence only comes with extensive practice (e.g., Hayes, 1985; Ericsson, Krampe, Tesche-Romer, 1993). In denying the critical role of practice one is denying children the very thing they need to achieve real competence. The instructional task is not to 'kill' motivation by demanding drill, but to find tasks that provide practice while at the same time sustaining interest. Substantial evidence shows that there are a number of ways to do this: 'learning-from-examples,' . . . is one such procedure that has been extensively and successfully tested in school situations."
Marzano, Pickering, & Pollock, 2001, 138	"Some skills are not amenable to discovery learning. For example, consider the skills of addition, subtraction, multiplication, and division. To have students discover the steps involved in these computational procedures makes little sense."
Jordan, Kaplan, & Hanich, 2002, 596	" . . in third grade children taught with a traditional approach made more progress than did children taught with a problem-centered approach. The finding held for all achievement groups."
Karp & Howell, Oct. 2004, 122	"As demonstrated in reading instruction, methods that rely heavily on constructivist approaches are sometimes not as effective for the learning-disabled population as are approaches that focus on more explicit instruction (Torgesen, 1998)."
Bryant, Hartman, & Kim, 2003, 151	"Reviews of research have revealed that students with LD benefit from a combined model of academic instruction that includes both explicit and strategic instructional procedures."
Ontario Ministry of Education, 2005, 40	"The knowledge about the world that children bring into the classroom is a strong determinant of their learning across a number of content areas. Children who have a solid knowledge base in a particular area are better able to understand what they hear and read on that topic, are better able to link old knowledge with new information or produce new learning, and are better able to use strategies to remember what they have learned (Bjorklund, 2005). Children can differ in their prior knowledge for a variety of reasons. . . . What is important to keep in mind is that general knowledge for new learning may need to be restated and reinforced for some children and actually taught to others."

Researcher(s)	Findings/Conclusions
Klein, 2005, 16	"Only a minority of states explicitly require knowledge of the standard algorithms of arithmetic for addition, subtraction, multiplication, and division. Instead, many states do not identify any methods for arithmetic, or worse, ask students to invent their own algorithms or rely on *ad hoc* methods. . . . Specialized methods for mental math work well in some cases but not in others, and it is unwise for schools to leave students with untested, private algorithms for arithmetic operations. Such procedures might be valid only for a subclass of problems. The standard algorithms are powerful theorems and they are standard for a good reason: they are guaranteed to work for all problems of the type for which they were designed."

Some writers on the math war issues are beginning to call for a balanced approach to teaching mathematics, understanding that the truth is likely to be somewhere in the middle, rather than on one extreme side or the other, just as it is in reading. A major problem is that much of existing research does not look at the effects of curriculum and instruction on struggling learners, per se, just on the general population. When research that specifically examines instructional effects on those who struggle (which this study includes), one generally sees different findings. The "discovery" or constructivist approach may, for example, work well with learners who have no learning difficulties or disabilities; but it is absolutely clear from numerous studies that direct instruction works best for students who struggle. Another problem is that general educators rarely see the specialized journal articles that examine effective curriculum and strategies for students with difficulties and/or disabilities. Many relevant studies are found in medical, neurological, and psychological journals, which are generally not accessible to education practitioners. What they read, therefore, leads them in a direction that provides good recommendations for practice as it relates to general education, but which never helps those who fail and fail and fail. Elucidating that research is a major purpose of this study on "why *MLS* works." In Chapter IV more research on the importance of the balanced approach to mathematics curriculum is provided.

Summary of Causes of Mathematics Difficulties

Students with mathematics difficulties, as opposed to mathematical disabilities, are those students who suffer the psychological effects caused by a cultural environment that does not value (or under-values) mathematics; or who, perhaps, are victims of stereotype threat that encourages disengagement; or whose fears of mathematics result in mathematical phobia or anxiety; or who, as a result of an unmet psychological need (frequently manifested by low self-esteem), have poor motivation for learning. These effects are real and pernicious, without doubt causing major problems in mathematics achievement. Students may, either temporarily or long-term, have mathematics difficulties due to other life events, such as parental divorce, death of a parent or other close family member, an upsetting family move to another community, a parent's deployment in the military, a parent's loss of employment, and so on. These kinds of experiences are not addressed in this study.

Students with mathematics difficulties may also be students for whom the structure of their home language causes problems in understanding one of the basic mathematics concepts, the base-10 system. They may be, as well, one of the millions of English-language learners enrolled in American schools, who struggle to learn mathematics at the same time as they learn English, who are sometimes confused by mathematics vocabulary, and who may also be confused by prior instruction in algorithms that differ from the algorithms taught in most American schools. Because these immigrant children are likely to come from poverty, they doubly suffer the negative effects of economic disadvantage, including poorly educated parents who may not understand American schools' expectations or who may not see value in learning mathematics and science.

A third major cause for mathematics difficulties, usually compounded with one of the other factors, is inadequate instruction. Economically disadvantaged children, whether American citizens or recent immigrants, tend to lack preschool education, informal or formal, which prepares them well for kindergarten. Many children, again due to poverty, mobility, negligence, poor motivation, or illness have poor attendance in school, missing many days, weeks, or even months of instruction during the school year. Inadequate instruction also exists when students in need do not receive the extended day, week, or year education that they need, or when they do not have access to meaningful and effective interventions to accelerate their learning. They are the children denied the opportunity to learn (see Chapter VIII).

One finds in the research more references to inappropriate instruction than to any other topic as the reason for low achievement in mathematics. The findings do not necessarily indict teachers for their failures. Rather, schools of education are the culprit in some studies for inadequately preparing teachers for teaching mathematics, especially at the elementary and middle school levels. Also, universities are simply not producing enough certified and well-qualified mathematics teachers to fill school needs. State departments of education receive blame for the poor quality of their mathematics standards, for approving flawed textbooks for use in schools, for poor assessment systems which limit timely and focused feedback to teachers and students, and for weak certification requirements. School districts are frequently targeted for criticism for not offering and requiring more high quality professional development for mathematics teachers. Recently, state legislatures are blamed for providing inadequate funding for all levels of education and for under-valuing the fields of mathematics and science in their appropriation formulas.

Teachers do not escape, however. They are criticized for holding on to traditional practices, for their failures in translating research into practice, for their not being proactive in updating their mathematics knowledge and skills, and for not customizing instruction to individual student needs. Their professional organization, the National Council of Teachers of Mathematics (NCTM), does not escape blame either, given its influence on mathematical standards and on the selection of instructional strategies. Even the National Science Foundation (NSF) has received its share of scrutiny for its role in the current status of mathematics achievement at both the K-12 and university levels. Many publishers are also to blame, especially those that emphasize "coverage" over "mastery," and those that provide inadequate and varied opportunities for students to practice new learning.

Inappropriate curriculum and instruction are the focus of those on either side of the so-called math wars. Extremists debate whether concepts or skills should be emphasized, and both sides disagree with those calling for a more balanced approach. The debate gets very hot between those who advocate a discovery or constructivist methodology for teaching mathematics as opposed to those who advocate a more direct instructional approach, especially for students who struggle. The wars themselves consume energy and paralyze policy makers so that progress in finding solutions to the nation's dilemma is slowed, if not doomed, and the research is ignored.

MLS as a solution for the varied cultural and motivational negative effects on student learning in mathematics, for second-language learners, and for victims of inadequate and/or inappropriate instruction will be discussed and documented in subsequent chapters.

Chapter III: Mathematics Disabilities

". . . the brain is an information processing organ made marvelously powerful not by its mystery, but by its complexity—by the enormous number, variety, and interactions of its nerve cells" (Kandel, 2006, 9).

Overview

Chapter I provided a general introduction to the study. Chapter II discussed the research on mathematics difficulties (as opposed to mathematics disabilities). In brief, mathematics difficulties are those relating to culture and values, to stereotype threat, to mathematics phobia, to general poor motivation to learn caused by unmet psychological needs, to problems confronted by English-language learners, and to issues resulting from inadequate or inappropriate instruction. The "math wars" are a debate on what constitutes inappropriate instruction, and those issues were discussed. In other words, mathematics difficulties occur among general education students and second-language learners. The manifestations of difficulties may appear similar to those of disabilities, but none involve actual learning disabilities in themselves, although they may certainly occur in combination with disabilities. For example, according to Elbaum and Vaughn (2003), "Self-concept has particular relevance to students with learning disabilities. Learning disabilities have been consistently linked to poor self-concept" (p. 229).

Fletcher, Morris, and Lyon (2003) make the following distinction between learning difficulties and learning disabilities:

> When exclusionary criteria are applied, LD [learning disability] represents a subgroup of "unexpected" underachievement. It is differentiated from expected underachievement due to emotional disturbance, disadvantage, cultural and linguistic diversity, and inadequate instruction (Kavale & Forness, 2000) (p. 35).

Reading disabilities are complex since the location of the disability may originate in the central executive or in the language or visuospatial system, similar to mathematics disabilities. Yet the study of mathematics disabilities, the focus for Chapter III, seems more complex and varied since mathematics has so many domains and subdomains. According to Geary, Hamson, and Hoard (2000), "Research on learning disabilities in mathematics (MD) has progressed more slowly than reading disabilities (RD) research. One impediment to research on MD is the complexity of the domain of mathematics and the resulting wide array of cognitive deficits that could contribute to this form of LD" (p. 236). The mathematical domains include arithmetic, obviously, but also algebra, geometry, statistics and probability, calculus, etc. Each one of these domains has its own set of potential disabilities. Mathematics instruction, of course, focuses on the development of mathematical concepts and procedures (the supporting competencies). There are sometimes interferences in learning those concepts and procedures due to disorders in the central executive; and there are also possible disorders in the language and visuospatial systems for both information representation and information manipulation. Any of these disabilities affects mathematics achievement. Dehaene, Piazza, Pinel, and Cohen (2005) provide ideas about the complexity of processing numbers, just one small area of the realm of mathematics—and, therefore, the variety of possible problems in doing so:

> The tricode model of number processing predicts that, depending on the tasks, three distinct systems of representation may be recruited: a quantity system (a nonverbal semantic representation of the size and distance relations between numbers, which may be category specific), a verbal system (in which numerals are represented lexically, phonologically, and syntactically much like any other type of word), and a visual system (in which numbers can be encoded as strings of Arabic numbers (p. 434).

A disability in the central executive or either the language or visuospatial system would affect, therefore, number processing.

Geary and Hoard (2005, p. 260) have constructed a useful model for understanding the complexity of mathematics disabilities, as follows:

<table>
<tr><td colspan="4">Mathematical Domain
(e.g., Base-10 Arithmetic)</td></tr>
<tr><td colspan="4">Supporting Competencies</td></tr>
<tr><td colspan="2">Conceptual
(e.g., base-10 knowledge)</td><td colspan="2">Procedural
(e.g., columnar trading)</td></tr>
<tr><td colspan="4">Underlying Cognitive Systems</td></tr>
<tr><td colspan="4">Central Executive
Attentional and Inhibitory Control of Information Processing</td></tr>
<tr><td colspan="2">Language System</td><td colspan="2">Visuospatial System</td></tr>
<tr><td>Information Representation</td><td>Information Manipulation</td><td>Information Representation</td><td>Information Manipulation</td></tr>
</table>

Wright (1996) contributes his explanation about the complexity of studying mathematics disabilities:

> There is no single mathematics disability. In fact, mathematics disabilities are as varied and complex as those associated with reading. Furthermore, there are some arithmetic disabilities which can exist independent of a reading disability and others which do not. One type of learning disability affecting mathematics can stem from an individual's difficulty processing language, another might be related to visual spatial confusion, while yet another could include trouble retaining math facts and keeping procedures in the proper order. While extremely rare, there are some learners who cannot successfully compare the lengths of two sticks and others who have almost no ability to estimate. Finally, some

> people experience emotional blocks so overwhelming as to preclude their ability to think responsibly and clearly when attempting math, and these students are disabled, as well (p. 1).

Wright's examples, although pre-dating the Geary and Hoard model, almost perfectly exemplify it. For example, a "difficulty processing language" is located in the language system. The disability related to "visual spatial confusion" resides, obviously, in the visuospatial system. "Trouble retaining math facts and keeping procedures in the proper order" reflect disabilities in the central executive. These topics are generally researched in the field of neuropsychology, "a science that examines alternations in mental processes produced by brain damage" (Kandel, 2006, 121).

Using Geary and Hoard's model, Chapter III will summarize the research relating, first, to mathematics disabilities only (MD), or mathematics learning disabilities (MLD), and the ways in which *dyscalculia,* the general term for mathematics disabilities, is defined and diagnosed. Research and theory about how disabilities affect the learning of the various mathematics domains and their supporting competencies, conceptual and procedural, will be discussed. Preceding an explanation of the "underlying cognitive systems" will be a general discussion of dyscalculia. Then, specific research on each of the underlying cognitive systems, including the central executive and the language and visuospatial systems will be reported and discussed, as they pertain to achievement in mathematics. There is much evidence that dyslexia, usually considered a reading disability, affects several different areas of mathematics. The effects of dyslexia on mathematics achievement are perhaps less serious than those of a mathematics disability, but they are nevertheless an issue. This discussion will be a subtopic under the general area of the language system.

The next section will include discussion of the more specific, but low-incidence, mathematics disabilities caused by Turner syndrome, Fragile X syndrome, Gerstmann's syndrome, and spinal bifada. More serious than MD (or MLD) are comorbid reading and mathematics disabilities (MD-RD or MRD), the occurrence of both reading (including dyslexia or vision/hearing impairments) and mathematics disabilities in one person. Most serious, of course, are the very real possibilities that any of these students with mathematics disabilities is also a potential victim of the issues outlined in Chapter II that, at a minimum, result in difficulties, and combined present very challenging problems. It is not uncommon, for example, for a child with disabilities also to be from a low socio-economic family that places little importance on mathematics and science; to confront the issues of home language interference in understanding mathematics concepts, vocabulary, and/or algorithms; to be a victim of stereotype threat and/or mathematics phobia; to have a low sense of self-efficacy; and to have had both inadequate and inappropriate instruction. Knowing this reality makes it clear why there is a critical need for a program such as *MLS*. It is also clear that without an appropriate intervention for such students, there is no hope of high school graduation.

Definitions of Dyscalculia

The general term used to describe mathematical disabilities is *dyscalculia*. Table 19 includes definitions of dyscalculia as determined from the varied perspectives of researchers in the area of mathematics disabilities. There is general agreement from these writers that dyscalculia is a neuropsychological dysfunction, a processing problem of some kind or another, which generally is the cause of learning problems in both mathematics and reading. Research, of course is continuing, and Rivera (1997) notes that it "is being approached from different perspectives, including developmental, neurological and neuropsychological" (p. 2).

Table 19: Definitions of Dyscalculia

Researcher(s)	Findings/Conclusions
Pennington, 1991, xii	"By definition, a learning disorder involves dysfunction in one or more neuropsychological systems that affect school performance."
Sousa, 2001, p. 139	"The condition that causes persistent problems with processing numerical calculations is often referred to as dyscalculia."
Dowker, 2004, 14	"The term 'developmental dyscalculia,' implying a specific disorder of mathematical learning, appears to have been popularised by Kosc (1974, 1981); though there was some earlier research on related problems (Kinsbourne and Warrington, 1963)."
D'Arcangelo, 2002, 85	Interview with Brian Butterworth: "Dyscalculia is a condition a child is born with that affects the ability to acquire the usual arithmetic skills. Dyscalculia students may show difficulty understanding even simple number concepts and, as a consequence, will have problems learning the standard number facts and procedures. Even when dyscalculia students can produce the correct answer or the correct method, they may do so mechanically and without confidence because they lack an intuitive grasp of numbers that the rest of us possess. Dyscalculia is rather like a dyslexia for numbers—but unlike dyslexia, little is currently known about its prevalence, causes, or treatment. Dyscalculia often appears in conjunction with other learning difficulties—including dyslexia, dyspraxia, and attention deficit disorders—but most dyscalculia students will have cognitive and language abilities in the normal range and may indeed excel in nonmathematical subjects."
Shaley & Gross-Tsur, 2001, 338	"Children who present with difficulty in learning arithmetic and who fail to achieve adequate proficiency in this cognitive domain despite normal intelligence, scholastic opportunity, emotional stability, and necessary motivation have developmental dyscalculia. Some have trouble learning the arithmetic tables; others never comprehend algorithms of addition, subtraction, multiplication, and division; whereas others have problems understanding the concept of numbers or cannot write, read, or identify the correct word to the numeral."
Levine & Schwartz, n.d., 3	"A neurodevelopmental dysfunction may exist because of a lack of sufficient use of that function, because of cultural influences, because of inadequate or ineffective teaching in the past, or in fact, as a result of genetic or acquired central nervous system lesions."
Scruggs & Mastropieri, 2002, 160	"Further, measures of cognitive processing, including attention, memory, and linguistic processes, although not necessarily directly related to neuropsychological dysfunction, have revealed processing deficits in students with learning disabilities."
Geary & Hoard, 2005, 253	"A learning disability can result from deficits in the ability to represent or process information in one or all of the many mathematical domains (e.g., geometry) or in one or a set of individual competencies within each domain. The goal is further complicated by the task of distinguishing poor achievement due to inadequate instruction from poor achievement due to an actual cognitive disability."

Researcher(s)	Findings/Conclusions
Sousa, 2001, 141	"Learning deficits can include difficulties in mastering basic number concepts, counting skills, and processing arithmetic operations as well as procedural, retrieval, and visual-spatial deficits (Geary, 2000). As with any learning disability, each of these deficits can range from mild to severe."
Lyon, 1996, 68	"Children identified as manifesting LD in mathematics can demonstrate deficits in arithmetic calculation, mathematics reasoning, or both. In general, authorities agree that approximately 6% of the school population have difficulties in mathematics that cannot be attributed to low intelligence, sensory deficits, or economic deprivation."
Raborn, 1995, 2	"Students with learning disabilities who are struggling in math generally have average or above average intelligence. However, math ability may be restricted by substantial differences in the areas of attention, perception, visual-motor abilities, language processing, memory, reading/writing, and in the use of cognitive strategies. These significant differences will stand out in each language used by a bilingual student with a learning disability. If differences appear only in English, then those differences are probably due to the challenges of learning a new language."
Geary & Hoard, 2005, 259	"The overall pattern suggests that the memory-retrieval deficits of children with MD/RD [mathematics disabilities/reading disabilities] or MD only reflect a cognitive disability and not, for instance, a lack of exposure to arithmetic problems, poor motivation, a low confidence criterion, or low IQ."
Noel, Rousselle, & Mussolin, 2005, 192	"Neuropsychological data also provide converging evidence for a role of the parietal lobe in number magnitude representation and in dyscalculia."

Dyscalculia's Frequency

Just as in reading, there is near consensus among researchers that approximately six percent of the population has a mathematics disability. Table 20 includes reports on frequency from several researchers.

Table 20: Frequency of Dyscalculia

Researcher(s)	Findings/Conclusions
Mazzocco & McCloskey, 2005, 272	"Despite a lack of consensus in how we define MD, most researchers report a prevalence of 5 to 8% in school-age children (Badian, 1983; Shalev, Auerbach, Manor, & Gross-Tsur, 2000)."
Noel, Rousselle, & Mussolin, 2005, 191	"About 6% of school-aged children have major difficulties in mathematics. . . . As number magnitude seems to be one of the roots of this learning, Butterworth (1999) has proposed that a dysfunction of that representation might well be one of the possible causes of mathematics disabilities."
Sousa, 2001, 139	"About 6 percent of school-age children have some form of difficulty with processing mathematics. This is about the same number as children who have reading problems."
Butterworth, 2005, 455	"Severe difficulties in learning about numbers and arithmetic are probably as widespread as disorders of literacy development (dyslexia). The best prevalence estimates for each lie between 3.6% and 6.5%. Studies in the U. K. have revealed that poor mathematical skills are more of a handicap in the workplace than poor literacy skills (Brynner & Parsons, 1997)."

Origins of Dyscalculia

As the research shows in Table 21, there is disagreement about the origins of dyscalculia. Shaley and Gross-Tsur (2001) state that ". . . not all researchers agree that developmental dyscalculia is a genetic, biologically based brain disorder (p. 339). Rather, they would place the origins within the topics discussed in Chapter II under mathematical difficulties: “Other etiologies implicated in its genesis are environmental deprivation, poor teaching, low intelligence, and mathematical anxiety” (p. 339).

Landerl, Bevan, and Butterworth (2004) offer another hypothesis. They write that “Developmental dyscalculia is likely to be the result of the failure of these brain areas to develop normally, whether because of injury or because of genetic factors” (p. 121). Root (1994) writes that “Learning disabilities are probably inherited” (p. 1). In summary, likely origins of dyscalculia include environmental conditions, injury, or genetics.

Table 21: Origins of Dyscalculia

Researcher(s)	Findings/Conclusions
Geary & Hoard, 2005, 260	“. . . the complexity of the field of mathematics results in a very large number of potential sources of MD [mathematics disabilities].”
Landerl, Bevan, & Butterworth, 2004, 121	“Neuropsychological evidence indicates that numerical processing is localized to the parietal lobes bilaterally, in particular the intra-parietal sulcus (Dehaene, Piazza, Pinel, & Cohen, 2003), and is independent of other abilities.”
Root, 1994, 1	“Learning disabilities are probably inherited; it is thought that they are caused by a neurological malfunction or processing glitch which renders written text-deciphering, sound-symbol connections and/or the sequencing of information very difficult (Saltus, 1992, p. 29, 31). A learning disability is not indicative of less intelligence. In fact, people who have a learning disability are often very bright, even gifted, people. It is true, however, that their short circuit or processing glitch causes them to see things differently and sometimes obscures their intelligence (Vail, 1987, p. xiv). While they cannot be cured, they can be taught compensatory strategies.”
Dehaene, Piazza, Pinel, & Cohen, 2005, 449	“. . . when a child is dyscalculic, other family members are also frequently affected, suggesting that genetic factors may contribute to the disorder.”
BBC News, 2006, 1	“A research paper published in the Proceedings of the National Academy of Sciences in the US shows a separate part of the brain is used for counting. The researchers, in California and London, say the area that processes numbers has two functions—counting ‘how many’ and knowing ‘how much.’ Prof. Brian Butterworth of UCL said this could be key to diagnosing dyscalculia. . . . . A different part of the brain was being used—instead of counting it was trying to assess how much colour was present. ‘By comparing these two types of stimulus, we identified the brain activity specific to estimating numbers of things,’ Prof. Butterworth said. ‘We think this is a brain network that underlies arithmetic and may be abnormal in dyscalculics.’” (1).

Diagnosis of Dyscalculia

Although much is known about dyscalculia and its manifestations, there is still controversy about its identification, especially since it may (and is likely to) exist along with one or more of the difficulties identified in Chapter II. Another problem in diagnosis is that the manifestations of mathematical difficulties are very similar to those of mathematical disabilities. Some of the same kinds of errors and misconceptions are likely among students who struggle in mathematics. A third problem is that the different domains of mathematics make diagnosis even more complex. A learner who struggles with fact retrieval, perhaps due to faulty phonological processing, may have no trouble with geometry concepts. Or a learner with disabilities in the visuospatial system will undoubtedly have trouble with geometry concepts, but not with learning other concepts, nor with fact retrieval. What is generally agreed upon is that dyscalculia is a neurological dysfunction and that it can be identified by observing whether a student has ongoing problems in mathematics over time. In other words, a student who performs poorly one year and then does well another year does not have dyscalculia, even if he or she reverts to poor performance the third year. Table 22 provides the conclusions of various researchers as to the ways to identify and diagnose students with dyscalculia.

Table 22: Diagnosis of Dyscalculia

Researcher(s)	Findings/Conclusions
Kroesbergen, 2002, 2.1	"Studies have shown most math difficulties to have a relatively early onset (i.e., problems emerge between the ages of five and seven years with the learning of basic skills; Schopman & Van Luit, 1996)."
Geary & Hoard, 2005, 254	"Unfortunately, measures that are specifically designed to diagnose MD [mathematical disabilities] are not available; thus, most researchers and practitioners rely on standardized achievement tests, often in combination with IQ scores. A score lower than the 25th or 30th percentile on a mathematics achievement test combined with a low-average or higher IQ score are common criteria for diagnosing MD. However, a lower than expected (based on IQ) achievement score does not, in and of itself, indicate the presence of MD."
Geary & Hoard, 2005, 254	"Many children who have lower than expected achievement scores across successive academic years . . . often have some form of memory or cognitive deficit, and thus a diagnosis of MD is often warranted. Many of these children do show year-to-year improvements in achievement and show more persistent deficits in some areas, such as fact retrieval."
Landerl, Bevan, & Butterworth, 2004, 100	"A range of terms for referring to developmental maths disability has emerged, along with different criteria. Geary and colleagues use the term 'mathematical disabilities' and include all children who fall below the 30th percentile (Geary, Hoard, & Hamson, 1999) or 35th percentile (Geary, Hoard, & Hamson, 2000) on the Woodcock-Johnson Mathematics reasoning test (Woodcock & Johnson, 2001). . . . "Shaley, Manor, and Gross-Tsur (1997), who have carried out the most extensive study of this condition, use the criterion of two grades below chronological age."
Pennington, 1991, 121)	"Children with specific math and handwriting problems tend to come to clinical attention later than children with dyslexic, ADHD, or autism spectrum disorder, because their learning disorder does not disrupt school performance as noticeably in the early school years . . . . A very typical referring symptom is that the child is not completing or turning in homework assignments, and that the child has become locked into an oppositional struggle with parents and teacher over written work. So the initial presentation may suggest an emotional or motivational problem."
Kroesbergen, 2002, 2.1	"During kindergarten and first grade, children typically develop number sense, which then grows along the lines of the various Piagetian operations (e.g., number conservation, classification, seriation) and in combination with various counting skills. A basic understanding of the arithmetic operations is established at this time (Correa, Nunes, & Bryant, 1999). The first category of interventions thus focuses on these preparatory arithmetic skills. . . . "The next step is to learn the four basic mathematical operations (i.e., addition, subtraction, multiplication, and division). Knowledge of these operations and a capacity to perform mental arithmetic also play an important role in the development of children's later math skills (Mercer & Mercer, 1992; Van Luit & Naglieri, 1999). Most children with math-related learning disorders are unable to master the four basic operations before leaving elementary school and thus need special attention to acquire the skills. A second category of interventions is therefore aimed at the acquisition and automatization of basic math skills.

Researcher(s)	Findings/Conclusions
	"Mastery of the basic operations, however, is not sufficient. Students must also acquire problem-solving skills (e.g., flexibility and adaptability) in addition to the basic computational skills (i.e., the development of automaticity; Carnine, 1997; Goldman, 1989). . . . The third category of interventions addresses problem-solving skills."
D'Arcangelo, 2002, 86	Interview with Brian Butterworth: "Research shows that we think about numbers as displayed in a line in our head, a kind of mental representation of numbers. Now, when you ask people if they have a number line, most are not conscious of it. Perhaps only 15 percent of people are conscious of having a number line. In most people, this number line seems to go from left to right."
D'Arcangelo, 2002, 86	Interview with Brian Butterworth: "Dyscalculic students seem to have an impaired sense of number size. This may affect tasks involving estimating numbers in a collection and comparing numbers. Dyscalculic students can usually learn the sequence of counting words but may have difficulty navigating back and forth, especially in 2s, 3s, or more. They may also find it especially difficult to translate between number words whose powers of 10 are expressed by new names, such as 'ten,' 'hundred,' or 'thousand' and numerals whose powers of 10 are expressed by the same numerals but in terms of place value, such as 10, 100, and 1,000. These students may be competent at reading and writing numbers, though some dyscalculic students have problems with numbers over 1,000, even in 6th grade."
D'Arcangelo, 2002, 86	Interview with Brian Butterworth: "We suspect that when we can map the parietal lobes with great precision, we will see that separate areas do the separate arithmetical operations—addition, subtraction, multiplication, and division. Each of these operations can be selectively affected by brain damage without the others being affected."
Dowker, 2004, ii	"The general evidence is that arithmetical difficulties are part of a continuum of ability. However, a few individuals do have specific difficulties with arithmetic, which do not resemble anything observed in the general population: for example, they may be unable to recognize small quantities of objects (even as low as two or three) without counting them. The term 'dyscalculia' is sometimes reserved for such individuals, though it is sometimes used for any individual with relatively specific mathematical difficulties."
Dowker, 2004, 12	"There are certain forms of brain damage and of genetic disorder (e.g. Williams syndrome) which not only lead to general intellectual impairment, but to disproportionate difficulties in arithmetic."

Disabilities Specific to Mathematics Domains

If one reviews the first section of the model created by Geary and Hoard (2005, p. 260) that depicts the areas in which mathematics disabilities can occur, he or she sees that students may have mathematical disabilities specific to one or more of the various domains, or subdomains, under the mathematics umbrella, exemplifying again the complexity of understanding mathematics disabilities.

<table>
<tr><td colspan="2" align="center">Mathematical Domain
(e.g., Base-10 Arithmetic)</td></tr>
<tr><td colspan="2" align="center">Supporting Competencies</td></tr>
<tr><td align="center">Conceptual
(e.g., base-10 knowledge)</td><td align="center">Procedural
(e.g., columnar trading)</td></tr>
</table>

Geary (n.d.) explains as follows:

> One of the difficulties in studying children with MD is the complexity of the field of mathematics. In theory, MD could result from difficulties in the skills that comprise one or many of the domains of mathematics, such as arithmetic, algebra, or geometry. Moreover, each of these domains is very complex, in that each has many subdomains, and a learning disability can result from difficulties in understanding or learning basic skills in one or several of these subdomains (p.1).

Table 23 includes the observations of other experts on the kinds of disabilities that affect various mathematical domains.

Table 23: Disabilities Specific to Mathematics Domains

Researcher(s)	Findings/Conclusions
Geary, 2004, 4	"In theory, a learning disability can result from deficits in the ability to represent or process information in one or all of the many mathematical domains (e.g., geometry) or in one or a set of individual competencies within each domain. The goal is further complicated by the task of distinguishing poor achievement due to inadequate instruction from poor achievement due to an actual cognitive disability (Geary, Brown, & Samaranayake, 1991)."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 369	"Some researchers have suggested that individuals with math disabilities have a deficit in some aspect of central executive processing that is domain-general. . . . Researchers have also found that children with math disabilities may have domain-specific deficits, showing reduced ability to process information that is specifically numerical. . . ."
Geary, 2003b, 199	"The complexity of the field of mathematics makes the study of any associated learning disability daunting."

Mathematics's Supporting Competencies: Concepts and Procedures

Cognitive psychologists have long made a "distinction between two different types of propositional networks within linguistic thought: declarative networks and procedural networks" (Marzano, 1998, p. 17), so Geary and Hoard's model is in line with the research on both of these areas of learning. According to Marzano's study of the research, "the distinction between declarative and procedural knowledge, or more simply, content knowledge and process knowledge is one of the most basic in terms of guiding educational practice" (Snow and Lohman, 1989, p. 266). "Declarative knowledge," according to Marzano (1998), "is informational in nature" (p. 18), while "procedural knowledge" is "process knowledge, both mental and physical" (p. 18). Geary and Hoard's (2005) model on mathematics disabilities, therefore, includes two supporting competencies in mathematics: conceptual and procedural. Siegler (2003) identifies the supporting competencies similarly:

> . . . from the preschool years onward, children learn abstract mathematical concepts and principles, as well as procedures and facts. Fairly often, however, they either fail to grasp the concepts and principles that underlie procedures or they grasp relevant concepts and principles but cannot connect them to the procedures. Either way, children who lack such

> understanding frequently generate flawed procedures that generate systematic patterns of errors (p. 221).

The interconnectedness and critical importance of *both concepts and procedures* in curriculum are repeated throughout this study of the scientific evidence grounding *MLS*. *MLS'* content focuses primarily on concept development *and* procedures, including fluency in fact retrieval. Chapter IV will include more research related to the importance of concepts and procedures in mathematics curricula in general, and specifically in *MLS*, and Chapters V-VI will include discussion of the research-based strategies that are used in *MLS* to develop concepts and procedures.

Table 24 presents the conclusions of various researchers relative to disabilities in learning mathematical concepts. Some focus on the resulting poor performance of learners without strong conceptual understandings; others also note that procedural errors are frequently the result of poor conceptual understandings—again the recognition that both of the supporting competencies are necessary to develop proficiency in any mathematics domain.

Table 24: Disabilities in Learning Mathematical Concepts

Researcher(s)	Findings/Conclusions
Irish, 2002, 3	"Many students with learning and cognitive disabilities struggle with the attainment of math concepts and performance in mathematics (Carnine, Jones, & Dixon, 1994; Parmar, Cawley, & Frazita, 1996). Most students with learning disabilities (LD) tend to make mathematic progress at a rate that is approximately one-half that of their average achieving peers (Cawley & Miller, 1989), and students with mild cognitive disabilities (CD) progress through the developmental levels in the same sequence but at a slower rate and level (Zigler, 1969). Students with LD struggle with most aspects of learning in mathematics (Mastropieri, Bakken, & Scruggs, 1991; Mastropieri, Scruggs, & Shiah, 1991). By definition individuals with CD and LD cannot keep up with their regularly achieving peers (Cawley & Frazita, 1996; Ohio Department of Education, 1982; Zentall, 1990; Zentall & Ferkins, 1993) and their understanding and demonstration of basic concepts is typically weak (Barron, Bransford, Kulewicz, & Hasselbring, 1989)."
Geary, 2004, 6	"In summary, many children with MLD, independent of their reading achievement levels or IQ, have a poor conceptual understanding of some aspects of counting. These children understand most of the inherent counting rules identified by Gelman and Gallistel (1978), such as stable order and cardinality, but they consistently err on tasks that assess order irrelevance or adjacency from Briars and Siegler's (1984) perspective. It is not currently known whether the poor counting knowledge of children with MLD/RD or MLD only extends beyond the second grade."
Dehaene, Piazza, Pinel, & Cohen, 2005, 445	". . . addition performance cannot dissociate from both subtraction and multiplication. . . . That is to say, a patient cannot be impaired in addition but not in subtraction nor in multiplication (since the latter would imply that both the verbal and the quantity circuits are intact), nor can a patient show preserved addition with impaired subtraction and multiplication (since the latter would imply that both systems are impaired)."
Landerl, Bevan, & Butterworth, 2004, 122	"We suggest that the key deficit in developmental dyscalculia is a failure to represent and process numerosity in a normal way. Numerical expressions do not seem to have the same meaning for these children, as is evidenced by the relative difficulty they have with the number comparison and dot counting."

Researcher(s)	Findings/Conclusions
Landerl, Bevan, & Butterworth, 2004, 122	"In conclusion, the most likely candidate for an underlying cause of dyscalculia is a congenital failure to understand basic numerical concepts, especially the idea of numerosity, a capacity which is independent of other abilities."
Siegler, 2003, 222	"Much of children's difficulty in fractional arithmetic arises from their not thinking of the magnitude represented by each fraction. This is evident in children's errors in estimating the answer to 12/13 + 7/8. On a national achievement test, fewer than one-third of U.S. 13-year-olds accurately estimated the answer to this simple problem (Carpenter, et al., 1981). Yet how could adding two numbers that were each close to 1 result in a sum of 1, 19, or 21?"
Miller & Mercer, 1997, 4	"Many students with disabilities have histories of academic failure that contribute to the development of learned helplessness in math (Parmar & Cawley, 1991). It is postulated that learned helplessness in math results from youngsters repeatedly trying to solve problems when they have little or no understanding of mathematical concepts. . . ."
National Research Council, 2001, 199	" . . . research has shown that it is difficult to develop procedural fluency with multidigit arithmetic without an understanding of the base-10 system. If such understanding is missing, students make many different errors in multidigit computation."
Geary, 2003b, 206-207	"In addition to working memory, a poor understanding of the concepts underlying a procedure can also contribute to a developmental delay in the adoption of more sophisticated procedures and reduce the ability to detect procedural errors."
Sherman, Richardson, & Yard, 2005, 19	"In general, conceptual misunderstandings occur when students lack fundamental understanding and experience with positional systems (Kamil, 1986). Learners struggle with trading groups for collections of groups, such as regrouping 10 tens for one hundred. There is a lack of understanding of the place value structure, that is, multiplying each place value position to the left of a number by the base (such as 10) and dividing each place to the right of the decimal point by the base."
Woodward & Montague, 2002, 22	". . . an extensive analysis of worksheets from over 400 middle school students with learning disabilities from three school districts suggests that the majority of students will have not mastered algorithms for a beginning operation like subtraction, a skill that most of the students had been practicing for five or six years (Woodward & Howard, 1994). Lack of mastery may be attributed to the highly procedural nature of the instruction that occurs in special education classrooms (Parmar & Cawley, 1991). Without a substantive and persistent link to the conceptual underpinnings of these algorithms, the chances of error increase considerably (Hasselbring, Bottge, & Goin, 1992; Hiebert, 1986)."

These findings make it evident, therefore, that a quality mathematics intervention must focus, at least in part, on concept development, as *MLS* does (see Chapter IV for discussion.

Marzano (1998) makes a point that is repeated consistently in the research on mathematical procedures:

> One distinguishing characteristic of procedural networks is that their effectiveness is a function of the extent to which they have been internalized to the level of automaticity. . . . the more an individual has practiced the mental process of long division (i.e., a procedural network) until he can use it with little conscious effort, the more useful the procedure will be to him (p. 19).

Table 25 includes the findings of several researchers relating to disabilities that affect students' ability to learn a variety of mathematical procedures, the second of the supporting competencies.

Table 25: Disabilities in Learning Mathematical Procedures

Researcher(s)	Findings/Conclusions
Spear-Swerling, n.d., 1	"Scientific investigators interested in learning disabilities have identified several patterns that may be found in youngsters with math disabilities. Some of these children have difficulties that revolve primarily around automatic recall of facts, coupled with good conceptual abilities in mathematics; this pattern characterizes some children with reading disabilities. Another common pattern involves difficulties with computational algorithms; yet a third pattern involves visual-spatial difficulties, such as difficulty lining up columns or with learning spatial aspects of math, such as geometry."
Jones, Wilson, & Bhojwani, 1997, 151	". . . McLeod and Armstrong found that secondary students with LD had difficulty with basic operations, percentages, decimals, measurement, and the language of mathematics."
Dowker, 2004, 8	"Bryant, Bryant, and Hammill (2000) found that several difficulties were common in children with mathematical weaknesses, but that the commonest problem was a difficulty in carrying out multi-step arithmetic."
Kroesbergen, 2002, 4	"A second characteristic is that students with difficulties learning math often show inadequate use of strategies to compute answers or solve word problems. This can be explained at least in part by the aforementioned memory deficits, which produce slower development of the relevant strategies than in normal achieving students (Rivera, 1997) . . . ."
Geary, n.d., 4	"The other consistent finding is that many children with MD use immature problem-solving procedures to solve simple arithmetic problems, that is they use procedures that are more commonly used by younger children without MD."
Mazzocco & McCloskey, 2005, 271	"Examples of math-specific skills are counting, cardinality, arithmetic fact retrieval, and calculation procedure skills; these may be differentially spared or deficient in persons with different MD subtypes."
Butterworth, 2005, 460	"Geary (1993) notes that DD [developmental dyscalculics] children have two basic functional, or phenotypic, numerical deficits: (1) the use of developmentally immature arithmetical procedures and a high frequency of procedural errors; (2) difficulty in the representation and retrieval of arithmetic facts from long-term semantic memory."
Landerl, Bevan, & Butterworth, 2004, 100	"A second feature of children with dyscalculia is difficulty in executing calculation procedures, with immature problem-solving strategies, long solution times and high error rates (Geary, 1993)."
Landerl, Bevan, & Butterworth, 2004, 101	"Geary (1993) suggests that procedural problems are likely to improve with experience, whereas retrieval difficulties are less likely to do so. Geary proposes that this dissociation emerges because procedural problems are due to lack of conceptual understanding, while retrieval difficulties are the result of general semantic memory dysfunction. However, it is possible that both difficulties result from a lack of conceptual understanding. It may be easier for a child to memorise one or two meaningless procedures than the multitude of arithmetic facts (from simple number bonds to multiplication tables) which, without understanding of cardinality, are simply unrelated word strings."
Geary & Hoard, 2005, 261	". . . a common procedural deficit of children with MD [mathematical disabilities] involves use of developmentally immature strategies, such as sum counting and finger counting, and miscounting when using these procedures to solve simple arithmetic problems. Potential sources of these procedural deficits include (a) a poor conceptual understanding of counting concepts. . . or (b) poor working memory/central executive resources."

Researcher(s)	Findings/Conclusions
Stotsky, S., 2005, 2	"Children who do not master the standard algorithms begin to have problems as early as algebra I."
Geary, 2003b, 208	"Disruptions in the ability to retrieve basic facts from long-term memory might, in fact, be considered a defining feature of arithmetic disabilities. Most of these individuals can, however, retrieve some facts, and disruptions in the ability to retrieve facts associated with one operation (e.g., multiplication) are sometimes found with intact retrieval of facts associated with another operation (e.g., subtraction), at least when retrieval deficits are associated with overt brain injury (Pesenti, Seron, & Van Der Linden, 1994)."

Just as strongly as the research indicates the importance of concept development is the emphasis on procedural fluency in learning mathematics—again, one of the strong features of *MLS*. (See discussions of procedures in Chapters IV, V, and VI.)

Disabilities in Underlying Cognitive Systems

Geary and Hoard's (2005) model for understanding mathematics disabilities includes "underlying cognitive systems" as origins for learning problems. The first of these systems is central executive, which governs "attentional and inhibitory control of information processing." Kandel's (2006) research also indicates that "the brain stem regulates attentiveness" (p. 44).

Underlying Cognitive Systems
Central Executive Attentional and Inhibitory Control of Information Processing

Central executive is that system that enables people to establish goals, to organize, to sequence, to follow directions, to stay focused, to inhibit extraneous information or distractions, to stay on task, to self-monitor, to multi-task, to control emotional responses—what some psychologists call "habits of mind," all essential to effective and efficient learning. Marzano (1992) reflects that

> The mental habits of self-regulation, critical thinking, and creative thinking permeate virtually every academic task students undertake. Being, or not being, self-regulated, critical, and creative affects how well students acquire and integrate knowledge. Being, or not being, self-regulated, critical, and creative affects how well they extend and refine knowledge. And it affects how well they make use of their knowledge. To this extent, the habits of mind are like attitudes and perceptions. To this extent, when they are negative or weak, they hamper students' ability to learn; when they are positive or strong, they improve students' ability to learn (pp. 134-135).

Marzano writes about the need to teach and reinforce good habits of mind for all students. He says,

> . . . students rarely see these habits of mind being used in the world around them. Few people plan and manage resources well. Few people seek clarity or accuracy. Few people work at the edge, rather than the center, of their competence. In fact, it is rather remarkable how infrequently human beings use these mental habits (p. 135).

If the general populace seems weak in the use of central executive, then learners with disabilities in this system are even more dramatically dysfunctional.

Table 26 includes the research findings on the effects on mathematics achievement of a dysfunction in the central executive system.

Table 26: Effects of Central Executive Disabilities on Mathematics Achievement

Researcher(s)	Findings/Conclusions
McGuinness, 1997, 169	"When children are highly distractible, overly disruptive, and unable to stay 'on task,' this usually means they can't do the task."
Butterworth, 2005, 459	"Even when they think they understand something, the slightest distraction causes them to lose track."
Miller & Mercer, 1997, 5	"Attention deficits: 1. Student has difficulty maintaining attention to steps in algorithms or problem solving. 2. Student has difficulty sustaining attention to critical instruction (e.g., teacher modeling). Memory problems: 1. Student is unable to retain math facts or new information. 2. Student forgets steps in an algorithm. 3. Student performs poorly on review lessons or mixed problems. 4. Student has difficulty telling time. 5. Student has difficulty solving multi-step problems."
Miller & Mercer, 1997, 6	"In addition to the general learner characteristics. . ., students with learning disabilities also have difficulty with cognitive and metacognitive processes. . . . Specifically, these students are described as having difficulty in (1) assessing their abilities to solve problems, (b) identifying and selecting appropriate strategies, (c) organizing information, (d) monitoring problem-solving processes, (e) evaluating problems for accuracy, and (f) generalizing strategies to appropriate situations. . . ."
Levine, n.d., 3	"A kid may look lazy or she has lost motivation. Some kids look lazy when they really have attentional difficulties that make it extremely hard for them to concentrate."
Levine, n.d., 3	"Most of the time, when kids are bored in school, it is either because they are having trouble with their attention or because they don't fully understand what is going on."
Kroesbergen, 2002, 4	"A third general characteristic of students with difficulties learning math is deficits in other metacognitive regulation processes such as the organization, monitoring, and evaluation of information (Mercer, 1997). As a result of these deficits, the students often produce mistakes showing the incorrect application of solution . . . . "In addition to these general (meta)-cognitive characteristics of students with difficulties learning math, they frequently have other problems such as attention deficits or motivational problems."
Geary, 2004, 9	"As with all competencies that engage working memory, deficits in the central executive, such as poor attentional control, can also disrupt the executive of mathematical procedures (Hitch, 1978)."

Researcher(s)	Findings/Conclusions
Geary & Hoard, 2005, 260	"The central executive controls the attentional and inhibitory processes needed to use procedures during problem solving, and much of the information supporting conceptual and procedural competencies is likely to be represented in the language of visuospatial systems, although a distinct modular system for arithmetic has been proposed."
Geary, 2003a, 458	"The most consistently found deficit in children with MD is in the ability to quickly and accurately retrieve basic arithmetic facts from long-term memory (LTM). These children retrieve fewer facts than their academically normal peers do, and they quickly forget many of the facts that they do learn. When they can retrieve facts from LTM, children with MD make many more retrieval errors and sometimes show unusual reaction time patterns, in relation to their academically normal peers (Geary, 1993). For many of these children, the retrieval deficit appears to reflect a persistent, perhaps lifelong, disability (e.g., Ostad, 1997). The source of this deficit is currently unknown, although there appear to be two contributing factors. Some of these children appear to have difficulties inhibiting irrelevant associations during the retrieval process . . . . For instance, when asked to determine the sum of 3 + 6, many of these children appear to recall 4, 7, and 9; 4 and 7 are the numbers following 3 and 6 in the counting string. These three 'answers' then compete for expression, which, in turn, disrupts reaction times and results in more retrieval errors (Geary et al., 2000).
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 369	"Children with arithmetic disabilities have been shown to have particular difficulty with task switching. . . and with inhibiting irrelevant information."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 363	"Processes that have been attributed to the central executive include inhibition of irrelevant information, task switching, information updating, goal management, and strategic retrieval from long-term memory. . . ."
Ontario Ministry of Education, 2005, 42	"A child's ability to exhibit self-regulatory behaviours is an important component of academic success (Zimmerman, 2000). Self-regulatory behaviours or executive functions are those cognitive processes that support strategic and goal-oriented behaviour. These cognitive processes can include both cognitive control functions (e.g., planning, organizing, monitoring) and emotional control (regulating emotional responses) (Gioia & Isquith, 2004).
Geary & Hoard, 2002, 109	"Research on the working memory deficits of children with MD/RD or MD only is still in preliminary stages, but suggests the following: The primary deficit of children with MD only may involve the central executive. When solving simple arithmetic problems, the result is retrieval deficits due to the intrusion of irrelevant associations and poor skill at using counting procedures during problem solving, presumably due to difficulties in monitoring the act of counting."
Dehaene, Piazza, Pinel, & Cohen, 2005, 447	". . . attentional and numerical systems are dissociable."
Landerl, Bevan, & Butterworth, 2004, 104	"Other conditions which have been associated with dyscalculia are ADHD (Badian, 1983; Rosenberg, 1989; Shalev et al., 1997), poor hand-eye co-ordination (Siegel & Feldman, 1983), poor memory for non-verbal material (Fletcher, 1985), and poor social skills (Rourke, 1989)."
Levine & Schwartz, n.d., 2	"An individual's neurodevelopmental function is likely to contribute to learning across a range of performance areas. For example, the ability to retain sequences of data in short-term memory is a neurodevelopmental function that plays a role in following directions, acquiring procedural knowledge in mathematics, and remembering a person's telephone number."
Tannock & Martinussen, 2001, 15	"Current theories propose that the behavioral symptoms of ADHD are not primary features of the disorder but are attributes to underlying deficits in cognitive control processes that guide both behavior and cognitive functioning.

Researcher(s)	Findings/Conclusions
Geary, 2004, 12	"More recent studies of children with MLD have suggested a second form of retrieval deficit—specifically, disruptions in the retrieval process due to difficulties in inhibiting the retrieval of irrelevant associations."
Kandel, 2006, 311	"The brain's capacity for processing sensory information is more limited than its receptors' capacity for measuring the environment. Attention, therefore, acts as a filter, selecting some objects for further processing. . . . This focusing of the sensory apparatus is an essential feature of all perceptions."
Kandel, 2006, 313	". . . selective attention is critical to the unitary nature of consciousness."

Information Processing

Geary and Hoard's (2005) model describing mathematics disabilities also includes two areas of information processing under the general heading of "underlying cognitive systems":

Language System		**Visuospatial System**	
Information Representation	Information Manipulation	Information Representation	Information Manipulation

Kandel (2006), a 2000 Nobel Prize winner in Physiology/Medicine, explains the origin of the research that developed the theory of information processing and continues to validate it and expand scientific understandings about how people learn:

> In the 1970's cognitive psychology, the science of mind, merged with neuroscience, the science of the brain. The result was cognitive neuroscience, a discipline that introduced biological methods of exploring mental processes into modern cognitive psychology (p. 7).

Educators are generally familiar with some of the basic concepts of information processing. According to Sternberg (2003), "Information processing theorists seek to understand cognitive development in terms of how people of different ages process information (i.e., decode, encode, transfer, combine, store, retrieve it), particularly when solving challenging mental problems" (p. 462). He further explains that "Any mental activity that involves noticing, taking in, mentally manipulating, storing, combining, retrieving, or acting on information falls within the purview of information processing approaches" (p. 462).

New information comes to the brain through one or more of the senses (visual, auditory, kinesthetic, olfactory, tactile). Simply put, it is very temporarily parked in a storage area called short-term memory. It is then quickly filtered to determine whether it is related to any previous knowledge or skill, whether it makes sense given what else is known, and how well it fits into what is valued by the learner. If the new information is not filtered out, given opportunities to practice or rehearse it—to process it—it then enters into long-term memory for more permanent storage. Kandel (2006) explains as follows:

> For a memory to persist, the incoming information must be thoroughly and deeply processed. This is accomplished by attending to the information and associating it meaningfully and systematically with knowledge already well established in memory (p. 210).

There is another storage area in the brain termed working memory. Working memory is the area where stored knowledge/skill from long-term memory is retrieved for temporary consideration, as well as where short-term memory is held for consideration through conscious use of strategy (such as verbal rehearsal of multiplication facts). There are various theories and models about how the information processing model actually works, where in the brain the different information is stored, and how connections between what is learned are made, but research is moving those theories to converge.

One thing that is known is that storage space is limited in both short- and working-memory. Research has established that the average person can hold only about nine items at the most in short-term memory and that the information quickly decays (Sternberg, 2003, 155). That is why social security numbers are only nine digits, telephone numbers are only seven (or ten with the area code), and zip codes are only five (or nine with the suffix). Through memory devices such as chunking or clustering, more items can be remembered. The ten-digit telephone number is chunked into three sets—the three-digit area code, the three-digit prefix, and then the four-digit number. The length of time that items stay in short-term memory is typically only seconds—perhaps up to a couple of minutes, unless there is input or output interference, and then the life of the information diminishes rapidly (Sternberg, p. 157).

The goal of *MLS* and, indeed, of all instruction is to move new information and skills into long-term memory as efficiently as possible so that it can be retrieved at will and applied to new situations. One of the theories about how that is done is called the "levels-of-processing framework," originally proposed, according to Sternberg, by Fergus Craik and Robert Lockhart (1972) (p. 158). This framework sees knowledge along a continuum "in terms of depth of encoding" (p. 159). In other words, "the deeper the level of processing, the higher, in general, the probability that an item may be retrieved" (p. 159). Deeper levels of processing means encoding information in multiple modalities, not just in the modality of the students' preference or the teacher's preference. It is easier to retrieve information from a modality if it has been encoded there, so learning in multiple modalities provides the learner more flexibility in retrieval of anything stored in long-term memory.

The levels-of-processing framework includes three levels: physical, acoustic, and semantic. As applied to mathematics, the physical level includes the features of number representations and shapes—or the visuospatial system as described in the Geary and Hoard (2005) model. The sounds of number names, shapes, and so forth comprise the acoustic level, which is a part of the language system and includes phonological processing. The semantic level includes the meaning of mathematics vocabulary and the conceptual understandings, also located in the language system. Chapter VI includes the research on multi-sensory processing and the specific ways in which it is used in *MLS*. Table 27 includes definitions of information processing, making it clear that the root cause of many (but not all) learning disabilities in mathematics is faulty sensory processing in the language and visuospatial systems.

Table 27: Definitions of Information Processing

Researcher(s)	Findings/Conclusions
National Center for Learning Disabilities, n.d., 1	"While there are several different and often overlapping types of information processing, two important groups are: visual processing (visual discrimination, visual sequencing, visual memory, visual motor processing, visual closure, and spatial relationships) and auditory processing (auditory discrimination, auditory memory, and auditory sequencing)."
National Center for Learning Disabilities, n.d., 1	"An information processing disorder is a deficiency in a person's ability to effectively use the information the senses have gathered. It is NOT the result of hearing loss, impaired vision, an attention deficit disorder or any kind of intellectual or cognitive deficit.
Kandel, 2006, 79	". . . the nature of the information conveyed depends on the type of nerve fibers that are activated and the specific brain systems to which these nerve fibers are connected. Each class of sensation is transmitted along specific neural pathways, and the particular kind of information replayed by a neuron depends on the pathway of which it is a part. In a sensory pathway, information is transmitted from the first sensory neuron—a receptor that responds to an environmental stimulus such as touch, pain, or light—to specific and specialized neurons in the spinal cord or in the brain. Thus visual information is different from auditory information because it activates different pathways."
Sternberg, 2003, 462	"Any mental activity that involves noticing, taking in, mentally manipulating, storing, combining, retrieving, or acting on information falls with the purview of information processing approaches."
National Center for Learning Disabilities, n.d., 1	"Though information processing disorders are often not named as specific types of learning disabilities, they are seen in many individuals with learning disabilities and can often help explain why a person is having trouble with learning and performance. The inability to process information efficiently can lead to frustration, low self-esteem and social withdrawal, especially with speech/language impairments."
Kroesbergen, 2002, 2	"The behavioral and cognitive frameworks constitute the major paradigms for studying the phenomenon of human learning. "Behaviorists recognize the existence of several different stages of learning: acquisition, proficiency, maintenance, generalization, and adaptation. Given the behaviorist's emphasis on the environment as a critical factor for learning, considerable emphasis is also placed on the teacher's arrangement of the classroom for learning. . . . One of the essential components of the behavioral approach to learning is direct instruction. The key principle underlying direct instruction is that both the curriculum materials and the teacher presentation of these materials must be very clear and unambiguous. This includes an explicit step-by-step strategy, development of mastery at each step in the learning process, strategy corrections for student errors, gradual fading of teacher-directed activities and increased independent work, use of systematic practice with an adequate range of examples, and cumulative review of newly learned concepts. "The cognitive approach involves the study of the human mind, and developed a model for how people receive, process, and recall information. . . . The main theory within the cognitive approach is the information-processing theory, which construes learning as the process of obtaining, coding, and remembering information."
Kandel, 2006, 116	"In a large sense, learning and memory are central to our very identity. They make us who we are."

Language System Disabilities

Table 28 includes the conclusions of researchers relating to the incidence of language system disabilities and their effects on mathematics achievement. Many of the references relate to issues involving working memory deficits and their effects on the application of procedural knowledge and in problem solving. Geary (Feb. 2004) explains as follows:

> Although the relation between working memory and difficulties in executing arithmetical procedures is not yet fully understood, it is clear that children with MLD have some form of working memory deficit (Hitch & McAuley, 1991; McLean & Hitch, 1999; Siegel & Ryan, 1989; Swanson, 1993) . . . . this deficit appears to involve information representation and manipulation in the language system—that is, the systems that support the representation and articulation of number words and that support associated procedural competencies, such as counting (p. 9).

Table 28: General Effects of Language System Disabilities

Researcher(s)	Findings/Conclusions
Bell, 2003, 2	"Mathematics is the essence of cognition. It is thinking (dual coding) with numbers, imagery, and language; reading/spelling is thinking with letters, imagery, and language. Both processes, often mirror images of each other, require the integration of language."
Stern, 2005, 458	"In mathematics, as in any other subject, language is a vehicle for thought; therefore, students need many opportunities to put newly discovered concepts into their own words."
Geary, 2004, 9	"Working memory may also contribute to the tendency of children with MLD to undercount or overcount—the source of their counting procedure errors (Geary, 1990; Hanich et al., 2001)—during the problem-solving process. Such miscounting can occur if the child loses track of where he or she is in the counting process—that is, how many fingers he or she has counted and how many remain to be counted. These deficits could be due to difficulties with information representation in the language, specifically the phonetic-articulatory system, or from a deficit in accompanying executive processes, such as attentional control (see McLean & Mitch, 1999). If the phonetic representations of number words fade more quickly or do not achieve typical levels of acoustical fidelity, then manipulating these presentations in working memory, as with counting, will be difficult for children with MLD (Geary, 1993)."
Dehaene, Piazza, Pinel, & Cohen, 2005, 439	". . . small parietal lesions can severely impact on the understanding of numbers and their relations while sparing other aspects of language and semantics. . . . In many cases, the deficit can be extremely incapacitating. Patients may fail to compute operations as simple as 2 + 2, 3 – 1, or 3 x 9."
Bruer, 1993, 104	"Students with learning disabilities have the usual response to word problems; they don't understand them, and they often use key-word strategies to guess which mathematical operation to apply. But these students also have other difficulties. In sixth grade, many have not learned elementary math facts and use counting procedures, like preschool children, to solve simple problems. Many of these children are poor readers, which makes word problems even more difficult. Often teachers think that solving word problems is beyond these students' capabilities."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 371	"Research consistently shows that storage capacity is a factor in mathematics performance."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 367	". . . phonological, visual, and spatial codes all seem to be implicated in mathematical tasks."
Ontario Ministry of Education, 2005, 38	"Many children with exceptionalities are slow in processing information. In general, they may have difficulty keeping up with the pace of language spoken and instructions delivered in the classroom. They may be slow at reading words and text. Their basic arithmetic skills may lack fluency. They may take more time than expected to copy information from the board or a book, and their written work may be laborious."

Researcher(s)	Findings/Conclusions
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 370	". . . children with math disabilities find speeded math tasks particularly difficult."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 371	"Children may fail on some problems due to capacity limitations, despite adequate conceptual knowledge . . . ."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 372	". . . a role for interactions between developing storage capacity and processing efficiency and consequent changes in working memory capacity is supported for mathematical cognition. . . ."
Erlauer, 2003, 13	"The working memory can hold a small amount of information just long enough to determine if it is knowledge that is important or worthy of being remembered for a longer period of time."
Fazio, 1999, 421	". . . the primary difficulty of preschool children with SLI [specific language impairment] was remembering and retrieving counting words in the correct sequence. It appears that the serial nature and the phonological characteristics of the number word sequence placed heavy demands on information processing resources."
Fazio, 1999, 421	"The findings of the second study suggested that memory problems of children with SLI [specific language impairment] contributed to delays in learning new rote material."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 369	"Math disabilities have been linked to deficits in working memory. . . . A role for a variety of possible links between math disability and working memory is supported by the multi-dimensional nature of math disabilities."
Landerl, Bevan, & Butterworth, 2004, 121	"In summary, the dyscalculic children identified in this study demonstrated general deficits in number processing, including accessing verbal and semantic numerical information, counting dots, reciting number sequences and writing numbers."
Wolfe, 2001, 92	"Working memory allows us to integrate current perceptual information with stored information, and to consciously manipulate the information (think about it, talk about it, and rehearse it) well enough to ensure its storage in long-term memory."
Barnes, Smith-Chant, & Landry, 2005, 303	"Models of math disability specify subtypes that are characterized by deficits in different core cognitive and neuropsychological processes and that are related in different ways to reading."
Siegler, 2003, 225	"Another key difference involves working memory capacity. Learning of arithmetic requires sufficient working memory capacity to hold the original problem in memory while computing the answer so that the problem and answer can be associated. However, children labeled as mathematically disabled cannot hold as much numerical information in memory as age peers (Geary, Bow-Thomas, & Yao, 1992; Koontz & Berch, 1996)."
Ontario Ministry of Education, 2005, 40-41	"The most common memory difficulty experienced by children with special needs is in what is called working memory (Siegel, 1994). Working memory refers to a 'mental workspace' in which the student can store and manipulate information for brief periods of time in order to perform another cognitive activity. When working memory is limited, the student will have difficulty keeping in mind multiple pieces of information while carrying out a task. He or she may not be able to carry out the task or monitor performance for errors. "Working memory plays a role in a range of academic activities such as mental calculation, math problem solving, language and reading comprehension, and writing (Baddeley, Emslie, Kolodny, & Duncan, 1998; Bull & Scerif, 2001; Daneman & Carpenter, 1980; Gathercole & Pickering, 2000; Swanson, 1999). ". . . Difficulties in long-term retention may . . . reflect incomplete learning of new concepts, which makes these concepts vulnerable to forgetting. Or such difficulties may reflect insufficient practice with new concepts and skills, which is what consolidates new learning in memory.

Researcher(s)	Findings/Conclusions
Geary, 2003a, 459	". . . Donlan found that young children with SLI [specific language impairment] had age-appropriate number and arithmetic skills in some areas (e.g., conceptual understanding of counting) and deficits in others. Moreover, the deficits largely involved numerical processes, such as the use of verbal counting procedures to count objects, that are dependent on the language system, in general, and the phonological loop, in particular."
Geary & Hoard, 2002, 107	". . . RD is associated with disrupted phonetic processing (e.g., Morrison et al., 1998), specifically poor activation of phonetic representations of familiar information."
Geary & Hoard, 2002, 107-108	". . . children with RD, independent of their mathematics achievement scores, are slower at or have more difficulties in accessing familiar information in long-term memory, in keeping with findings that the phonetic system is disrupted in most children with RD (e.g., Morrison et al., 1998).
Sousa, 2001, 94	"Recent studies of young children with language-learning difficulties indicate that they may have a dysfunction in brain-timing mechanisms, which makes processing of certain speech sounds difficult. Researchers discovered that by using computer-processed language programs that pronounced words more slowly, some children (ages 5 to 10) were able to advance their reading levels by two years after just four weeks of training. This improvement was maintained for at least a year."
Miles, T., 1992a, 7	"A likely interpretation seems to be that an adequate immediate memory is a necessary condition for mathematical progress; and since dyslexics are known to be weak in this area their weakness at mathematics would in that case not be at all surprising."
Marzano, 1998, 13	"It is probably safe to say that the linguistic mode is the one that receives the most attention from an educational perspective. Knowledge is most commonly presented linguistically and students are most commonly expected to respond linguistically."

Dyslexia and Mathematics

Because so many celebrities, including Tom Cruise, Jay Leno, and Whoopi Goldberg, have been willing in recent years to discuss their struggles in school due to dyslexia, virtually everyone knows something about this disorder. And virtually everyone believes it to result in achievement problems in reading, writing, and/or spelling. And that is, of course, true.

Popular media extend these understandings. The July 28, 2003, issue of *Time* includes a feature section on dyslexia. Gorman writes: ". . . a growing body of scientific evidence suggests there is a glitch in the neurological wiring of dyslexics that makes reading extremely difficult for them" (p. 52). And he continues:

> Fortunately, the science also points to new strategies for overcoming this glitch. The most successful programs focus on strengthening the brain's aptitude for linking letters to the sounds they represent (p. 52).

And *Time* is right as well. CEI long ago figured that out, and that is one of the major reasons, according to staff, that *Essential Learning Systems (ELS)* was created almost 20 years ago.

These understandings are also institutionalized in educational policy. For instance, the Texas Education Agency's 2001 *Dyslexia Handbook* defines "dyslexia" as "a disorder of constitutional origin manifested by a difficulty in learning to read, write, or spell, despite conventional instruction, adequate instruction, and sociocultural opportunity" (p. 1). Mississippi's definition in their *Dyslexia Handbook* is almost exactly the same.

It may come as a surprise to many, therefore, that dyslexic students almost invariably also struggle with mathematics—not all mathematics, but some areas of mathematics. The International Dyslexia Association (1998) says that "Not all individuals with dyslexia have problems with mathematics, but many do" (p. 1). The problems that are manifested, of course, are those cited in the literature on disabilities that lie chiefly in the language system, but also in the central executive, and visuospatial system.

One major area of mathematics difficulty among dyslexics is vocabulary—related, of course, to reading and intricately related to concept development. Mathematical terms may be especially challenging (E. Miles, 1992b, p. 59) since they frequently do not hold the same meanings as those same terms in informal language (e.g. "mean," "median," "acute," "difference," "odd," etc.). Another problem is the complexity of mathematical terms. According to Barton and Heidema (2002), "mathematics tests contain more concepts per word, per sentence, and per paragraph than any other kind of text. In addition, these concepts are often abstract, so it is difficult for readers to visualize them" (p. 2).

Again, related to reading, is the problem of decoding. In mathematics students must decode not only words in a text or in a word problem, but they must also decode numeric and nonnumeric symbols (Barton & Heidema, 2002, p. 2; Miller & Mercer, 1997, p. 6). Miller and Mercer (1997) point out that "Irrelevant numerical and linguistic information in word problems is especially troublesome for many students with learning disabilities" (p. 6).

Several researchers have found that virtually all dyslexics have difficulties in fact retrieval, especially in learning multiplication tables (Chinn & Ashcroft, 1992, pp. 98-99; Dowker, 2004, p. 11; T. Miles, 1992a, pp. 5, 11, 13; LeFevre, DeStefano, Coleman, & Shanahan, 2005, p. 370; Geary & Hoard, 2005, p. 261; Pennington, 1991, p. 68).

Another issue for dyslexics is sequencing. At the most basic level, Kibel (1992) notes the following: "If mathematics is taught through the medium of language, if children are taught what to do and expected to remember a sequence of verbal instructions, then dyslexic children are going to find this hard. We are asking them to rely on an area in which we know they are cognitively weak" (p. 44). Dowker (2004) makes similar points. He advises that "language difficulties will directly affect the child's ability to benefit from oral or written instruction. . ." (p. 12). Kibel provides specifics:

> Dyslexics have difficulty with sequencing. In mathematics, the algorithms are often long sequences of fairly meaningless operations, and these usually have to be memorized in words. Children forget. They mix operations. They often resort to rows of tiny dots and tally marks in an attempt to find a way around the difficulty (p. 52).

Similarly, Pennington (1991) notes that dyslexics sometimes "missequence numbers they write," just as they sometimes "missequence" letters in a word (p. 112). The answer to this problem is the use of manipulatives. Kibel (1992) notes that using words to explain a procedure only makes things worse and adds to the student's confusion. Rather, he recommends, "It may be better to present the problem in a concrete form and allow him to see the relationships in this way" (p. 55).

Direction is a related problem to sequencing for learners with dyslexia. For instance, E. Miles (1992b) explains that

> Particular difficulties will also arise from the dyslexic's confusion over direction and his general inflexibility of approach. In following a text in a reading book, the pupil has been taught to move from left to right. In mathematics, he must be flexible, depending on the operation required . . . ." (p. 63).

According to E. Miles, "Difficulty over direction often gets dyslexics into trouble over the signs for 'greater than' and 'less than,' namely, > and <" (p. 67). Henderson (1992) adds that both sequencing and direction problems are "problems of short-term memory" (p. 71).

"Position is even more important in mathematics than it is in spelling," (p. 63) writes E. Miles (1992b). Students must learn that the meaning of a number depends upon its position in a string of numbers. Henderson (1992) warns that "Another difficulty for dyslexics is the recognition of the decimal point within a number. One thing that can go wrong is that the comma "dividing off the thousands is often mistaken to be the part which he finds most difficulty" (p. 73).

In summary, then, dyslexic students will obviously have problems with decoding and vocabulary since they are closely related to reading. Their low performance in mathematics also indicates a need for an effective mathematics intervention to address fact fluency—especially in multiplication—as well as sequencing, direction, and position.

McEwan (2000, p. 72) summarizes a study done in California about the rise in mathematics achievement scores when there had been no special efforts in that area. Rather, the teachers had been engaged for more than two years in a massive reading improvement initiative, and that work not only improved reading performance, but also mathematics performance. CEI has seen that phenomenon occur many times, according to Lesley Mullen, service manager. "Struggling students, regardless of label, often improve across the curriculum after engagement in an *ELS* lab," she said.

Table 29 includes some of the research findings relating to the causes of dyslexia.

Table 29: Causes of Dyslexia

Researcher(s)	Findings/Conclusions
Kujala, Karma, Ceponiene, Belitz, Turkkila, Tervaniemi, & Naatanen, 2001, 6	"Our results support this view that difficulties in dyslexia are based, at least to some extent, on the dysfunction of general sensory discrimination rather than on a deficit specific to phonological processing."

Researcher(s)	Findings/Conclusions
Kujala, Karma, Ceponiene, Belitz, Turkkila, Tervaniemi, & Naatanen, 2001, 2	"Neural dysfunctions underlying dyslexia are still largely unknown despite decades of research. Dyslexia has been identified as a problem of phonological processing . . ., although other difficulties like those in visual processing have also been reported. . . . Dyslexic individuals might actually suffer from a more general auditory-perception problem, which may underlie their difficulties in phonological perception . . . . For example, some authors suggest that these individuals have problems in processing temporal aspects of the speech signal, such as rapid acoustic transitions or tone-order reversals . . . . However, even some other aspects of sounds, such as rhythm or pitch, are problematic for individuals with dyslexia . . . . The evidence suggesting that these individuals have dysfunctions also in their nonlinguistic auditory and visual perception . . . supports the view that a general sensory-processing disorder is involved."

The effects of dyslexia and other reading disabilities on mathematics achievement become apparent in elementary school, as Table 30 shows.

Table 30: General Effects of Dyslexia on Mathematics Achievement

Researcher(s)	Findings/Conclusions
Jordan, Kaplan, & Hanich, 2002, 594	". . . it appears that children who start out with specific reading difficulties are at risk for developing secondary or associated mathematics difficulties as they progress through elementary school."
Geary, n.d., 5	"It appears that many—perhaps more than ½--children with MD also have difficulties learning how to read and that many children with RD also have difficulties learning basic arithmetic. In particular, children and adults with RD often have difficulties retrieving basic arithmetic facts from long-term memory."
Butterworth, 2005, 461	"Although there is a high comorbidity between numeracy and literacy disabilities, it is unclear why this should be. One possible line of argument here is that there will be a range of numerical and arithmetical tasks that depend on language, and that dyslexia is usually a deficit in language abilities that affects phonological processing. . ., which is known to reduce working memory capacity. . ., which in turn may affect lexical learning as well."
Jordan, Kaplan, & Hanich, 2002, 594	"How do reading abilities influence children's growth in mathematics? . . . Children who do not read well have less access to language, at least in its written form. Some areas of mathematics, such as word problems and number combinations, may be mediated by language (Hanich et al., 2001); Jordan, Levine, & Huttenlocher, 1995). . . . MD-only children performed better than MD-RD children on mathematics tasks that have a basis in language but not on those that rely on 'purer' numerical understanding (e.g., numerical magnitudes). "In contrast, mathematics abilities do not seem to have a significant influence on reading growth. Children with RD only grew at the same rate as children with MD-RD in reading, when IQ and income level were held constant."
E. Miles, 1992b, 58	"To speak of dyslexic children having mathematical difficulties may obscure the fact that many of these difficulties are of a linguistic nature and are therefore not unexpected in view of their particular weaknesses in the literacy field."
Garnett, 1998, 6	"Language difficulties, even subtle ones, can interfere with math learning. In particular, many LD students have a tendency to avoid verbalizing in math activities, a tendency often exacerbated by the way math is typically taught in America. Developing their habits of verbalizing math examples and procedures can greatly help in removing obstacles to success in mainstream math settings."

Researcher(s)	Findings/Conclusions
Henderson, 1992, 71	"When dealing with a dyslexic pupil the teacher should be fully conversant with the effect that his language difficulties are having on his mathematics. In the knowledge that sequencing and direction problems are problems of short-term memory will all be contributing to the pupil's learning difficulties, a teacher should be continually on the look-out for problematic areas and be ready to help with ideas and suggestions."
Henderson, 1992, 75	"Because of the dyslexic's distinctive weaknesses, the symbol/language connection needs continually to be talked about."
T. Miles, 1992b, 83	"If they [dyslexics] have been taught to merely memorize rules for operating with symbols then they are likely to find such memorization extremely difficult; and, as a further consequence, any sense of enjoyment or excitement at the elegance and beauty of mathematics will almost certainly be missing."
D'Arcangelo, 2002, 85	Interview with Brian Butterworth: "The parts of the brain that process words are different from parts of the brain that process numbers. We store words in two areas, Wernicke's area in the left temporal lobe, at least in most right-handers, and Broca's area, in the left frontal lobe. Numbers are stored in the parietal lobe—not that far away, but far enough to be a separate system. No part of the brain is specialized at birth for reading because reading is a very recent skill for which the brain adapts the language areas. The brain, however, does seem to have evolved special circuits for numbers. There's an important difference between those two types of learning. Mathematics is built on a specific innate basis, and reading is not. It's quite important for teachers to remember that when children are learning mathematics, they are using distinctly different brain areas than they use when learning to read."
Dowker, 2004, ii	"Mathematical difficulties often (by no means always) co-occur with dyslexia and other forms of language difficulty. In particular, people with dyslexia usually experience at least some difficulty in learning number facts such as multiplication tables."
International Dyslexia Association, 1998, 1	"Not all individuals with dyslexia have problems with mathematics, but many do."
Miller & Mercer, 1997, 6	". . . reading, language, and handwriting disabilities can have a strong negative influence on math performance. The heterogeneity of students with learning disabilities is apparent within math disabilities; that heterogeneity becomes even more of an issue when students without disabilities, students at risk, and students with learning disabilities and mild retardation participate continuously in the same math lessons."
International Dyslexia Association, 1998, 1	"Too frequently and too readily, individuals with dyslexia who have difficulty with mathematics are misdiagnosed as having dyscalculia—literally trouble with calculating, a neurologically based disability. True dyscalculia is rare (Steeves, 1983). 1. We know that for individuals with dyslexia, learning mathematical concepts and vocabulary and the ability to use mathematical symbols can be impeded by problems similar to those that interfered with their acquisition of the written language (Ansara, 1973). 2. Additionally, we know that the learning of mathematical concepts, more than any other content area, is tied closely to the teacher's or academic therapist's knowledge of mathematics and to the manner in which these concepts are taught (Lyon, 1996). 3. Therefore, there are individuals with dyslexia who will exhibit problems with mathematics, not because of their dyslexia or dyscalculia, but because their instructors are inadequately prepared in mathematical principles and/or in how to teach them."
Fayol & Seron, 2005, 11	"To summarize, the currently available data suggest that, in western cultures, the Arabic code is initially learned in relation to the verbal code. However, the Arabic code very quickly becomes independent of the verbal code. In normal subjects, this independence is manifested in the ability to perform better, or differently, with the former compared to the latter. It can also be seen in the vastly superior performances achieved by dysphasic children when using the Arabic code. It has been shown to exist in adult patients through the presence of double dissociations."

Researcher(s)	Findings/Conclusions
Fayol & Seron, 2005, 11	"These conclusions lead us to raise two issues. The first relates to the possibility that the Arabic code is, from the outset, associated with the analogue representation without any mediation via the verbal code. If this is the case, it would be conceivable to design a specific mode of teaching the Arabic code for dysphaic children who would then no longer be hampered by the effects induced by their language problem."
Dowker, 2004, 11	"Yeo (2001) is a teacher at Emerson House, a school for dyslexic and dyspraxic primary school children, and has written extensively about the mathematical difficulties of some dyslexic children. She reports that while many dyslexic children have difficulties only with those aspects of arithmetic that involve verbal memory, some dyslexic children have more fundamental difficulties with 'number sense.' They comprehend numbers solely in terms of quantities to be counted and do not understand them in more abstract ways, or perceive the relationships between different numbers. Yeo suggests that the counting sequence presents so much difficulty for this group that it absorbs their attention and prevents them from considering other aspects of number."
Miles & Miles, 1992, xi	". . . the difficulties experienced by dyslexics in mathematics are manifestations of the same limitation which also affects their reading and spelling. From this it follows that the appropriate teaching techniques are likely to involve the same basic principles. In the words of Ansara (1973, 120): 'The insights the therapist brings to the teaching of language skills to a dyslexic student may be especially helpful in the teaching of basic mathematics."
T. Miles, 1992a, 8	"If one is talking about dyslexia one is talking about a constellation of difficulties. . . . It is clear that written language and school mathematics share a lot of common features."
T. Miles, 1992a, 8-9	". . . all dyslexics have difficulties of some kind with mathematics (as part and parcel of their problems with language and memory) but . . . there is considerable variation in the extent to which these difficulties are overcome."
Hitti, 2006, 2	"According to the International Dyslexia Association, some people with dyslexia have issues with learning to speak, organizing written and spoken language, learning letters and their sounds, memorizing number facts, spelling, reading, learning a foreign language, and doing math correctly."
Dowker, 2004, 12	"Grauberg (1998) has written a book on her experiences of teaching mathematics to pupils with language difficulties. She notes that pupils with language difficulties tend to have difficulties in particular with: 1. Symbolic understanding. This includes difficulty in understanding how one item can 'stand for' another item or items, and effects can range from difficulties in understanding how a numeral can present a quantity to difficulties in understanding how a coin of one denomination may be equivalent to a set of coins of a smaller denomination. . . . 2. Organization. Children with language difficulties often have difficulties with organizing items in space or time, which may, for example, affect their ability to arrange quantities in order; to organize digits spatially on a page; and to 'talk through' a problem, especially a word problem." 3. Memory. Poor short-term and long-term verbal memory are frequent characteristics of individuals with language difficulties and will affect learning to count, remembering number facts, and keeping track of one step in an arithmetic problem while carrying out subsequent steps." In addition, language difficulties will directly affect the child's ability to benefit from oral or written instruction, and to understand the language of mathematics."

More details about the effects of disabilities in the language system in general and of dyslexia specifically are found in Chapter IV. These effects establish the research base for CEI's decisions relating to *MLS* content, student acquisition of both concepts and fact fluency.

Visuospatial System Disabilities

The other cognitive system that, when affected by a neuropsychological dysfunction or disability that causes problems in learning mathematics, is the visuospatial system. Table 31 includes the findings of researchers in this area. This disability is manifested in several ways. Most obviously, it affects students' ability to learn geometry. Earlier manifestations include such problems as understanding direction, in lining up numbers on the page, and in copying correctly. Students with disabilities in the visuospatial system need clean, uncluttered computer screens (see Chapter V), as in the *MLS* design, so that they are not confused, and they must be explicitly taught that equations can be depicted both horizontally and vertically, again as is done in *MLS*.

Table 31: Effects of Visuospatial Disabilities on Mathematics Achievement

Researcher(s)	Findings/Conclusions
Miller & Mercer, 1997, 5	"Visual-Spatial deficits: 1. Student loses place on the worksheet. 2. Student has difficulty differentiating between numbers (e.g., 6 and 9; 2 and 5; or 17 and 71), coins, the operations symbols, and clock hands. 3. Student has difficulty writing across the paper in a straight line. 4. Student has difficulty relating to directional aspects of math, for example, in problems involving up-down (e.g., addition), left-right (regrouping), and aligning of numbers. 5. Student has difficulty using a number line. Motor disabilities: 1. Student writes numbers illegibly, slowly, and inaccurately. 2. Student has difficulty writing numbers in small spaces (i.e., writes large)."
Kandel, 2006, 299	". . . space is indeed the most complex of sensory representations."
Kandel, 2006, 315	"O'Keefe. . . found clear differences in the way women and men attend to and orient themselves to the space around them."
Sousa, 2001, 141	"Individuals with visual processing weaknesses almost always display difficulties with mathematics."
Spear-Swerling, n.d., 1	" . . . yet a third pattern involves visual-spatial difficulties, such as difficulty lining up columns or with learning spatial aspects of math, such as geometry."
Garnett, 1998, 5	"A small number of LD students have disturbances in visual-spatial-motor organization, which may result in weak or lacking understanding of concepts, very poor 'number sense,' specific difficulty with pictorial representations and/or poorly controlled handwriting and confused arrangements of numerals and signs on the page. Students with profoundly impaired conceptual understanding often have substantial perceptual-motor deficits and are presumed to have right hemisphere dysfunction . . . ."
Garnett, 1998, 5	"It is important to recognize that average, bright, and even very bright youngsters can have the severe visual-spatial organization deficits that make developing simple math concepts extremely difficult."
Fuson & Abrahamson, 2005, 214	"The Apprehending Zone Model foregrounds the agency of student body and sensory perception in assimilating and linking mathematical formats, situational attributes, and relations among all of these. In our design research classrooms, teachers taught and students learned ratio and proportion by tacitly internalizing-externalizing dynamic visuo/body-sensed schematic images that systematically linked the word-problem situations with the spatial-numerical mathematical formats and solution methods."
Campbell & Epp, 2005, 356	". . . there is ample evidence that format (Arabic numerals, written number words, and dots) has large effects on the specific errors that people produce."

Researcher(s)	Findings/Conclusions
Fias & Fischer, 2005, p. 43	"More systematic studies have supported the anecdotal reports by demonstrating a tight correlation between mathematical and visuo-spatial skill. In the clinical field, learning disorders establish a similar association between visuo-spatial and mathematical disabilities. Evidence from brain imaging provides further support for a link between numbers and space."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 367	". . . researchers have speculated that visual-spatial codes are central to mental calculation, as well as to other mathematical tasks such as geometry."
Geary & Hoard, 2005, 263	". . . spatial deficits have been associated with misalignment of numbers when setting up arithmetic problems (e.g., writing 45 x 68 out horizontally) and in interpreting the positional base-10 meaning of the numbers."
Fias & Fischer, 2005, 43	". . . spatial and number processing are intimately connected."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 367	". . . participants showed evidence of using visual-spatial codes when problems (e.g., 34 + 9) were presented vertically but relied more on verbal codes when problems were presented horizontally."
Geary & Hoard, 2005, 260	"The central executive controls the attentional and inhibitory processes needed to use procedures during problem solving, and much of the information supporting conceptual and procedural competencies is likely to be represented in the language of visuospatial systems, although a distinct modular system for arithmetic has been proposed."
Brysbaert, 2005, 38	"Arabic numbers can in principle be recognized like objects (or pictures of them); the stimulus is decomposed into a structural description of perceptual features, which activates the corresponding semantic information."
Brysbaert, 2005, 38-39	"All in all, recent research on the recognition of small Arabic numerals has revealed a rather intriguing picture. First, digits activate their meaning faster than words and also seem to require semantic mediation for further processing. In this respect, their processing is closer to that of picture recognition than to that of word processing."
Wiliams & Lecluyse, 1990, 121	". . . the visual processing of disabled readers is characterized by a longer integration time and a slower processing rate for both simple and word-like stimuli."
Sylwester, 1995, 2	". . . dyslexia may be at least partly a result of a coordination problem in the timing of the fast and slow visual pathway systems."
Bell, 2003, 2	"Visualizing numerals is one of the basic cognitive processes necessary for understanding math."
Bell, 2003, 2	"While imaging numerals is important to mathematical computation, another aspect of imagery is equally important: concept imagery. Understanding, problem solving, and computing in mathematics require another form of imagery—the ability to process the gestalt (the whole)."
Bell, 2003, 3	". . . not all children create mental imagery as they work with concrete manipulatives. For these children, the process of turning the concrete experience into imagery must be consciously stimulated."
Geary, 2003b, 209	"There is . . . evidence that some children with arithmetic disabilities who show broader deficits in mathematics may have a deficit in visuospatial competencies."

Other Mathematics Disabilities

Dyscalculia is the general term given to mathematics disabilities, which may be a result of inheritance, injury, or environmental factors, according to some. There are some, although low-incidence, more specifically-named mathematics disabilities, all genetic, about which educators must be informed in order to understand the importance of diagnosis and prescription in an intervention program such as *MLS*. Table 32 includes information about Turner syndrome, which affects females. According to Mazzocco and McCloskey (2005), it is a "sporadic chromosome abnormality that occurs in approximately 1:2000 to 1:5000 live female births (Rieser & Underwood, 1989). It results from complete or partial absence of one of two X chromosomes normally present in a female" (p. 269). These girls, according to Sousa (2001), "usually display dyscalculia, among other learning problems (Mazzocco, 1998)" (p. 141).

Table 32: Turner Syndrome

Researcher(s)	Findings/Conclusions
Mazzocco & McCloskey, 2005, 276	"Considered together, these findings are consistent with earlier studies of selective executive dysfunction in girls with Turner syndrome and suggest difficulty inhibiting strongly prepotent responses and a lack of organization in rapid retrieval of information but intact skills in planning ahead to execute a strategy and intact ability to shift response set."
Mazzocco & McCloskey, 2005, 277	"Functional neuroimaging results also implicate weak executive function skills in girls with Turner syndrome."
Mazzocco & McCloskey, 2005, 278	". . . visual working memory—versus visual, spatial, or memory skills per se—may be a hallmark deficit in girls with Turner syndrome."
Mazzocco & McCloskey, 2005, 279	"Several findings pointing to cognitive deficiencies in girls with Turner syndrome take the form of slower performance. . . . These findings raise the possibility that girls with Turner syndrome suffer from a general slowing of cognitive processing and that this slowing underlies some of the observed deficits on cognitive tasks, including math tasks."

Fragile X syndrome, also a chromosome abnormality, affects both males and females, but males more severely. This disorder, according to Mazzocco and McCloskey (2005) is "recognized as the leading known cause of mental retardation" (p. 270). Table 33 includes research findings on its effects on mathematics achievement.

Table 33: Fragile X Syndrome

Researcher(s)	Findings/Conclusions
Mazzocco & McCloskey, 2005, 271	"As is typical with X-linked disorders, fragile X syndrome affects males more severely than females; approximately 50% of females with fragile X have mental retardation (Rousseau et al., 1994) versus nearly 100% of males (Bailey, Hatton, & Skinner, 1998). Females without mental retardation may have borderline to average levels of intellectual ability."
Mazzocco & McCloskey, 2005, 281	"Although research on fragile X is even more limited than current studies of Turner syndrome, the findings consistently indicate that fragile X syndrome is a risk factor for MD; the risk for poor math achievement is evident across studies that utilize a wide range of math measures . . . during early childhood, adolescence, and adulthood."
Mazzocco & McCloskey, 2005, 281	". . . whereas poor math achievement in one year does not necessarily indicate future poor math achievement in the general population, among girls with fragile X poor math achievement at one time point is a stronger indicator of future math performance. This has important implications for immediate and sustained interventions for young girls with fragile X syndrome."
Mazzocco & McCloskey, 2005, 281	"The math difficulties reported in fragile X appear in the absence of comorbid reading disability. Although many children with fragile X do have reading disabilities, these difficulties are less apparent in females with fragile X who do not have mental retardation."
Mazzocco & McCloskey, 2005, 283	". . . our findings suggest that girls with fragile X may have inefficient working memory skills not accounted for by low FSIQ [full-scale IQ] and relatively low thresholds for working memory demands."

Researcher(s)	Findings/Conclusions
Barnes, Smith-Chant, & Landry, 2005, 299-300	"There are a number of neurodevelopmental disorders associated with problems in math cognition, such as spinal bifida, fragile X syndrome, and Turner syndrome."
Kandel, 2006, 67	". . . fragile x syndrome affects dendrites."

Spinal bifida, according to Barnes, Smith-Chant, and Landry (2005) is "the most common severely disabling birth defect in North America" (p. 300). It is caused, they say, by a "complex pattern of gene/environment interactions." Children with this disability most often suffer from mathematics difficulties, while only about two percent have decoding problems. Table 34 includes the research findings on this abnormality and its effects on mathematics achievement.

Table 34: Spinal Bifida

Researcher(s)	Findings/Conclusions
Barnes, Smith-Chant, & Landry, 2005, 300	"SBM [spinal bifida myelomeningocele] is the most common severely disabling birth defect in North America and occurs in 0.5—0.7 per 1,000 live births. It arises from a complex pattern of gene/environment interactions, which produce a neural tube defect that is associated at birth with distinctive physical, neural, and cognitive phenotypes. The physical phenotype of SBM, with its spinal cord defect and orthopedia sequelae, is what is most commonly associated with this developmental disorder. The spinal dysraphism produces impairment of lower and upper extremity coordination, often with significant paraplegia and limited ambulation. Less well known and less well studied is the neural phenotype of SBM that involves significant disruption of brain development. The failure of neuroembryogenesis is associated with anomalies in the regional development of the brain, especially the corpus callosum, midbrain and techtum, and cerebellum."
Barnes, Smith-Chant, & Landry, 2005, 301	"The cognitive phenotype of SBM involves a modal profile of preserved and impaired cognitive and academic skills. As a group, children with SBM are stronger in language and weaker in perceptual and motor skills. . . . In terms of academic competencies, math is impaired relative to word-recognition skills, and writing problems are common."
Barnes, Smith-Chant, & Landry, 2005, 301	"SBM has long been associated with math difficulty, based on studies of standardized achievement scores on tests of written or mental computation. . . . In a sample of over 300 children with SBM, only 2% had problems in word decoding alone, 20% had problems in both reading and math, and 21% had problems in math but not reading. . . . Even among children with SBM who were not math disabled, reading was typically better developed than math. Of interest was the finding that over a fifth of the sample had specific math disability; that is, math disability without comorbid reading disability. . . . Available figures for math disability in the general population are largely based on European studies that use more stringent cut points than those used in North America. In any event, those studies suggest a prevalence rate for math disabilities of about 6 percent in the general school-age population. . . and a population rate of specific math disability between 1-2%. . . ."

Another genetic cause of mathematics disabilities is Gerstmann's syndrome. In this case, there are clusters of deficits, some of them causing problems similar to dyscalculia, but others causing problems such as right-left disorientation and dysgraphia. Table 35 summarizes those research findings.

Table 35: Gerstmann's Syndrome

Researcher(s)	Findings/Conclusions
Dehaene, Piazza, Pinel, & Cohen, 2005, 439	". . . calculation impairments often co-occur with other deficits, forming a cluster of deficits called Gerstmann's syndrome. . ., which comprises agraphia, finger agnosia, and left-right distinction difficulties (to which one may often add constructive aphasia)."
Sousa, 2001, 141	"Because the parietal lobe is heavily involved with number operations, damage to this area can result in difficulties. Studies of individuals with Gerstmann syndrome—the result of damage to the parietal lobe—showed that they had serious problems with mathematical calculations as well as right-left disorientation, but no problems with oral language skills (Suresh and Sebastian, 2000)."
Dehaene, Piazza, Pinel, & Cohen, 2005, 448	"Deficits of number processing should be observed in the case of early left parietal injury or disorganization. Developmental dyscalculia is relatively frequent, affect 3-6% of children. . . . We predict that a fraction of those children may suffer from a core conceptual deficit in the numerical domain. Indeed, a 'developmental Gerstmann syndrome' has been reported. . . . In those children, dyscalculia is accompanied by most or all of the following symptoms: dysgraphia, left-right disorientation, and finger agnosia, which suggest a neurological involvement of the parietal lobe. Interestingly, even in a sample of 200 normal children, a test of finger knowledge appears to be a better predictor of later arithmetic abilities than is a test of general intelligence."
Landerl, Bevan, & Butterworth, 2004, 103	"Another set of deficits which are associated with developmental dyscalculia are finger agnosia, dysgraphia, and difficulties with left-right discrimination. Taken together this symptom complex constitutes developmental Gerstmann's syndrome."
Noel, Rousselle, & Mussolin, 2005, 192	". . . digital agnosia measured at entry level in elementary school may predict disabilities in learning mathematics 1 year later. Furthermore, these learning problems might specifically affect number magnitude processing, leaving intact more verbal numerical abilities (such as arithmetical facts and transcoding)."
Fias & Fischer, 2005, 43	". . . the processing of numerical magnitudes and of visuo-spatial information are functionally connected. Patient studies further confirm the close link between visuo-spatial processing and basic number processing. A particular example is Gerstmann syndrome, which is characterized by the co-occurrence of left-right confusion, finger agnosia, and dyscalculia."
Dehaene, Piazza, Pinel, & Cohen, 2005, 443	". . . imaging studies in normals confirm that distinct sites of activation underlie performance in simple multiplication and subtraction. . . . Second, all patients in whom subtraction was more impaired than multiplication had left parietal lesions and/or atrophy, most often accompanied by Gerstmann's syndrome."

There are, of course, multiple other genetic issues that affect academic performance, such as Down's Syndrome, Autism, Cerebral Palsy, and traumatic brain injury. This study, however, focuses on those areas most closely associated with specific mathematical disabilities.

Comorbid Disabilities

Much more serious than mathematics difficulties, much more serious than dyscalculia alone, and much more serious than dyslexia alone are the problems that learners have when they carry both reading and mathematics disabilities (comorbidity). These learners present very complex manifestations, making remedial efforts more complex and difficult—and slower. Table 36 displays some of that research and the effects of comorbidity on mathematics achievement.

Table 36: Comorbid Reading and Mathematics Disabilities

Researcher(s)	Findings/Conclusions
Landerl, Bevan, & Butterworth, 2004, 104	"Shalev et al. (1997) found that children with comorbid maths and reading difficulties were more profoundly impaired than children with specific maths problems on subtraction and division and had lower verbal IQ scores."
Shaley & Gross-Tsur, 2001, 338	"Interestingly, children who were comorbid for dyslexia were impaired more profoundly on arithmetic skills and neuropsychologic tests than children with developmental dyscalculia alone or developmental dyscalculia and ADHD."
Geary, 2004, 7	"In first and second grades, children with MLD only and especially children with MLD/RD committed more counting errors and used the developmentally immature counting-all procedure more frequently than did the children in other groups. Moreover, in keeping with models of typical arithmetical development, the children in the RD-only and typically achieving groups showed a shift from first grade to second grade, from heavy reliance on finger counting to verbal counting and retrieval. The children in the MLD/RD and MLD-only groups, in contrast, did not show this shift but instead relied heavily on finger counting in both grades."
Geary, 2004, 8	"Unlike the use of counting strategies, it appears that the ability to retrieve basic facts does not substantively improve across the elementary school years for most children with MLD/RD and MLD only. When these children do retrieve arithmetic facts from long-term memory, they commit more errors and often show error and reaction time (RT) patterns that differ from those found with younger, typically achieving children (Barrouillet et al., 1997; Fayol, Barrouillet, & Marinthe, 1998; Geary, 1990; Geary & Brown, 1991; Rasanen & Ahonen, 1995). . . . These patterns suggest that the memory retrieval deficits of children with MLD/RD or MLD only reflect a cognitive disability and not, for instance, a lack of exposure to arithmetic problems, poor motivation, a low confidence criterion, or low IQ (Geary et al., 2000)."
Geary & Hoard, 2005, 256	". . . many children with MD/RD [mathematics disabilities/reading disabilities] had difficulties holding information in working memory."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 369	". . . children who had both math and reading disabilities showed a different pattern of deficiencies than children who had only math disabilities. The former had profound problems that were probably linked to their phonological processing deficits, whereas the latter appeared to have specific difficulties with numerical magnitudes and visual-spatial processing."
Barnes, Smith-Chant, & Landry, 2005, 302	"The presence or absence of a comorbid reading disability is central to several models of math disability, as reading disability plus math disability is associated with different cognitive markers than math disability alone. . . . Children with comorbid reading and math disability are characterized by deficits in verbal and visual working memory. . . and phonological processing. . . . In contrast, specific math disability has been associated with difficulties in visual memory, visual-spatial working memory. . ., and visual-spatial function. . . . Within the domain of mathematical processing itself, recent studies show that the pattern of impairment on specific math skills is also related to the presence of a reading disability. . . . Children with math disability, regardless of their reading status, have difficulty with numerical estimation and rapid retrieval of number facts. Those with both reading and math disabilities have particular difficulty in areas of math assumed to be mediated by language, such as word problems and verbal counting."
Robinson, Menchetti, & Torgesen, 2002, 1	"A two-factor theory is proposed in an attempt to explain the difficulty that children with math disabilities have in mastering the basic number facts. The theory is based on the premise that weak cognitive representations lead to poorer retrieval of information from long-term memory. Two groups of children are discussed: those with math disabilities alone (MD) and those with co-morbid math and reading disabilities (MD/RD). It is proposed that weak phonological processing abilities underlie the learning difficulties of MD/RD children, and that weak number sense is a causal factor in the math-fact learning difficulties of MD only and some MD/RD children."

Researcher(s)	Findings/Conclusions
Geary & Hoard, 2002, 109	"Children with MD/RD appear to have similar deficits in executive functions but these appear to be more severe than those evident in children with MD only or RD only. Children with MD/RD also appear to have more specific deficits in phonetic memory, specifically low activation of the associated long-term memory representations when phonetic information, such as number words, is encoded into working memory."
Geary & Hoard, 2005, 259	". . . research results on more complex forms of arithmetic are beginning to emerge and appear to support the separation of MD only and MD/RD groups. In comparison to MD only children, children with MD/RD show more pervasive deficits as problem complexity increases from simple operations to complex, multistep problems, although children in both groups demonstrate performance below normal peers."
Geary & Hoard, 2005, 254	". . . 5% to 8% of school-age children exhibit some form of MD [mathematics disability]. Many of these children have comorbid disorders, including reading disabilities (RD), spelling disability, attention deficit hyperactivity disorder (ADHD), or some combination of these disorders."
Jordan, Hanich, & Kaplan, May/June 2003, 834	"In early elementary school, children with mathematics difficulties (MD) who are good readers progress faster in mathematics achievement than do children with comorbid MD and reading difficulties (RD), independent of their intelligence, income level, ethnicity, and gender (Jordan, Kaplan, & Hanich, 2002). In contrast, children with RD who are good in mathematics and children with comorbid RD and MD progress at about the same rate in reading achievement. Although reading abilities influence growth in mathematics achievement, mathematics abilities do not influence growth in reading achievement."
Barnes, Smith-Chant, & Landry, 2005, 300	"In many math disability models. . . , the presence or absence of a comorbid reading disability is related to the type of math disability that arises. . . . In math-disabled children with no neurological disorder, reading and math deficits typically co-occur . . . and specific math disabilities, that is math disability without reading disability, are relatively less common. . . ."

Summary of Mathematical Learning Disabilities

Chapter III has presented an overview of mathematics disabilities, since at least some understanding of them and their manifestations relative to mathematics are important to educators seeking learning solutions or interventions that will improve achievement. The chapter was organized using a model created by Geary and Hoard (2005, p. 260). The first part of the chapter discussed dyscalculia in general, the umbrella term for mathematical disabilities. It then moved to a discussion on the complexity of mathematical disabilities and their effect on the varied mathematics domains, as well as on the two supporting competencies—concepts and procedures. Next, the disabilities in the central executive and in the language (including dyslexia effects) and visuospatial systems were discussed, along with their manifestations.

This section was followed by an analysis of the various forms of specific mathematics disabilities and their effects on mathematics achievement. Turner syndrome, Fragile X syndrome, spinal bifida, and Gerstmann's syndrome were included. The final section concerned comorbid reading and mathematics disabilities and their most serious effects on mathematics achievement. This study did not include effects on mathematics achievement resulting from such conditions as Autism, Downs Syndrome, Cerebral Palsey, or mental retardation since they are sometimes, but not necessarily identified with specific mathematics disabilities.

Chapter IV begins the documentation of the research leading to CEI's decisions relating to the content focus of *MLS*: concept development and fact fluency.

Chapter IV: Research Findings that Ground *MLS'* Content

"One of the most robust findings of research is that conceptual understanding is an important component of proficiency, along with factual knowledge and procedural facility (Bransford, Brown, & Cocking, 1999)." (Bohan, 2002, 35)

Overview

Chapter II focuses on the research on mathematics difficulties and their effects on mathematics achievement. In Chapter III the discussion concerned the complex variety of mathematics disabilities, the effects that reading disabilities have on mathematics achievement, and comorbid mathematics and reading disabilities and their effects on achievement.

Chapter IV begins the documentation of why *MLS* works as an intervention for struggling learners—the ones with difficulties and the ones with disabilities. Its focus is on *MLS'* content and begins with a brief overview of mathematics cognition, definitions of mathematics, descriptions of early school mathematics, and expanded research on students who struggle to learn. Understanding of what mathematics is, especially at the foundational levels, was critical in the original design of *MLS* and continues to be important as the program is enhanced and improved.

The chapter then moves to the wealth of research findings on manifestations of difficulties or disabilities in mathematics, the documentation of research behind CEI's decision to emphasize concept development (including problem solving) and fact fluency in the design of *MLS,* and the ongoing research that supports those original decisions.

Each of these sections ends with a description of *MLS'* application of the research—how the content addresses each area of identified learning problem. Understanding the areas in which learners are most likely to struggle, whether they are difficulties or disabilities, also highly influenced design decisions.

In analyzing the content of *MLS*, it is important to note that *MLS* makes no attempt to be a comprehensive mathematics program that is grade-level specific. It is instead a component of the larger mathematics program in a school—used either as prevention in the early introduction of critical concepts and as insurance that all students learn to fluency their mathematics facts; as remediation to re-teach the critical concepts already introduced by the regular classroom teacher and using the fluency component for practice; or as a cognitive intervention for students who have already failed and who need for any of a variety of reasons something truly different that addresses the root causes of the failure to learn mathematics.

All is not lost if students do fail to gain mastery in elementary school of the prerequisite knowledge and skills. A well-implemented *MLS* lab in a middle or high school, or even for adults can accelerate their learning in the critical areas of need, making it possible for them to move into more sophisticated mathematics, including algebra, with success. The prevention of failure is always preferable, of course, so CEI recommends that intervention occur at the earliest signs of difficulties or disabilities.

MLS' designers sought to identify the most common problem areas and to prioritize those having the most influence not only on early achievement, but in subsequent years. The preponderance of evidence in the early 1990's, as currently, clearly indicates that concept development *and* fact fluency are those priorities. Without mastery in these identified areas, students not only struggle in elementary school, but they likely will not be able to succeed in algebra or other areas of advanced study. Although concept development and fact fluency are major emphases, *MLS* addresses all five of the critical strands required for *mathematical proficiency* (National Research Council, 2001):

1. Conceptual understanding—comprehension of mathematical concepts, operations, and relations
2. Procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
3. Strategic competence—ability to formulate, represent, and solve mathematical problems
4. Adaptive reasoning—capacity for logical thought, reflection, explanation, and justification
5. Productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (p. 5).

These five strands reflect, say the writers, "Our analysis of the mathematics to be learned, our reading of the research in cognitive psychology and mathematics education, our experience as learners and teachers of mathematics, and our judgment as to the mathematical knowledge, understanding, and skill people need today. . ." (p. 5). They add that "The most important observation we make about these five strands is that they are interwoven and interdependent. This observation," they say, "has implications for how students acquire mathematical proficiency, how teachers develop that proficiency in their students, and how teachers are educated to achieve that goal" (p. 5).

This research synthesis was published about seven years after the initial design team for *MLS* was constituted by CEI. CEI staff members often speak of the prescience of the *ELS* and *MLS* program designers since in both cases, the programs not only reflected the research that was current at the time, but they continue to reflect cutting-edge research on what works with all those students who struggle to learn. Also, the National Research Council (2001) report is about mathematics for all children, not just struggling learners, and yet the research on difficulties and disabilities identify essentially the same areas for emphasis. This chapter will document that *MLS* incorporates the first four critical strands in its content. Other chapters document that *MLS* includes components that address the other strand—productive disposition. (See Chapter II discussions about cultural value of mathematics and about the negative effects on achievement if students have poor motivation to learn mathematics and/or low senses of self-efficacy. See Chapter III discussion of habits of mind. See Chapter VII for a discussion of the motivational component of *MLS*.) Appropriately, the *MLS* components are "interwoven and interdependent," just as they must be according to research findings.

Beginnings of Mathematical Cognition

There are, interestingly, varied opinions about what mathematics as a discipline is. One thing that is known, however, is that very young children (and even many animals) seem to be born with some understanding of mathematics. In contrast, although very young children are wired for language, they do not appear to have the innate ability to read, unless taught. Table 37 includes some of those research findings relating to early mathematics cognition.

Table 37: Mathematical Cognition in Young Children

Researcher(s)	Findings/Conclusions
Brysbaert, 2005, 31-32	". . . numerical knowledge is represented separately from many other types of semantic knowledge in the brain."
Fayol & Seron, 2005, 5	"A number of sets of research claim to have found that preverbal infants possess a mental representation of small quantities. . . . To summarize, both newborns and animals seem to be able to mobilize two different systems for the processing of quantities. One of these is precise and is limited by its absolute set size (e.g., 1, 2, and 3), while the other is extensible to very large quantities, operates on continuous dimensions, and yields an approximate evaluation in accordance with Weber's law."
Brysbaert, 2005, 23	". . . Antell and Keating (1983) reported that newborns who were habituated to successive displays with two elements each (and, therefore, barely looked at them anymore), showed increased interest when a display with three elements was presented. Using a similar habituation technique, Xu and Spelke (2000) reported that 6-month-olds can discriminate between 8 and 16 items but not between 8 and 12."
Dehaene, Piazza, Pinel, & Cohen, 2005, 447	"Our hypothesis is that the cultural construction of arithmetic is made possible by pre-existing cerebral circuits that are biologically determined and are adequate to support specific subcomponents of number processing. . . ."
Bisanz, Sherman, Rasmussen, & Ho, 2005, 143	"The foundations of arithmetic emerge well before school begins, and preschool children often display striking knowledge of arithmetic facts, procedures, and concepts prior to entering school."
Garnett, 1998, 3	"Many younger children who have difficulty with elementary math actually bring to school a strong foundation of informal math understanding. They encounter trouble in connecting this knowledge base to the more formal procedures, language, and symbolic notation system of school math. . . ."

Definitions of Mathematics

Before a decision is made about the content of an effective mathematics intervention, one needs some understanding of what mathematics is as a domain or a K-12 discipline. The mathematics standards developed by the National Council of Teachers of Mathematics (2000) are themselves one definition of the discipline of mathematics. The National Research Council (2001) frequently referenced those standards in their report, *Adding It Up*, which focuses on elementary school mathematics. They also, however, pose a rhetorical question near the beginning of the book about what would constitute *proficiency* in mathematics. Traditionally in American schools, the answer to that question has given heavy importance to mastery of mathematical procedures. Although there are some advocates for that view today and although there are advocates for the total focus on conceptual understandings, most mathematics educators have come to see that mathematics has to encompass more than procedures—and more than concepts (see Chapter II discussions of "inappropriate instruction" and "math wars").

It is no accident that conceptual understandings and procedural fluency are identified by the National Research Council (2001) as the first two critical components of effective mathematics curriculum. Students who struggle have problems in each of these areas of "supporting competencies," as discussed in Chapter III with reference to the model constructed by Geary and Hoard (2005, p. 260):

Mathematical Domain (e.g., Base-10 Arithmetic)	
Supporting Competencies	
Conceptual (e.g., base-10 knowledge)	**Procedural** (e.g., columnar trading)

Table 38 includes definitions of mathematics provided by a number of other researchers. Each quotation references one or more of the five critical strands. All in all, they establish that mathematics is both content and procedures, but it is also reasoning, problem-solving, thinking, and visualizing.

Table 38: Definitions of Mathematics

Researcher(s)	Findings/Conclusions
Battista, 1999, 428	"Mathematics is first and foremost a form of reasoning. In the context of reasoning analytically about particular types of quantitative and spatial phenomena, mathematics consists of thinking in a logical manner, formulating and testing conjectures, making sense of things, and forming and justifying judgments, inferences, and conclusions. We do mathematics when we recognize and describe patterns; construct physical and/or conceptual models of phenomena; create symbol systems to help us represent, manipulate, and reflect on ideas; and invent procedures to solve problems."
Sousa, 2001, 144	"Mahesh Sharma (1989) and other mathematics educators have suggested that the following seven skills are prerequisites to successfully learning mathematics. They are the ability to 1. Follow sequential directions. 2. Recognize patterns. 3. Estimate by forming a reasonable guess about quantity, size, magnitude, and amount. 4. Visualize pictures in one's mind and manipulate them. 5. Have a good sense of spatial orientation and space organization, including telling left from right, compass directions, horizontal and vertical directions, etc. 6. Do deductive reasoning, that is, reason from a general principle to a particular instance, or from a stated premise to a logical conclusion. 7. Do inductive reasoning, that is, come to a natural understanding that is not the result of conscious attention or reasoning, easily detecting the patterns in different situations and the interrelationships between procedures and concepts."

Researcher(s)	Findings/Conclusions
Kroesbergen, 2002, 6	"The mathematics curriculum starts with the development of number sense in kindergarten and first grade. At this time, a basic understanding of the arithmetic operations is to be established (Correa, Nunes, & Bryant, 1999). Formal math instruction usually begins with addition and subtraction, and then proceeds to multiplication and division before such more advanced skills as fractions, decimals, and percentages are taught. Mathematics has a logical structure, which means that the mastery of lower-level math skills is essential for learning higher-order skills."
Butterworth, 2005, 456	". . . mathematics, even in the early grades of schooling, comprise a wide variety of skills, including counting, estimating, retrieving arithmetical facts (number bonds, multiplication tables), understanding arithmetical laws such as commutativity of addition and multiplication (but not subtraction and division), knowing the procedures for carrying and borrowing in multidigit tasks, being able to solve novel word problems, and so on."
Dowker, 2004, ii	"In order to study the nature of the arithmetical difficulties that children experience, and thus to understand the best ways to intervene to help them, it is important to remember one crucial thing: arithmetic is not a single entity, but is made up of many components. These include knowledge of arithmetical facts; ability to carry out arithmetical procedures; understanding and using arithmetical principles such as commutativity and associativity; estimation; knowledge of mathematical knowledge; applying arithmetic to the solution of word problems and practical problems; etc."
Miller & Mercer, 1997, 9	". . . students need to acquire money skills; time skills; measurement skills; and an ability to add, subtract, multiply, and divide in order to function effectively in daily living."

An important part of understanding definitions of mathematics is knowing something of the contribution that Arabic numerals have played in mathematics history. Brysbaert (2005) offers an explanation of why it was that the Romans never developed algebra and why, instead, its origin comes from Arabs. Arabic numerals use the base-10 system and include the use of place value in denoting numerals. Those two basic concepts are requisite to learning mathematics. Brysbaert's analysis follows:

> The invention and application of Arabic (actually Hindi) numerals has further advanced the human numerical competence (Ifrah, 1998). It is widely assumed that the use of Roman numerals prevented the Romans from attaining a mathematical sophistication that matches the sophistication they reached in other knowledge areas (just try to solve the problem CMIX times LD). Interesting features of Arabic numerals are the use of a base 10 throughout . . and the use of place coding. Units are always written rightmost, tens are second, hundreds third, and so on. This way of coding required the invention of the digit 0, for instance, to represent 909 (nine hundreds and nine units, no tens). The power of Arabic notation can be seen in the fact that even for simple arithmetic problems involving the addition or multiplication of single digits, participants are much faster and more accurate when the numerals are presented as digits rather than as words, even when the words are spoken (pp. 32-33).

Early School Mathematics

Several researchers have studied the ways that children developmentally acquire mathematical understandings. Research in this area verifies that children's sense of number and quantity and their early recognition of sets and attempts to count, classify, and sort objects constitute beginning mathematics. Miller (2003), a former professor in early childhood education at Oakland University in Michigan and a native of New Zealand, conducted research on beginning mathematics for many years and assisted in developing the New Zealand government's *Beginning School Mathematic* (1995) program. His research led him to believe that the beginnings of mathematics, before acquisition of counting and sorting skills, for instance, were in children's understandings of prepositions, especially those relating to spatial relationships, such as *above, under, over, around, behind, to, from, at, in front of, with*, and *beside*. Table 39 includes additional research findings.

Table 39: Early School Mathematics

Researcher(s)	Findings/Conclusions
Gardner, 1985, 129	"For it is confronting objects, in ordering and reordering them, and in assessing their quantity, that the young child gains his or her initial and most fundamental knowledge about the logical-mathematical realm."
Gardner, 1985, 130	"The ability to group together objects serves as a 'public manifestation' of the child's emerging knowledge that certain objects possess specifiable properties in common. It signals, if you like, the recognition of a class or *set.*
Gardner, 1985, 135	"The mathematician Brian Rotman indicates that the whole of contemporary mathematics takes for granted and rests on the notion of counting . . . on the interpretation that occurs in the message 1, 2, 3."
NAEYC, n.d.b, 1	"Throughout the early years of life, children notice and explore mathematical dimensions of their world. They compare quantities, find patterns, navigate in space, and grapple with real problems such as balancing a tall block building or sharing a bowl of crackers fairly with a playmate."
National Research Council, 2001, 191	"U. S. children progress through a sequence of multiplication procedures that are somewhat similar to those for addition. They make equal groups and count them all. They learn skip-count lists for different multipliers (e.g., they count 4, 8, 12, 16, 20, . . . to multiply by four). They then count on and count down these lists using their fingers to keep track of different products. They invent thinking strategies in which they derive related products from products they know."
Fayol & Seron, 2005, 6	". . . there are actually two numerical systems in the human brain, one corresponding to a discrete and exact representation which is used for small numbers, while another approximate one is involved in the representation of large numbers."
Brysbaert, 2005, 33	"A first robust finding is that the processing is more demanding for larger numbers than for smaller numbers."
Brysbaert, 2005, 33	"A second robust finding in Arabic numeral processing is that when two numbers are processed together, processing times are influenced by the distance between the numbers. This is particularly clear when both numbers have to be compared, as it is much easier to say which digit is the smaller for the pair 2-8 than for the pair 2-3. More precisely, decision times are a function of the logarithm of the distance between the two numbers. Another distance-related effect that has been described is the number priming effect. A target digit is recognized faster when it follows a (tachistoscopically presented) prime with a close value than when it follows a prime with a more distant value."
Spear-Swerling, n.d., 1	At these grade levels [k-4], general education instruction in mathematics should include development of the following math-related abilities: concepts and reasoning (e.g., basic number concepts, meaning of operations such as addition, geometric concepts); automatic recall of number facts (e.g., memorization of basic addition facts such as 3+4 so that children know answers instantly instead of having to count); computational algorithms (the written procedure of series of steps for solving more complex types of calculation (e.g., for two digit addition with regrouping, calculation starts in the right-hand column and tens are 'carried'

Researcher(s)	Findings/Conclusions
	from the ones to the tens column); functional math (e.g., practical applications such as time and money); and verbal problem solving (e.g., solving word problems)."
National Research Council, 2001, 181	"Computation with whole numbers occupies much of the curriculum in the early grades, and appropriate learning experiences in these grades improve children's chances for later success."
Brysbaert, 2005, 35	"A third major finding about the processing of Arabic numerals is that the semantic magnitude information of the numeral is activated more rapidly than is the case for verbal numerals."

Students Who Struggle to Learn Mathematics

Even though human beings are apparently "wired" for mathematics and even though young children manifest some very sophisticated understandings, some learners lag behind. Learners who struggle to learn mathematics may be of any age, of course. They are frequently found in the *NCLB* subgroups: racial/ethnic minorities, limited English proficient, economically disadvantaged, and learning disabled. Chapters II and III discussed in some detail the variety of mathematical difficulties and disabilities among learners that cause them to struggle. Table 40 includes additional research on this topic, including attention to secondary students.

Table 40: Students Who Struggle

Researcher(s)	Findings/Conclusions
Sousa, 2001, 2	"Who are special needs students? . . . the term 'special needs' refers to students • diagnosed and classified as having specific learning problems, including speech, reading, writing, and mathematics disorders • enrolled in Title I programs • not classified for special education or Title I, but still struggling with problems affecting their learning, such as those with sleep deprivation."
Fuchs & Fuchs, 2001, 85	". . . 6th graders with LD compute basic addition facts no better than nondisabled 3rd graders."
Kroesbergen & Van Luit, 2003, 98	"Students with difficulties learning math include all students who have more trouble with learning math than their peers, students who perform at a lower level than their peers, and students who need special instruction to perform at an adequate level. . . . all students with mathematics difficulties require special attention (Geary, 1994). These students have special educational needs, need extra help, and typically require some type of specific mathematics intervention. . . ."
Balfanz, McPartland, & Shaw, 2002, 12-13	"A continuum of extra-help needs exists for high school students. The first group in this continuum consists of a very small percentage of students (5-10%) who are in need of intensive and massive extra help. Such students are those who enter ninth grade testing at the third or even second grade level and still need to learn elementary level skills. Next along the continuum, there are a considerably larger number of students who have mastered the most basic skills but lack or have only weakly learned intermediate skills. These students can decode but read with limited fluency. They can add, subtract, and multiply whole numbers, but struggle with fractions and decimals. These are the students who test at the fifth and sixth grade levels and essentially enter high school without the benefit of a middle school education."
Balfanz, Legters, & Jordan, 2004, 1	"The recent TIMSS [Third International Mathematics and Science Study] study shows that cities that educate primarily high-poverty students typically have performance levels equal to those in developing countries (Mullis et al., 2001)."

Manifestations of Mathematics Difficulties and Disabilities

In addition to needing to know what mathematics is as a discipline in order to design an effective mathematics intervention, it is also important to know the manifestations of mathematics difficulties and disabilities; that is, the kinds of errors or misconceptions that learners make and the kinds of problems they have in developing understandings. Many of those manifestations were presented in the tables in Chapters II and III on mathematics difficulties and disabilities. Table 41 includes the lists compiled by several researchers, which can be read as a summary of the research reviewed for Chapters II-III.

It is important to note that learners with difficulties make many of the same errors as those with disabilities. Dowker (2004) comments that ". . . it appears that distinguishing specific arithmetic difficulties from difficulties associated with low IQ is important from the point of view of understanding a child's general educational needs, but may not be crucial to planning arithmetical intervention as such" (p. 13). While that reality of similar errors makes identification of disabilities more complex, it actually simplifies the design of mathematics interventions. *MLS*, as such, was designed for all those students who struggle, regardless of the origin of their problems and regardless of the manifestations of their difficulties or disabilities, according to David Merryweather, a long-time employee of CEI.

Table 41: Manifestations of Mathematics Difficulties/Disabilities

Researcher(s)	Findings/Conclusions
Bryant, Bryant, & Hammill, 2000	"What do we know about characteristics of students with math problems? • Fails to verify answers and settles for first answer. • Cannot recall number facts automatically. • Takes a long time to complete calculations. • Makes "borrowing' (i.e., regrouping, renaming) errors. • Counts on fingers. • Reaches 'unreasonable' answers. • Calculates poorly when the order of digit presentation is altered. • Orders and spaces numbers inaccurately in multiplication and division. • Misaligns vertical numbers in columns. • Disregards decimals. • Fails to carry (i.e., regroup) numbers when appropriate. • Fails to read accurately the correct value of multi-digit numbers because of their order and spacing. • Misplaces digits in multi-digit numbers. • Misaligns horizontal numbers in large numbers. • Skips rows or columns when calculating."
Spear-Swerling, n.d., 1	"Scientific investigators interested in learning disabilities have identified several patterns that may be found in youngsters with math disabilities. Some of these children have difficulties that revolve primarily around automatic recall of facts, coupled with good conceptual abilities in mathematics; this pattern characterizes some children with reading disabilities. Another common pattern involves difficulties with computational algorithms; yet a third pattern involves visual-spatial difficulties, such as difficulty lining up columns or with learning spatial aspects of math, such as geometry."

Researcher(s)	Findings/Conclusions
Kroesbergen & Van Luit, 2003, 98	"Studies have shown most math difficulties to have a relatively early onset. . . . The first category of interventions thus focuses on these preparatory arithmetic skills. . . . The next step is to learn the four basic mathematical operations (i.e., addition, subtraction, multiplication, and division). Knowledge of these operations and a capacity to perform mental arithmetic also play an important role in the development of children's later math skills (Mercer & Miller, 1992; Van Luit & Naglieri, 1999). Most children with math-related learning disorders are unable to master the four basic operations before leaving elementary school and thus need special attention to acquire the skills. A second category of interventions is therefore aimed at the acquisition and automatization of basic math skills."
Garnett, 1992, 1	"Manifestations of math learning disabilities: • conceptual understanding (Kosc, 1974) • counting sequences (Baroody, 1986) • the written number symbol system (Russell & Ginsburg, 1984) • the language of math (Nesher, 1982) • basic number facts (Fleischner, Garnett, & Shepherd, 1982) • procedural steps of computation (Cohn, 1971) • application of arithmetic skills (Algozzine et al., 1987) • problem solving (Fleischner, Nuzum, & Marzola, 1987; Montague & Bos, 1986) • how arithmetic is taught in our schools (Nielsen, 1990; Stevenson, 1987)"
Bryant, n.d.b, 4	"Skills ranked as most problematic for students with LD and math weaknesses: • Has difficulty with word problems. • Has difficulty with multi-step problems. • Has difficulty with the language of math."
Karp & Howell, Oct. 2004, 120	"Potential barriers for students with special needs: Memory: visual memory, verbal/auditory memory, working memory (Mastropieri & Scruggs, 1998; Thornton, Langrall, & Jones, 1997; Wilson & Swanson, 2001) Self-regulation: excitement/relaxation, attention, inhibition of impulses (Lyon & Krasnegor, 1996; Swanson, 1996) Visual processing: visual memory, visual discrimination, visual/spatial organization, visual-motor coordination (Badian, 1999; Ginsburg, 1997; Rourke & Conway, 1997; Thornton, Langrall, & Jones, 1997) Language processing: expressive language, vocabulary development, receptive language, auditory processing (Cawley et al., 1998; Ginsburg, 1997) Related academic skills: reading, writing, study skills (Deshler, Ellis, & Lenz, 1996) Motor skills: writing legibly, aligning columns, working with small manipulatives, using one-to-one correspondence, writing numerals (Miller & Mercer, 1997; Rourke & Conway, 1997) "Students with learning disabilities usually experience a dramatic deficit in one or more of these areas. These deficits create a roadblock between the student and the learning of skills and concepts. A teacher cannot be effective in teaching until barriers to students' learning are removed."
Lochy, Domahs, & Delazer, 2005, 482	". . . data from the developmental field suggest possible benefits of a drill approach to transcoding. Indeed, when learning to transcode, children have particular difficulties at the syntactical level, especially with sum relationships. . ., which manifest by errors such as 'one hundred two' written 1002."

These summaries of the kinds of barriers that many students encounter in learning mathematics all have their origins in either an area of learning difficulties discussed in Chapter II or a result of a learning disability as described in Chapter III. Both concepts and skills are included in the lists, for a lack of understanding of a foundational concept inevitably causes problems in a student's use of appropriate procedures in problem solving. Learning concepts well helps to reinforce learning of procedures (skills), and learning procedures well helps to reinforce the understanding of concepts.

Description of MLS Content

MLS, reflecting the research findings on manifestations of mathematics difficulties and disabilities, the identification of the most common problem areas, the essential prerequisite knowledge and skills for success not only in arithmetic, but more advanced mathematics, and the content and strategies that result in effective interventions, emphasizes *concept development* and *fact fluency*.

The *MLS* concept development scope and sequence, which follows, includes extensive instruction and practice in problem solving involving both whole numbers and fractions. Each "phase" incorporates four steps in the lesson sequence: tactile (using manipulatives on a working mat), illustrative (seeing semi-concrete representations on the computer screen), abstract (including problem-solving applications), and assessment (measuring for mastery). Students must master each phase by scoring at least at the 80% level before proceeding to the next phase.

Table 42: *MLS* Concept Building Scope and Sequence

Unit	Level	Phase	
Unit 1: Understanding Numbers	Level 1: Defining Numbers	Identification 0-10	
		Recognition 0-10	
		Identification 11-20	
		Recognition 11-20	
	Level 2: Numbers 0-20	Patterns & Counting 0-20	
		Comparisons 0-20	
	Level 3: Numbers 21-99	Place Value 21-99	
		Patterns & Counting 21-99	
		Comparisons 21-99	
	Level 4: Numbers 100-999	Place Value 100-999	
		Patterns & Counting 100-999	
		Comparisons 100-999	
Unit 2: Number Operations	Level 1: Addition	Single Digits	**(Advance to Addition Fluency)**
		Double Digits	
		Triple Digits	
	Level 2: Subtraction	Single Digits	**(Advance to Subtraction Fluency)**
		Double Digits	
		Triple Digits	
	Level 3: Multiplication	Single Digits	**(Advance to Multiplication Fluency)**
		Single & Double Digits	
		Double Digits	
	Level 4: Division	Single Digits	**(Advance to Division Fluency)**
		Single & Double Digits	
		Double Digits	

Unit	Level	Phase
Unit 3: Using Whole Numbers	Level 1: Money	Pennies, Nickels, and Dimes
		Pennies, Nickels, Dimes, and Quarters
	Level 2: Time	To the Hour
		In Hours and Minutes
	Level 3: Estimation	Rounding to the Nearest Ten
		Rounding to the Nearest Hundred
Unit 4: Understanding Fractions	Level 1: Fraction Identification	Less Than One or Equal To One
	Level 2: Equivalent Fractions	Using Larger or Smaller Denominators
	Level 3: Comparing Fractions	Common Denominators
		Different Denominators
	Level 4: Simplifying Fractions	Simplified Numerators Equal to One
		Simplified Numerators Greater than One
	Level 5: Converting Fractions	Improper Fractions to Mixed Numbers
		Mixed Numbers to Improper Fractions
Unit 5: Fraction Operations	Level 1: Addition	Common Denominators
		Different Denominators
	Level 2: Subtraction	Common Denominators
		Different Denominators
	Level 3: Multiplication	Whole Numbers and Fractions
		Fractions and Whole Numbers
	Level 4: Division	Common Denominators
		Fractions and Whole Numbers
		Different Denominators

Concept development lessons are organized under five unit headings with levels:

1. Understanding Numbers (Defining Numbers, Numbers 0-20, Numbers 21-99, and Numbers 100-999)
2. Number Operations (Addition, Subtraction, Multiplication, and Division)
3. Using Whole Numbers (Money, Time, and Estimation)
4. Understanding Fractions (Fraction Identification, Equivalent Fractions, Comparing Fractions, and Converting Fractions)
5. Fraction Operations (Addition, Subtraction, Multiplication, and Division)

Each concept lesson is taught using the concrete—semiconcrete (illustrative)—abstract sequence recommended by researchers (see Chapter V for research and discussion). There is a Concept Building Introduction that includes the use of manipulatives and a working mat provided by CEI with the software license. The Learn segment is the illustrative (semiconcrete) stage where the student tutor Digit models the use of representations of the manipulatives on the computer screen and places them on a depiction of the working mat. The student copies the model, reinforcing the original instruction. In the abstract stage, students deal with the actual numbers in a problem-solving lesson. Students are provided instruction both in the solving of decontextualized problems and word (or story) problems. The word problem lesson includes strategy instruction—for example, how to determine which operation to use, how to set up the equations, and how to eliminate irrelevant information. There are approximately 15 tasks included in the concept development component of *MLS*.

The Fact Fluency component of *MLS* incorporates powerful multi-sensory processing strategies (see Chapter VI for the efficacy research and discussion) in the facilitation of rapid and accurate retrieval of mathematics facts. It involves varied and engaging practice exercises (again, see Chapter VI) for whole number operations, providing students with adequate repetition to embed the learning into long-term memory and to facilitate retrieval. The Fact Fluency component reflects much of the same research used for the development of *Essential Learning Systems (ELS)*, especially the development of fluency in decoding. An additional ten tasks make up this component. An enhancement was added in 2005 with the design of the web-based activity center (WAC) on CEI's webpage. All students in a school using *MLS* have access to a fact fluency development game called *Digit's Widgets*.

The software includes ample support for the struggling student. For instance, the student can press the space bar for the instruction to be repeated. There is a provision to erase incorrect responses. When a student makes an error, he or she is allowed to try again. If he or she misses the answer a third time, the computer instructor, Digit, does an automatic review or re-teach. Also, every student gets immediate auditory feedback that notes incorrect responses, but in an encouraging way, and praises all correct responses. Appropriate academic English is used for all mathematical terms, and the use of those terms is consistent. Since it is important for students to learn to read and think about equations in both the vertical and horizontal formats, both are used throughout the program. Instructions are provided auditorily and also in a text box at the bottom of the screen.

The following sections return to the research evidence. This time, research on specific problem areas in mathematics is provided, along with an explanation of how each problem in addressed in the *MLS* program.

Specific Problem Areas in Mathematics

The specific kinds of errors that students with difficulties or disabilities make in their struggle to learn mathematics are categorized in this section. No intervention can do everything, so identifying the areas *most critical* to students being able to move forward in learning mathematics and be able to access the general education curriculum forms the basis of an effective intervention. In addition to those lists of manifestations found in the preceding section of this study, the following tables explicate more specific areas where errors are made.

Problems in Learning Mathematics Vocabulary and Concepts

Many students fail to achieve well in mathematics because they fail to develop conceptual knowledge. Chapter II documented several reasons why learners without disabilities may, nevertheless, fail to learn. A major reason is inappropriate instruction—instruction, perhaps, that emphasized a "skills only" approach, ignoring the importance of concepts. In Chapter III the discussion focused on learners with disorders or disabilities. Those students have difficulties due to faulty sensory processing, resulting in their not learning mathematical terms (concepts) or in their not remembering them. Table 43 includes some of those research findings resulting from both difficulties and disabilities. Findings also include the imperative that mathematics instruction, both core and for interventions, include both conceptual knowledge and procedural skills and mutually reinforcing activities. Both are required for mathematics proficiency. As Marshall (2006) states, "The bottom line is that research has shown that things our brain does not understand are more likely to be forgotten. It is part of our makeup" (p. 362).

Table 43: Learning Mathematics Concepts and Vocabulary

Researcher(s)	Findings/Conclusions
Lock, 1996, 6-7	"Teaching key math terms as a specific skill rather than an outcome of basic math practice is essential for students with LD."
Kibel, 1992, 45	"Strong kinesthetic and visual images should underlie mathematical terms. I always arrange for considerable overlearning of the language, and ensure that abstract terms are linked to a concrete base."
Marzano, 1998, 29	"At a practical level, it is fairly obvious that students must understand a certain amount of the basic vocabulary in a subject area before they can understand facts, generalizations, and concepts within a content area."
Kibel, 1992, 45	"Concepts should not be passed on ready-made. They should be allowed to grow in concrete situations and only later should formal written work take place."
Fuson, Kalchman, & Bransford, 2005, 234	"The areas of focus—whole number, rational numbers, and functions—were identified by Case and his colleagues as requiring major conceptual shifts. In the first, students are required to master the concept of *quantity*; in the second, the concept of *proportion* and relative number; and in the third, the concept of *dependence* in quantitative relationships."
Spear-Swerling, n.d., 1	"Because progress in math builds heavily upon previously learned skills, it is important for instruction to be clear, unambiguous, and systematic, with key prerequisite skills taught in advance. For instance, children should not be expected to develop automatic recall of addition facts if they do not understand the basic concept of addition or the meaning of the addition sign."
Donovan & Bransford, 2005, 7	"Using concepts to organize information stored in memory allows for much more effective retrieval and application. Thus, the issue is not whether to emphasize facts or 'big ideas' (conceptual knowledge); both are needed."
Young, n.d., 3-4	"Traditional mathematics tends to focus students on being able to duplicate what the teacher has taught rather than having to understand a concept. Students certainly need to know how to compute, but they must also understand why the computations work and when they should be applied. . . . Knowing the 'basic number facts' are essential, but it no longer stops there."
Fuson, Kalchman, & Bransford, 2005, 232	". . . *How People Learn* suggests the importance of both conceptual understanding and procedural fluency. . . . Recognition of the weakness in the conceptual understanding of students in the United States has resulted in increasing attention to the problems involved in teaching mathematics as a set of procedural competences. At the same time, students with too little knowledge of procedures do not become competent and efficient problem solvers. When instruction places too little emphasis on factual and procedural knowledge, the problem is not solved; it is only changed. Both are clearly critical."
Fuson, Kalchman, & Bransford, 2005, 232	". . . procedural knowledge and conceptual understandings must be closely linked."
Fuson, Kalchman, & Bransford, 2005, 232	"Developing mathematical proficiency requires that students master both the concepts and procedural skills needed to reason and solve problems effectively in a particular domain."

Researcher(s)	Findings/Conclusions
Ontario Ministry of Education, 2005, 74	"Mathematics instruction should focus on the rules and symbols of mathematics (procedural knowledge) and the understanding of concepts and the ability to see relationships (conceptual understanding). An example of procedural knowledge is knowing how to add and subtract; the related conceptual understanding is the recognition of all that is connected to the concept of addition—that it could mean combining two sets, is the reverse operation of subtraction, and is commutative."
Ontario Ministry of Education, 2005, 74	"Children with special needs show the most growth in mathematical understanding when instruction concurrently addresses both procedural knowledge and conceptual understanding. . . . Moreover, evidence suggests that learning foundational math skills leads to greater conceptual math knowledge and that learning more conceptual math knowledge affects learning of foundational skills (Rittle-Johnson, Siegler, & Alibali, 1998)."
Bohan, 2002, 35	"It is clear that mathematical competence involves both a set of skills and procedures and the conceptual understanding to apply these skills and to extend the body of understanding as new and challenging situations are encountered."
Sousa, 2001, 153	"Mathematical disorders often arise when students fail to understand the language of mathematics, which has its own symbolic representations, syntax, and terminology. Solving word problems requires the ability to translate the language of English into the language of mathematics. The translation is likely to be successful if the student recognizes English language equivalents for each mathematical statement. . . . Learning to identify and correctly translate mathematical syntax becomes critical to student success in problem solving."
Sousa, 2001, 153	"Language can be an obstacle in other ways. Students may learn a limited vocabulary for performing basic arithmetic operations, such as 'add' and 'multiply,' only to run into difficulties when they encounter expressions asking for the 'sum' or 'product' of numbers. Teachers can avoid this problem by introducing synonyms for every function."
O'Brien & Moss, 2004, 292	"The fact is that an exclusive emphasis on rote memory and rote performance of computational procedures at a time when every desktop computer can do billions of computations in a second is downright foolish. Of course, children should learn to add, subtract, multiply, and divide, and they should do so sensibly and efficiently. And while shopkeeper's arithmetic is important, it is a very small part of real math. . . . Above all, school mathematics should involve making sense."
Zemelman, S., Daniels, H., & Hyde, A., 1998, 90	"Once they have a good conceptual foundation for the meaning of the operations of addition and subtraction (and later, multiplication and division), students should memorize the basic facts so they are able to calculate mentally and estimate with ease."
Zemelman, S., Daniels, H., & Hyde, A., 1998, 90	"'Knowing' the math facts without true understanding of the underlying concepts guarantees serious problems with learning other concepts in the mathematics curriculum."
Whitehurst, n.d., 4	". . . conceptual understanding is a good thing because it can tie together mathematical tasks that might otherwise seem disconnected to a child."
Wu, n.d., 7	"The fact must be faced that, in mathematics, one cannot have understanding without technique. The two are intertwined."
National Research Council, 2001, 182	"More than just a means to produce answers, computation is increasingly seen as a window on the deep structure of number sense. Fortunately, research is demonstrating that both skilled performance and conceptual understanding are generated by the same kinds of activities. No tradeoffs are needed. . . . the activities that provide this powerful result are those that integrate the strands of proficiency."
Cawelti, 1999, 121	"Research suggests that students who develop conceptual understanding early perform best on procedural knowledge later. . . . Students without conceptual understanding are able to learn procedural knowledge when the skill is taught, but research suggests that students with low levels of conceptual understanding need more practice to acquire procedural knowledge."

Researcher(s)	Findings/Conclusions
Lochy, Domahs, & Delazer, 2005, 474	"Not only skill learning and efficient fact retrieval, but also conceptual learning, should be taken into account in planning rehabilitation."
Geary & Hoard, 260	". . . competencies in any given area of mathematics will depend on a conceptual understanding of the domain and procedural knowledge that supports actual problem solving."
Marzano, 1998, 30	"Although vocabulary terms and facts are important, generalizations help students develop a broad knowledge base because they transfer more readily to different situations."
Lochy, Domahs, & Delazer, 2005, 474	"Simple calculation can be perceived as a complex interaction between simple retrieval from memory, execution of procedures, and application of conceptual knowledge (Delazer, 2003). Thus, in addition to drill, the rehabilitation of simple calculation can, in principle, also concentrate on the improvement of procedural or conceptual knowledge."
Sherman, Richardson, & Yard, 2005, 4-5	"Students who complete algorithms with little understanding quickly forget or confuse the procedures. . . . Understanding is the underpinning of skill work (Clements, 1997; Piaget, 1965)."
Sherman, Richardson, & Yard, 2005, 5	"Understanding fundamental concepts and accurately completing algorithms contribute to becoming numerate (mathematically proficient)."
Lochy, Domahs, & Delazer, 2005, 476	". . . repeated exposure to the problem will be the appropriate method if the aim is a more efficient retrieval. But in most cases of calculation deficits, pure drill will not lead to well-connected and meaningful knowledge. Importantly, skills not supported by concepts remain error prone and inflexible."
Lochy, Domahs, & Delazer, 2005, 476	"As findings from developmental research suggest, advances in skills and conceptual knowledge may be seen in an iterative relation (Baroody, 2003). Conceptual knowledge may lead to advances in procedures, the application of which can lead to improved knowledge and so forth. Adopting this iterative view, intervention in one domain may lead to benefits in the other."
Barton & Heidema, 2002, 39	"In addition to factual knowledge and skills, conceptual understanding is essential to mathematics."
Ainsworth & Christinson, 2000, 19	"Developing students' conceptual understanding is at the heart of effective instruction."
Wu, 1999, 1	"'Facts vs. higher order thinking' is an[other] example of a false choice that we often encounter these days, as if thinking of any sort—high or low—could exist outside of content knowledge. In mathematics education, this debate takes the form of 'basic skills or conceptual understanding.' This bogus dichotomy would seem to arise from a common misconception of mathematics held by a segment of the public and the education community: that the demand for precision and fluency in the executive of basic skills in school mathematics runs counter to the acquisition of conceptual understanding. The truth is that in mathematics, skills and understanding are completely intertwined. In most cases, the precision and fluency in the executive of the skills are the requisite vehicles to convey the conceptual understanding. There is not 'conceptual understanding' and 'problem-solving skill' on the one hand and 'basic skills' on the other. Nor can one acquire the former without the latter."
Cawley, 2002, 2	"I support the proposition that outcomes for students should include both knowing about and doing mathematics and that both knowing and doing should be included as dependent variables in intervention research. Knowing about means the student comprehends the basic principles of the mathematics and knows that there is more than one way to explain the mathematics and that there is frequently more than one acceptable answer. Doing mathematics means the student can do mathematics in many different ways and apply a number of different strategies and mathematics principles to complete an item. Presently, the knowing is neglected and the doing is overemphasized. I do not believe that the number of correct computations is a sufficient basis on which to stipulate that an improvement in mathematics performance has occurred. Improvement indicators should include explanations of knowing, proving, and generalization."

Researcher(s)	Findings/Conclusions
Ball, Ferrini-Mundy, Kilpatrick, Milgram, Schmid, & Schaar, 2005, 2	"Mathematics requires careful reasoning about precisely defined objects and concepts. Mathematics is communicated by means of a powerful language whose vocabulary must be learned. The ability to reason about and justify mathematical statements is fundamental, as is the ability to use terms and notation with appropriate degrees of precision. By *precision*, we mean the use of terms and symbols, consistent with mathematical definitions, in ways appropriate for students at particular grade levels. We do not mean formality for formality's sake."
Bruer, 1993, 90	"For mathematics to be meaningful, conceptual knowledge and procedural skills have to be interrelated in instruction."
Bruer, 1993, 95-96	"Buggy arithmetic does not necessarily arise because children lack understanding of the number system. More often the problem is that children make no connection between their number knowledge and written arithmetic procedures. . . . Resnick found no correlation between children's number knowledge and their computational skills.
Bruer, 1993, 98	"With appropriate, explicit instruction to marry concepts and procedures, the children learned to manipulate symbols meaningfully."
Bruer, 1993, 98	". . . for many adults there is little connection between conceptual knowledge and procedural skills. Mathematics as a high-order cognitive skill, a skill that can be flexibly applied to novel problems, requires that concepts and skills be interconnected."
Ontario Ministry of Education, 2005, 73	"If students fail to develop a conceptual understanding at an early stage, they will continue to have difficulties learning new concepts until foundational concepts have been mastered."
Cawelti, 1995, 98	"Investigations have consistently shown that an emphasis on teaching for meaning [emphasis on the mathematical meanings of ideas, including how the idea, concept, or skill is connected in multiple ways to other mathematical entities] has positive effects on student learning, including better initial learning, greater retention, and increased likelihood that the ideas will be used in new situations. These results have also been found in studies conducted in classrooms in high-poverty areas."
Mercer & Mercer, 2005, 433	"The learning of concepts and rules also is germane to facilitating a student's understanding of math."
Griffin, 2005, 261	". . . the central conceptual structure for whole number has been found to be central to children's mathematics learning and achievement in at least two ways. First, . . . it enables children to make sense of a broad range of quantitative problems in a variety of contexts. . . . Second, it provides the base—the building block—on which children's learning of more complex number concepts, such as those involving double-digit numbers, is built . . . . Consequently, this network of knowledge is an important set of understandings that should be taught."
Klein, 2005, 22	"Prerequisites cannot be discarded. They are essential to mathematics. The failure to develop appropriate prerequisites and mathematical reasoning based on these prerequisites leads to the generations of mathematic standards into what might be described as mathematics appreciation."
Klein, 2005, 24	"Of particular importance is a coherent and thorough development of arithmetic in the early grades, both in terms of conceptual understanding and computational fluency. Without a solid foundation in this most important branch of mathematics—arithmetic—success in secondary school algebra, geometry, trigonometry, and pre-calculus is impossible."
Leinwand & Fleischman, 2004, 88-89	"Since the 1980's several studies have examined the role and impact of instrumental versus relational practices on student achievement outcomes. Although the research base is limited and should be replicated to validate the findings, results consistently point to the importance of using relational practices of teaching mathematics. In the existing research, students who learn rules before they learn concepts tend to score significantly lower than students who learn concepts first."

Researcher(s)	Findings/Conclusions
Leinwand & Fleischman, 2004, 89	"This month's column offers some research-based guidelines for mathematics instruction with the hope that they will support improved student achievement. The research message: Teach for meaning initially, or risk never getting students beyond a superficial understanding that leaves them unprepared to apply their learning."

***MLS* Application.** Clearly, given the wealth of available evidence, concept development is critically important in teaching mathematics; it is a major area of student difficulty, regardless of cause, in learning mathematics to the proficiency level; and it is, therefore, a primary focus in an effective mathematics intervention. *MLS,* from its inception, includes as an area of emphasis the development of foundational concepts, as is evident in the scope and sequence (Table 40) and other supporting documents (such as the Teacher's Manual), as well as in a review of the program itself.

MLS includes five broad concept areas: (1) Understanding Numbers, (2) Number Operations, (3) Using Whole Numbers, (4) Understanding Fractions, and (5) Fraction Operations. Further, the design includes a careful and consistent use of accurate mathematical terms in the instruction, directions, modeling, and feedback so that mathematics vocabulary is continually reinforced—especially important for English language learners (ELLs). The strategies used for concept development, including the concrete—semiconcrete—abstract lesson sequence and the use of manipulatives, are defined and the research documented in Chapter V.

In the next section, the research on several specific concepts is provided, along with a description of ways that *MLS* incorporates instruction on that topic.

Counting

Just as the research on the importance of concept development often includes admonitions that concept teaching has to include procedural skills (and the reverse), so do the lines blur on the difference between a concept and a procedure. For instance, counting is a concept, as is the long-division algorithm a concept, but both terms also suggest procedural skills—the "doing" of counting or division. No attempt was made to separate those out in this analysis since they are not separated in what occurs in a classroom, in the *MLS* program, nor, for that matter, in the research findings.

Table 44 includes the research findings related to counting. Counting, of course, is an essential prerequisite to learning elementary arithmetic. Also, one of the first signs that a learner is struggling is his/her use of inaccurate or immature counting strategies.

Table 44: Counting

Researcher(s)	Findings/Conclusions
Fayol & Seron, 2005, 15	"As rightly noted by Butterworth (1999), in all human culture, children use their fingers to count before they are systematically taught arithmetic in school."
T. Miles, 1992a, 6	". . . the reason why many dyslexics use their fingers or put marks on paper when doing calculations is because the requisite number fact is not immediately available to them by any other means. It is thus a typically dyslexic 'compensatory strategy.'"

Researcher(s)	Findings/Conclusions
Geary & Hoard, 2005, 258	"In comparison to their normal peers, children with MD [mathematical disabilities] often (a) rely on developmentally immature strategies, such as finger counting, (b) frequently commit counting errors, (c) use immature counting procedures (they often use *sum* counting rather than *min* counting), and (d) have difficulties retrieving basic facts from long-term memory."
Woodward & Montague, April 2002, 18	"Several studies suggest that students with learning disabilities tend to use immature strategies when they learn math facts. . . . Students with learning disabilities tended to use counting all strategies (i.e., laborious one-by-one counting to achieve answers) even after extended practice."
Fuchs & Fuchs, 2002, 2	". . . students with MD scored worse on counting large numbers, identifying multiples of large numbers, and using basic fact within calculations (Russell & Ginsburg, 1984). Counting difficulties and persistently deficient fact retrieval have been well documented for MD. . . ."
Siegler, 2003, 223	"Rather than adding by using the same strategy all of the time, children use a variety of strategies from early in learning, and continue to use both less and more advanced approaches for periods of many years. Thus, even early in first grade, the same child, given the same problem, will sometimes count from one, sometimes count from the larger addend, and sometimes retrieve the answer. Even when children master strategies that are both faster and more accurate, they continue to use older strategies that are slower and less accurate as well. This is true not just with young children, but with preadolescents, adolescents, and even adults (Kuhn, Garcia-Mila, Zohar, & Anderson, 1995; Schauble, 1996)."
Cordes & Gelman, 2005, 128	"Much of the early work on numerical knowledge involves one or another counting task."
Brysbaert, 2005, 23	"The finding that people spontaneously start to count numerosities larger than 4 shows how important symbolic representations are for human numerical cognition."
Siegler, 2003, 225	"Limited conceptual understanding of arithmetic operations and counting adds further obstacles to these children's learning of arithmetic (Hitch & McAuley, 1991; Geary, 1994)."
Geary, Feb. 2004, 7	"The development of procedural competencies is related in part to improvements in children's conceptual understanding of counting and is reflected in a gradual shift from the frequent use of counting all to counting on (Geary et al., 1992; Siegler, 1987)."
National Research Council, 2001, 72	"At first, school arithmetic is mostly concerned with the whole numbers: 0, 1, 2, 3, and so on. The child's focus is on counting and on calculating—adding and subtracting, multiplying and dividing. Later, other numbers are introduced: negative numbers and rational numbers (fractions and mixed numbers, including fine decimals). Children spend considerable effort learning to calculate with these less intuitive kinds of numbers."
Siegler, 2003, 224	". . . children actually learn better when they are allowed to choose the strategy that they wish to use. Immature strategies generally drop out naturally when students have enough knowledge to answer accurately without them. Even basic strategies such as counting fingers allow students to generate correct answers when forbidding use of the strategies would lead to many errors."
Fazio, 1999, 428	". . .current research suggests that counting speed is slow in children with SLI [specific language impairment] (Donlan, 1998)."
Verschaffel, Greer, & Torbeyns, 2006, 54	". . . low achievers and children with MD [math disabilities] use the same types of strategies as their normally achieving peers, but rely more often on immature counting strategies and less often on more efficient mental strategies than the latter."

***MLS* Application.** Levels 2, 3, and 4 of Unit 1, Understanding Numbers, include lessons relating to counting. Level 1 includes Identifying and Recognizing Numbers 0-10, Level 2 includes counting 0-20, Level 3 includes 21-99, and Level 4 includes 100-999. Students learn the base-10 system and the place value concept in Unit 1, along with the explicit instruction on counting.

Place Value

One of the fundamental concepts taught in arithmetic is the place value construct of the base-ten system. A deep understanding of place value facilitates students' acquisition of algorithms, for without it there is no meaning behind the procedures that are being taught. The base-ten system in itself is conceptual (declarative knowledge). That is why that concept is explicitly taught using the concrete-semiconcrete-abstract lesson sequence and why the manipulatives are used. Adding numbers, which requires the use of regrouping (which, of course, relies on an understanding of the base-ten concept), is procedural (procedural knowledge). Applying the algorithm is also procedural, although understanding which algorithm to apply depends upon conceptual understanding. Knowing which algorithm to use in problem-solving is strategic competence, and being able to explain and justify one's decisions is adaptive reasoning. The lessons and applications of the base-ten system, therefore, include four of the five critical strands from the National Research Council's (2001) *Adding It Up* that were discussed in this chapter's overview. Table 45 includes the research on place value.

Table 45: Place Value

Researcher(s)	Findings/Conclusions
Sherman, Richardson, & Yard, 2005, 17	"Place value is perhaps the most fundamental concept imbedded in the elementary and middle school mathematics curriculum."
Mathematics Standards Study Group, 2004, 1	"Whole number arithmetic and the place value system are the foundation for school mathematics with most other mathematical strands evolving from this foundation. This foundation should be the subject of most instruction in early grades."
Mathematics Standards Study Group, 2004, 2	". . . we cannot overemphasize the requirement of a firm foundation in arithmetic and the place value system, both as preparation for mastery of later school mathematics and as a model for the power of mathematical methods."
Sherman, Richardson, & Yard, 2005, 36	"Conceptual understanding of place value is possible when lessons are designed in a developmental learning sequence as follows: 1. Materials and diagrams are used to express multidigit numerals. 2. Numeric symbols are connected to material by writing the numerals that represent the quantity in the presence of manipulatives. 3. Number words are connected to numerals and materials that represent a quantity."
McEwan, 2000, 70	"Without a thorough understanding of the concept of numeration (i.e., the base-10 place value system), a student will be unable to succeed in mathematics."
McEwan, 2000, 71	"A thorough understanding of the base-10 system is important for three reasons (Geary, 1994, 44-46). First, students cannot grasp the conceptual meaning of spoken and written multidigit numbers without a thorough understanding of the base-10 system. "Second, understanding that multidigit numbers represent groups of 100s, 10s, and 1s influences the sophistication of the problem-solving strategies the student can use to solve complex arithmetic problems. "And, finally, understanding the base-10 system is important in order to be able to regroup and figure out place value. Fuson (1988) recommends that kindergarten children learn basic word names and be introduced to the base-10 system. Since number words are arbitrary, simply hearing the sounds associated with the words offers no clues about their meaning. So even children who understand that counting and quantity are related—and Gelman and Gallistel believe that it is innate—may have to memorize number names (Gelman & Gallistel, 1978)."

Researcher(s)	Findings/Conclusions
McEwan, 2000, 71-72	"the Big Ideas in Numeration 1. Understanding the base-10 system and place value are absolutely essential mathematical learnings. 2. Students should be carefully and thoughtfully introduced to place value in kindergarten. 3. Students should learn the number names and be able to count to 30 by the end of kindergarten. 4. Students who are having difficulties in mathematics in upper grades should be assessed both formally with a paper and pencil test and informally through interview and observation for their understanding of place value to determine if remediation is necessary."
Geary, n.d., 2	"The learning of basic number skills is much more complicated than many adults would assume. . . . The most difficult feature of the number system is its base-10 structure. . . . Coming to really understand the base-10 system is difficult for all children, but is essential, as this conceptual knowledge is important for the mastery of other domains. . . ."
Sherman, Richardson, & Yard, 2005, 37	"Understanding a place value system, base ten or non-base ten, is fundamental to computing and number sense. Remediation should be based on determining whether students' errors are based upon conceptual or rule misunderstandings. If the former is true, students can be assisted by bundling objects in groups of ten, trading them, and recording the trades in diagrams and with numerals. . . . If students' errors are the results of forgetting rules, activities that focus on paper-and-pencil games can be very helpful."
Sherman, Richardson, & Yard, 2005, 37	"The trading aspect of learning about place value is essential to conceptual development. Students bundle objects, exchange place value blocks, and indicate the trades on place value charts. Lastly, results are recorded with numerals and words. Skills can be practiced and sustained by providing students with frequent and targeted instructional feedback."
Sherman, Richardson, & Yard, 2005, 41	"Addition of whole numbers is represented in the physical world by the union of sets. The cardinal numbers of the sets involved are represented by addends. . . . As children work to understand the addition concept, developmental work with understanding the algorithm for combinations of two or more addends is begun. Children learn algorithmic understandings of place value and procedures of 'regrouping and renaming' by using the same developmental process that they use to learn the foundations of set unions to represent addition. First, learners combine sets of objects, such as base ten blocks, to demonstrate ability to combine addends and determine the sum. Then learners' drawings are connected to the manipulative work of unioned sets. . . . Finally, the symbols that represent the drawings and objects are written and the regrouping is indicated with tally marks to aid in visualizing the regrouping process."
Klein & Milgram, n.d., 4	"Prior to teaching long division a teacher has to be sure that students understand place value."
Sherman, Richardson, & Yard, 2005, 19	"Misunderstandings and errors are evident in student work when place value concepts and procedures are learned in isolation from previous knowledge and with little meaning (Baroody, 1990)."

***MLS* Application.** Place value lessons in *MLS* begin in Unit 1, Level 3 and continue through Level 4. The concept is, of course, reinforced in Unit 2 when the operations for addition, subtraction, multiplication, and division are taught. Strategic use of the manipulatives provide concrete experiences in the base-ten system. Students receive critical reinforcement and clear instruction in the nature of the base-ten system and the relevant coding used in common arithmetic algorithms for each number operation.

Number Sense

"Number sense," according to Gersten and Chard (1999), "is difficult to define but easy to recognize." Their definition follows:

> Number sense is an emerging construct (Berch, 1998) that refers to a child's fluidity and flexibility with numbers, the sense of what numbers mean and an ability to perform mental mathematics and to look at the world and make comparisons (p. 3).

They add that number sense "not only leads to automatic use of math information, but also is a key ingredient in the ability to solve arithmetic computations" (p. 4). They continue: "Knowing that 15 is much further away from 8 than 11 requires an awareness of 8 + 7 and 8 + 3. However, more than 100 basic addition facts must be memorized to automaticity before students can experiment with this type of interesting problem" (p. 4).

Gersten and Chard (1999) see "number sense" to be for mathematics what "phonemic awareness" is for reading—a critically important, but insufficient, prerequisite to learning the domain. They hypothesize that, just as an understanding that many reading problems are due to faulty sensory processing has led to improved prevention, remediation, and intervention reading strategies, the same could happen for mathematics if the role of number sense is understood by educators (p. 1).

There are numerous parallels. Students require fluency to recognize and decode words and to work efficiently with math facts to solve problems. Without automaticity, a learner's working memory is consumed in decoding or, for mathematics, in attempting to retrieve or calculate the math facts, leaving no available memory required for reading comprehension or for problem solving. Poor development of number sense results, then, in poor understanding of mathematics—its relevance and its concepts.

Gersten and Chard (1999) note that "there is increasing empirical support for its [number sense] relationship to underlying deficits in learning disabilities (Geary, 1993; McCloskey & Macaruso, 1995) and some support that instruction including number sense activities leads to significant reductions in failure in early mathematics (Griffin et al., 1994)." Then they add:

> Moreover, we submit that simultaneously integrating number sense activities with increased number fact automaticity rather than teaching these skills sequentially—advocated by earlier special education mathematics researchers such as Pellegrino and Goldman (1987)—appears to be important for both reduction of difficulties in math for the general population and for instruction of students with learning disabilities. It is also likely that some students who are drilled on number facts and then taught various algorithms for computations may never develop much number sense, just as some special education students, despite some phonics instruction and work on repeated readings/fluency and accuracy, fail to develop good phonemic awareness or any sense of the pleasure of reading (p. 4).

Table 46 includes other researchers' conclusions relating to number sense.

Table 46: Number Sense

Researcher(s)	Findings/Conclusions
Cawelti, 1995, 104	"'Number sense' is a construct that relates to having an intuitive feel for number size and combinations as well as the ability to flexibly work with numbers in problem situations in order to make sound decisions and reasonable judgments. Number sense involves being able to flexibly use the processes of mentally computing, estimating, sensing number magnitudes, moving between representation systems for numbers, and judging the reasonableness of numerical results."
Griffin, 2005, 260	"This research suggests that the following understandings lie at the heart of number sense. . . : (1) they know the counting sequence from 'one' to 'ten' and the position of each number word on the sequence (e.g., that 'five' comes after 'four' and 'seven' comes before 'eight'); (2) they know that 'four' refers to a set of a particular size (e.g., it has one fewer than a set of five and one more than a set of 3), and thus there is no need to count up from 'one' to get a sense of the size of this set; (3) they know that the word 'more' in the problem means that the set of four chocolates will be increased by the precise amount (three chocolates) given in the problem; (4) they know that each counting number up in the counting sequence corresponds precisely to an increase of one unit in the size of a set; and (5) it therefore makes sense to count on from 'four' and to say the next three numbers up in the sequence to figure out the answer. . . ."
Cawelti, 1995, 104	"Teaching mathematics with a focus on number sense encourages students to become problem solvers in a wide variety of situations and to view mathematics as a discipline where thinking is important."
Committee on *How People Learn,* 2005, 259	"In the NCTM standards, number sense is the major learning objective in the standard (numbers and operations) to which primary school teachers are expected to devote the greatest amount of attention."

***MLS* Application.** All five units in *MLS* emphasize numbers and operations, with the development of number sense being a major objective. *MLS'* core instructional methodologies provide adequate rehearsal with physical objects to create concrete understanding of individual numbers and how they grow and change using the various operations. Students develop deep number sense through relevant physical experiences. Also, the fluency component of *MLS*, which includes 10 or more different tasks does precisely what Gersten and Chard (1999) advocate. It intertwines the practice exercises to develop fluency and automaticity with activities that build number sense.

Algorithms

Algorithms constitute a set of mathematical procedures that are important in students becoming proficient in mathematics. In fact, algorithms are a major part of what the researchers at the National Research Council (2001) mean when they refer to the importance of "procedural fluency—skill in carrying out procedures accurately, efficiently, and appropriately" (p. 5). Marzano (1998) defines "algorithms" as "even more specific types of processes than tactics" (p. 34). He continues as follows:

> These processes normally do not vary in application, they have very specific outcomes, and frequently they must be learned to the level of automaticity to be useful. For example, many computing processes in mathematics and decoding passages in reading are algorithmic in nature (p. 34).

The specific steps of an algorithm are many times culturally determined (see Chapter II's discussion of cultural effects on learning mathematics for English-language learners). Table 47 presents evidence of the importance of algorithms in learning mathematics to a proficient level and on difficulties that many students have in learning and remembering algorithms.

Table 47: Algorithms

Researcher(s)	Findings/Conclusions
National Research Council, 2001, 103	"An algorithm is a 'precisely-defined sequence of rules telling how to produce specified output information from given input information in a finite number of steps.' More simply, an algorithm is a recipe for computation."
Committee on *How People Learn,* 2005, 233	". . . the less-advanced students in a classroom also need to be considered. It can be helpful for either a curriculum or teacher of such less advanced students to select an accessible method that can be understood and is efficient enough for the future, and for these students to concentrate on learning that method and being able to explain it."
T. Miles, 1992a, 12	"When one looks at the attempts of dyslexic students to do subtraction and addition, the overall picture is often that of a highly sophisticated person, well capable of quite complex logical reasoning, who is nevertheless severely restricted in his ability to give instant answers, and who therefore has to resort to strategies—often of his own devising—which are time-consuming and may sometimes involve considerable risk of error."
Sherman, Richardson, & Yard, 2005, 67	"Operations of addition and subtraction should be understood by the beginning of grade 3, with instruction focusing on strategies for computing with whole numbers."
Dehaene, Piazza, Pinel, & Cohen, 2005, 434	". . . recent behavioral studies have made clear that mental arithmetic relies on a highly composite set of processes, many of which are probably not specific to the number domain. For instance, studies of language interference in normal subjects suggest that language-based processes play an important role in exact but not approximate calculation (Spelke & Tsivkin, 2001). Likewise, concurrent performance of a language task interferes with multiplication but not subtraction (Lee and Kang, 2002). Such behavioral dissociations suggest that the neural bases of calculation must be heterogeneous."
McEwan, 2000, 76	"Students should be expected to master the standard algorithms for addition, subtraction, multiplication, and division of whole numbers at some agreed-upon point in their school careers. This might be a possible timetable (California State Board of Education, 1999, 16, 21): a. Standard algorithms for the addition and subtraction of multidigit numbers by end of fourth grade. b. Standard algorithms for multiplying a multidigit number by a two-digit number and for dividing a multidigit number by a one-digit number by the end of fourth grade. c. Standard algorithm for long division with multidigit divisors by the end of fifth grade."
Raimi, 2002, 2	". . . a child bereft of the 'regrouping' algorithm for multidigit subtraction has lost something deeper than a quick answer."
Fazio, 1999, 428	"Research on school-age children with language learning disorders suggests that two types of instruction foster procedural knowledge in arithmetic. One instructional technique is designed to make children more aware of the steps involved in various arithmetic operations."

Researcher(s)	Findings/Conclusions
Ball, Ferrini-Mundy, Kilpatrick, Milgram, Schmid, & Schaar, 2005, 3	"Students should be able to use the basic algorithms of whole number arithmetic fluently, and they should understand how and why the algorithms work. Fluent use and understanding ought to be developed concurrently. These basic algorithms were a major intellectual accomplishment. Because they embody the structure of the base-ten number system, studying them can reinforce students' understanding of the place value system. "More generally, an algorithm is a systematic procedure involving mathematical operations that uses a finite number of steps to produce a definite answer. An algorithm can be implemented in different ways; different recording methods for the same algorithm do not constitute different algorithms. The idea of an algorithm is fundamental in mathematics. Studying algorithms beyond those of whole number arithmetic provides opportunities for students to appreciate the diversity and importance of algorithms."
National Research Council, 2001, 197	"Some students need help to develop efficient algorithms . . ., especially for multiplication and division. Consequently, for these students the process of learning algorithms involves listening to someone else explain an algorithm and trying it out, all the while trying to make sense of it. Research suggests that students are capable of listening to their peers and to the teacher and making sense of an algorithm if it is explained and if the students have diagrams or concrete materials that support their understanding of the quantities involved."
Duverne & Lemaire, 2005, 399	"Overall, older adults appeared able to learn new arithmetic algorithms but less efficiently than their younger peers."
Klein & Milgram, n.d., 1	"There is a long standing consensus among those most knowledgeable in mathematics that standard algorithms of arithmetic should be taught to school children. Mathematicians, along with many parents and teachers, recognize the importance of mastering the standard methods of addition, subtraction, multiplication, and division in particular. . . . It is unfortunate that during the past decade, and even before, mathematics education leaders in the United States have called into question the practice of requiring elementary school students to master these standard algorithms. Long division has been especially targeted for de-emphasis, or even elimination from the school curriculum."
Akin, 2001, 4	"I believe that most mathematicians share my belief that systems like the Arabic number notation with the associated algorithms for multiplication and division, and the symbolisms of fractions and of algebra are really triumphs of human ingenuity and that to learn them is to acquire tools of great beauty as well as power. We strongly feel that their use should be encouraged rather than avoided."
Klein & Milgram, n.d., 3-4	". . . a committee of the American Mathematical Society (AMS), formed for the purpose of representing the views of the AMS to the National Council of Teachers of Mathematics, published a report which stressed the mathematical significance of the long division algorithm, as well as addressing other mathematical issues. An excerpt from this report published in the February 1998 issue of the *Notices of the American Mathematical Society* is illuminating: We would like to emphasize that the standard algorithms of arithmetic are more than just 'ways to get the answer'—that is, they have theoretical as well as practical significance. For one thing, all the algorithms of arithmetic are preparatory for algebra, since there are (again, not by accident, but by virtue of the construction of the decimal system) strong analogies between arithmetic of ordinary numbers and arithmetic of polynomials. The division algorithm is also significant for later understanding of real numbers."
Raimi, 2002, 2	"Children brought up on calculator computation are weak in understanding. They can be drilled in the meaning of the decimal notation, yes, but if in addition they learn the standard algorithms of arithmetic, with due care to the exposition that explains the meaning of 'carry the two' in terms of the place value of the 'two' in question, which might be 20 or .002 according to the case, they will have arrived at an internalization of the sense of our number system that no other way has been shown to accomplish."

<table>
<tr><th>Researcher(s)</th><th>Findings/Conclusions</th></tr>
<tr><td>Raimi, 2002, 3</td><td>“If the problem had been to subtract 178 from 3562, the old tried-and-true subtraction algorithm would have produced the answer without thought and without fail, while children ‘discovering their own algorithms’ have been seen floundering for hours trying to get an answer, coming home without learning a thing, and with a great dislike of math class.”</td></tr>
<tr><td>Ocken, 2001, 13</td><td>“Students who come equipped with symbolic manipulation skills are not guaranteed success in college mathematics. However, those who lack such skills face virtually certain failure in any math or physics course that has not been watered down by the evisceration of algebraic content.”</td></tr>
<tr><td>Ocken, 2001, 14</td><td>“Among the standard algorithms, the long division algorithm is perhaps the most important as preparation for higher mathematical study. Elementary school students deprived of exposure to and practice with that algorithm will be severely handicapped when they encounter applications and generalizations that surface at several stages of their ensuing mathematical education.”</td></tr>
<tr><td>Klein, 2005, 16</td><td>“Knowing the standard algorithms, in the sense of being able to use them and understanding how and why they work, is the most sophisticated mathematics that an elementary school student is likely to grasp. Students who have mastered these algorithms gain confidence in their ability to compute. They know that they can solve any addition, subtraction, multiplication, or division problem without relying on a mysterious black box, such as a calculator. Moreover, the ability to execute the arithmetic operations in a routine manner helps students to think more conceptually.”</td></tr>
<tr><td>Sherman, Richardson, & Yard, 2005, 43</td><td>“Students typically have a good grasp of additive reasoning by the third grade.”</td></tr>
<tr><td>Sherman, Richardson, & Yard, 2005, 89</td><td>“Multiplication of whole numbers is represented in the physical world by unioning multiple sets of equal cardinality. . . . Early work with multiplication should mainly be devoted to the conceptual understanding that multiplication is a shorthand notation for denoting multiple addition. Multiplication situations should be presented to children, and they should then use materials (beans, counters, etc.) to demonstrate the problem given and to generate an answer (a product).”</td></tr>
<tr><td>Sherman, Richardson, & Yard, 2005, 89-90</td><td>“It is essential that children understand that the multiplication problem expresses a relationship between the numbers involved and that they own the meaning of the symbolism—that the first factor in the problem denotes the number of sets and the second factor denotes the number of objects contained in each set. The product is then the total number of objects when the sets are joined (unioned).”</td></tr>
<tr><td>Siegler & Booth, 2005, 201</td><td>“It is . . . possible to classify computational strategies at a more specific level. The following is a list of the most common strategies for addition and multiplication:
1. Rounding: Converting one or both operands to the closest number ending in one or more zeroes (on 297 x 296, both multiplicands might be converted to 300).
2. Truncating: Changing to zero one or more digits at the right end of one or more operands (on 297 x 296, both multiplicands might be converted to 290).
3. Prior Compensation: Rounding the second operand in the opposite direction of the first before performing any computation (on 297 x 296, 296 might be rounded to 290 rather than 300 to compensate for the effect of rounding 297 to 300).
4. Postcompensation: Correcting after a computation has been done for distortion introduced by earlier rounding or truncation (on 297 x 296, subtracting 2% from the product after multiplying 300 x 300).”
5. Decomposition: Dividing numbers into simpler forms (on 282 x 153, multiplying 280 x 10 x 15).
6. Translation: Simplifying an equation, for example, by changing the operation (e.g. on 44 + 53 + 51 + 47, multiplying 50 x 4).
7. Guessing.
As might be expected, some of these strategies are used more often than others. Rounding is the most common approach; compensation tends to be the least frequently used.”</td></tr>
</table>

Researcher(s)	Findings/Conclusions
Mathematics Standards Study Group, 2004, 2	"Multi-digit arithmetic algorithms are a quintessential example of how a powerful mathematical theory is constructed. From single-digit addition facts, one derives the facts for subtraction and multiplication, and from multiplication comes division. Thus, a methodology is developed to add, subtract, multiply, and divide any two numbers. This theory extends naturally to the arithmetic of fractions and decimals. More complicated calculations in algebra and later in college mathematics all are done using further incremental extensions of these basic algorithms. For this reason we want to stress the importance of these algorithms for students as preparation for studying mathematics in high school and, for the majority, later in college."
Klein & Milgram, n.d., 7	"Just as multiplication of counting numbers is based on repeated addition, the inverse operation of division may be understood in terms of repeated subtractions."
Sherman, Richardson, & Yard, 2005, 90	"In grades 3-5, a central focus should be directed at helping children develop the conceptual meaning for whole number multiplication and division (NCTM, 2000). Multiplication and division can begin to have meaning in earlier grades by engaging children in problem situations that utilize multiplication concepts for solution. Further, developing fluency involves a connection and a balance between conceptual understanding and computational proficiency. Understanding without fluency can inhibit children's problem solving abilities (Thornton, 1990)."
Sherman, Richardson, & Yard, 2005, 113	"Division of whole numbers is represented in the physical world by partitioning and by measurement. Each of these conceptualizations can be represented in the physical world with real world examples."
Sherman, Richardson, & Yard, 2005, 114	"As children progress in conceptualizing division, work should focus more on the partitioning notion. Partitioning is the basis for understanding the traditional algorithm. Partitioning also forms the readiness foundations for later understanding of equations such as $3x = 14$. While many invented algorithms may allow children to perform division, the traditional algorithm is the most precise and efficient method, which is why it has evolved over the centuries as the preferred algorithm."
Sherman, Richardson, & Yard, 2005, 114	"For children in grades 3 through 5, a central focus should be the conceptual understanding of division (and multiplication) as well as the exploration of algorithmic strategies that are invented, recorded, and discussed. You, as the teacher, have the opportunity to provide a foundation for children to develop and understand efficient, accurate algorithms. The foundation of such algorithm conversations should be conceptual understanding of the operation (NCTM, 2000)."
Mathematics Standards Study Group, 2004, 2	"Mastering addition, subtraction, multiplication, and division facts needs to be an incremental, evolving process, which carefully extends previous knowledge and constantly lays a solid foundation for future knowledge. Informal multiplication can begin very early with counting by 2's, 3's, 4's, or 5's. And simultaneously division can start with finding how many groupings of 2's, 3's, 4's, or 5's can be made from a given pile of, say, sticks."
Mathematics Standards Study Group, 2004, 2	". . . from an early age, students need to be developing an understanding of the algebraic structure underlying arithmetic, e.g., that subtraction is the inverse of addition and later that division is the inverse of multiplication."
Mathematics Standards Study Group, 2004, 2	"Connecting multiplication with division is critical to developing a sound understanding of division; division is possibly the most important of the basic arithmetic processes since it leads to fractions and proportions, a topic which too many U.S. students have great trouble mastering."
Mathematics Standards Study Group, 2004, 2	"Missing number problems, such as 21 + ___ = 58 and 'Four times what is 12,' and practice with the distributive law, such as simplifying 37*42+63*42, set the stage for algebra."

***MLS* Application.** *MLS* carefully models and teaches to mastery the algorithms (addition, subtraction, multiplication, and division) for number operations and fraction operations. The fluency component includes varied and multiple tasks to develop automaticity in whole number operations. These skills, along with fact fluency, are further reinforced in the web-based activity, *Digit's Widgets*.

Units 2, 4, and 5 provide experiences that embed reliable algorithms for common mathematical processes. These are done through a 10-lesson instructional cycle for each concept. Students begin learning the algorithm using concrete objects placed on working mats designed to align with the computer screen and provide clear areas that will coincide with later placement or manipulation of the numbers within the algorithm. *MLS* instructions clearly present the link between manipulatives and completing the abstract steps. Students learn why they must perform each step in solving an equation.

Sequencing

Sequencing, of course, is involved in learning and remembering algorithms, but it includes other areas as well. Table 48 includes research evidence on the kinds of sequencing errors made by students who struggle, most frequently, it appears, among dyslexics.

Table 48: Sequencing

Researcher(s)	Findings/Conclusions
Henderson, 1992, 75	"For many pupils writing down a process in little stages is the easy part of a computation; for a dyslexic it is likely to be the part which he finds most difficult."
International Dyslexia Association, 1998, 1	"In understanding the complex nature of dyslexia, Ansara (1973) made three general assumptions about learning, in particular, for individuals with dyslexia. These assumptions affect the way one needs to provide instruction. They are: • learning involves the recognition of patterns which become bits of knowledge that are then organized into larger and more meaningful units; • learning for some children is more difficult than for others because of . . . deficits that interfere with the ready recognition of patterns; and • some children have difficulty with the organization of parts into wholes, due to . . . a disability in the handling of spatial and temporal relationships or to unique problems with integration, on, sequencing or memory."
T. Miles, 1992a, 7	". . . Ashcraft and Fierman (1982) have distinguished 'counting' from 'memory retrieval.' When children aged 9 and upwards (grades 3, 4, and 6) were presented with addition sums and had to say if the answer given was 'true' or 'false,' there appeared to be differences in their processing procedures at different ages. Reaction time patterns suggested that third grade is a transitional stage with respect to memory structure for addition—half of these children seemed to be counting and half retrieving from memory. It may be surmised that this finding that dyslexics, if they make the transition at all, do not do so until a somewhat later date."
Kibel, 1992, 52	"Dyslexics have difficulty with sequencing. In mathematics, the algorithms are often long sequences of fairly meaningless operations, and these usually have to be memorized in words. Children forget. They mix operations. They often resort to rows of tiny dots and tally marks in an attempt to find a way around the difficulty."
T. Miles, 1992b, 84	". . . problems with numbers are very common in dyslexia and they are in fact part of the same basic limitation which has made reading and spelling difficult."
Chinn & Ashcroft, 1992, 100	"Patterns can be seen as sequences. Such sequences can lead to small step, success-oriented solutions to problems."

Researcher(s)	Findings/Conclusions
Chinn & Ashcroft, 1992, 100	"The use of patterns can help provide a structure and organization in mathematics, reducing the load on memory, helping understanding and helping to develop concepts. Patterns provide motivation, since success is more likely as the logical momentum of the pattern leads the learner to the correct answer."
Kibel, 1992, 43-44	"Dylexics have difficulty with language. If mathematics is taught through the medium of language, if children are told what to do and expected to remember a sequence of verbal instructions, then dyslexic children are going to find this hard. We are asking them to rely on an area in which we know they are cognitively weak."
T. Miles, 1992a, 9	". . . dyslexics cannot easily recall strings of digits, whether presented auditorily or visually."

***MLS* Application.** Sequencing skills are taught in *MLS* through lessons on patterns, counting, the whole number and fraction algorithms, and in problem-solving applications that follow each set of abstract lessons. *Math Magic*, an individual or group activity that uses higher-order thinking skills to complete six intertwined equations, requires students to use estimation, logic, and sequencing skills.

Position and Direction

Closely related to algorithms and sequencing are position and direction in mathematics. Again, dyslexic learners have problems with these concepts. Evidence of these kinds of errors is provided in Table 49.

Table 49: Position and Direction

Researcher(s)	Findings/Conclusions
E. Miles, 1992b, 63	"Position is even more important in mathematics than it is in spelling."
E. Miles, 1992b, 64	"Particular difficulties will also arise from the dyslexic's confusion over direction and his general inflexibility of approach. In following a text in a reading book, the pupil has been taught to move from left to right. In mathematics, he must be flexible, depending on the operation required. . . . A dyslexic child has to understand explicitly in a way that may not be necessary for the more linguistically able members of the class, who simply accept that they have to work in a particular direction."
T. Miles, 1992a, 15	"There is also evidence that the difficulties experienced by dyslexics over 'left' and 'right' spill over into mathematics. A tiresome complication is that, of the four basic operations, three of them (addition, subtraction, and multiplication) require to be started on the right, whereas division has to be started on the left—as does writing across the page. Now unless dyslexics have an adequate understanding as to what is involved—so that whether one starts on the left or the right is simply something to be remembered—there is considerable risk that they will go wrong."
E. Miles, 1992b, 64	"In dealing with an equation, on the other hand, the mathematical equivalent of a sentence, he must be prepared to read it from left to right *or* right to left according to what he needs to do."
Henderson, 1992, 73	"Another difficulty for dyslexics is the recognition of the decimal point within a number. One thing that can go wrong is that the comma dividing off the thousands is often mistaken for the real decimal point."

Researcher(s)	Findings/Conclusions
T. Miles, 1992a, 17-18	"From the evidence cited the following are conclusions which can be accepted with a reasonable degree of confidence: (1) All or most dyslexics have mathematical difficulties of some kind, but these can be overcome to varying degrees and in some cases dyslexics can become extremely successful mathematicians. (2) They are likely to have problems in their immediate memory for 'number facts,' and where it is necessary they may resort to compensatory strategies such as counting on their fingers or putting marks on paper. (3) They have difficulty in learning their tables and, in reciting them, may lose the place or become confused. (4) They may also lose the place in adding up columns of numbers. (5) Their difficulties over 'left' and 'right' may affect their calculations. (6) They are helped if the basic concepts (addition, and so on) are introduced with concrete examples (adding and taking away blocks, for instance); otherwise the notation is far harder to understand."

***MLS* Application.** The lessons on place value and algorithms systematically teach position and direction. *MLS* reinforces position and direction by providing graphic work-space organizers that mirror the on-screen layout used in concrete lessons. Coaching and animations reinforce both the position and direction of work. The instructions provide clear, consistent referents to assist the student in tracking the position and direction of the algorithms' movement, e.g., "Take the orange tens cubes from the first row on your working mat. . . ." Decimals are given emphasis in the money unit. This choice places decimals within the context that would be most accessible and relevant to students.

Measurement

Table 50 includes research from the Mathematics Standards Study Group involving the importance of measurement in mathematics curricula.

Table 50: Measurement

Researcher(s)	Findings/Conclusions
Mathematics Standards Study Group, 2004, 2	"Measurement—in time, in money, in weight, and in physical dimensions (length, area, and volume)—arises as an extension of counting and provides contexts in which to practice arithmetic while also learning needed knowledge for daily life."
Mathematics Standards Study Group, 2004, 2	"Problems involving money lay a foundation for decimals."

***MLS* Application.** *MLS* includes specific lessons on the measurement of money and time in Unit 3.

Estimation

Researchers agree, by and large, that estimation is a skill that should be explicitly taught in mathematics for a variety of reasons. It is, of course, related to number sense, and without the ability to estimate, students have difficulty in self-monitoring—determining, that is, whether an answer to a problem is reasonable. Estimation skills are among those required for "adaptive reasoning—capacity for logical thought, reflection, explanation, and justification" (National Research Council, 2001, p. 5)—one of the five critical strands required for mathematical proficiency. Table 51 includes the evidence on this topic.

Table 51: Estimation

Researcher(s)	Findings/Conclusions
Siegler & Booth, 2005, 197	"Estimation is an important part of mathematical cognition, one that is pervasively present in the lives of both children and adults. . . . Estimation may be used more often in everyday life than any other quantification process."
Mercer & Mercer, 2005, 459	"Estimation has applications to every aspect of mathematics and is an essential part of an effective mathematics program for students with learning problems. . . ."
Siegler & Booth, 2005, 197	"In addition to its pervasive use, estimation is also important because it is related to other specific aspects of mathematical ability, such as arithmetic skill, and also to general measures of mathematical ability, such as achievement test scores."
Siegler & Booth, 2005, 197	"Yet another basis for the importance of estimation is practical—most school-age children are surprisingly bad at it, and even many adults are far from good at it. This limited proficiency, together with the pervasiveness of estimation in everyday life, its correlation and possible causal connection to general mathematical ability, and its embodying the type of flexible problem solving that is viewed as crucial within modern mathematics education, has led the National Council of Teachers of Mathematics to assign a high priority to the goal of improving estimation skills within each revision of its Math Standards since 1980."
Klein, 2005, 19	"Fostering estimation skills in students is a commendable goal shared by all state standards documents. However, there is a tendency to overemphasize estimation at the expense of exact arithmetic calculations."
National Research Council, 2001, 215	"Making estimates of exact answers is another form of computation that has its own special properties and uses in developing mathematical proficiency. Estimating before solving a problem can facilitate number sense and place-value understanding by encouraging students to use number and notational properties to generate an approximate result. Estimating is also a practical skill. It can guide students' use of calculators, especially in identifying implausible answers, and is a valuable part of the mathematics used in everyday life."

***MLS* Application.** *MLS* includes estimation in Unit 3 with specific lessons. For example, students learn to round to the nearest ten and to the nearest hundred. *Math Magic*, an individual or group activity that uses higher-order thinking skills to complete six intertwined equations, requires students to use estimation, logic, and sequencing skills.

Problem-Solving

Any mathematics teacher will affirm that a huge issue in students learning mathematics is problem-solving. The National Research Council (2001) uses the term "strategic competence—ability to formulate, represent, and solve mathematical problems" (p. 5) in its definition of five critical strands that are required for mathematical proficiency. Table 52 includes the research on this important topic.

Table 52: Problem-Solving

Researcher(s)	Findings/Conclusions
Sherman, Richardson, & Yard, 2005, 207	"The most common reason students report for their difficulty with problem solving is that they do not understand what is being asked of them. That is, the context of the problem does not make sense and is not clearly translatable to a number sentence."
Dowker, 2004, 8	"Russell and Ginsburg (1984) found that difficulties with word problem solving, as well as with memory for facts, characterized 9-year-old children who were described by their teachers as weak at arithmetic."
Fazio, 1999, 428	"An inability to solve simple math problems rapidly and accurately is cited as a frequent problem for fourth- to sixth-grade children with LD (Pellegrino & Goldman, 1987; Torgesen et al., 1987)."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 372	"In a sample of adolescents, the use of strategies, such as placing the numbers from a word problem into an equation, has been linked to mathematical talent. . . . This type of strategy could be used to isolate and structure problem-relevant information, reducing the total working memory load and decreasing interference from irrelevant words in the problem. Thus, changes in the mixture of solution procedures on arithmetic problems is likely to be one source of the improvements in performance with age that is related to working memory."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 365	"Problem complexity is the central variable in research on mathematical cognition. . . . There are at least three ways to operationalize problem complexity: (a) operand magnitude (e.g., 1 + 1 vs. 9 + 9); (b) the number of digits in the operands (i.e., 2 + 3 vs. 25 + 67); and (c) the presence or absence of carry operations (e.g., 23 + 41 vs. 29 + 46). We propose that all of these variations in complexity can be linked to working memory demands by considering the number of steps required to solve the problems."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 366	". . . as problems increase in complexity—multiple digits, more complex algorithms, and, therefore, more steps—they are likely to require increased working memory resources."
Bruer, 1993, 99	"Word problems are the black hole of middle school math; a lot of energy goes in and no light comes out."
Sherman, Richardson, & Yard, 2005, 211	"Instruction designed to assist students in problem solving is enhanced when students are taught specific strategies and receive frequent feedback. Moreover, when problems are integrated as a story or a real-life situation in daily lessons, problems are solved more intuitively and language becomes much more familiar."
Dixon, 2005, 393	"Past research shows that people can and do access previously solved problems when asked to generate a mathematical solution to a new problem. In this way, stored exemplars can act as a representation of mathematical structure. However, the successful use of exemplars depends heavily on the problem solver's ability to map the structure of the current problem to that of the stored problem."
Pennington, 1991, 102	"In terms of educational intervention, an emphasis on teaching metacognitive skills may be important, as ADHD children have less experience developing and applying such strategies to academic tasks. Such a metacognitive intervention has shown dramatic success in improving reading comprehension skills among poor comprehenders (Brown and Campione, 1986), and could be very helpful with ADHD children. Other metacognitive interventions would include teaching explicit algorithms and strategies for dealing with complex problems and assignments, whether they be long division problems or term papers."

Researcher(s)	Findings/Conclusions
Balfanz, McPartland, & Shaw, 2002, 17	"The National Research Council synthesis of research on learning mathematics highlights several core elements of extra help. On non-routine problems, students need to learn to slow down and ask themselves some guiding questions. Many students who are not skilled at mathematical reasoning are so, in part, because they do not allow themselves the time to reason. They quickly attempt to deduce which operations are called for and then apply them to the numbers in the problem without determining whether this is the appropriate solution. Students who have not mastered intermediate skills with rational numbers and integers need to develop conceptual understanding of these operations and learn the standard algorithms. Many students also need help learning the language and symbol systems of mathematics and understanding how mathematics terminology differs from everyday speech. Finally, students needing extra help in mathematics need sufficient guided practice both to internalize procedures and to learn how to apply their mathematical knowledge to non-routine problems."
Balfanz, McPartland, & Shaw, 2002, 17-18	"Bottge (2001), in a synthesis of research on students with learning difficulties, argues that effective instruction for low-achieving students in mathematics should involve several elements, including meaningful problems which engage students, explicit instruction in foundational knowledge and skills, use of students' informal knowledge and intuitions, and shared dialogue about challenging mathematical tasks.
Balfanz, McPartland, & Shaw, 2002, p.18	". . . organizing instruction to develop students' conceptual understanding can lead to significant gains in problem solving and mathematical reasoning skills . . . without a deterioration in students' basic computational skills."
Sherman, Richardson, & Yard, 2005, 215	"Problem solving is a creative process and a skill. It is the goal of mathematics."
Lock, 1996, 5	". . . six problem-solving strategies: 1. Read and understand the problem. 2. Look for the key questions and recognize important words. 3. Select the appropriate operation. 4. Write the number sentence (equation) and solve it. 5. Check your answer. 6. Correct your errors."
Kroesbergen, 2002, 5-6	"A second major problem confronting students with difficulties learning math is that many of them show deficits in the adequate use of strategies. For adequate strategy use, students must have an adequate repertoire of strategies (strategy acquisition) and also know just how and when to apply the various strategies (strategy application). In general, elementary school students with difficulties learning math rely more heavily on counting strategies than normally achieving students (Pellegrino & Goldman, 1987). An adequate repertoire of math strategies can be built in several ways; it should be noted, however, that students with difficulties learning math do not have an exhaustive repertoire of strategies and that teaching them only a limited (but effective) number of strategies may be sufficient (Jones, Wilson, & Bhojwani, 1997). The acquisition of many different strategies may only lead to confusion."
Klein, 2005, 20	"Problem solving is an indispensable part of learning mathematics and, ideally, straightforward practice problems should gradually give way to more difficult problems as students master skills. . . . Students should solve single-step problems in the earliest grades and deal with increasingly more challenging multi-step problems as they progress."
Ball, Ferrini-Mundy, Kilpatrick, Milgram, Schmid, & Schaar, 2005, 2	"Students must be able to formulate and solve problems. Mathematical problem solving includes being able to (a) develop a clear understanding of the problem that is being posed; (b) translate the problem from everyday language into a precise mathematical question; (c) choose and use appropriate methods to answer the question; (d) interpret and evaluate the solution in terms of the original problem; and (e) understand that not all questions admit mathematical solutions and recognize problems that cannot be solved mathematically."
Sherman, Richardson, & Yard, 2005, 205	"It is most important for students to realize that the reason they study and learn algorithmic rules and computation facts is actually for the purpose of solving problems."

Researcher(s)	Findings/Conclusions
Sherman, Richardson, & Yard, 2005, 205	"Clearly, problem solving is a process of thinking mathematically. The term has also been defined as 'strategic competence,' which describes the 'ability to formulate, represent, and solve mathematical problems' (Kilpatrick, Swafford, & Findell, 2001, 5)."
Barton & Heidema, 2002, 32-33	"Although research indicates that teachers define and implement problem-solving instruction in a variety of ways, mathematics educators, textbooks, and classroom resources typically rely on a view of problem solving based on Polya's four-step process for problem solving (see Polya, 1957, and Gay, 1999): 1. Understand the problem. . . 2. Devise a plan. . . . 3. Carry out the plan, checking (or proving) that each step is correct. 4. Examine the solution obtained. Check the result to make sure that it is reasonable or solves the problem."
Heaton, 2000, 5	"However a problem is solved, the aim is for students to construct powerful and reasonable understandings of why particular solutions and problem-solving methods make sense."
Ball, Ferrini-Mundy, Kilpatrick, Milgram, Schmid, & Schaar, 2005, 3	"Teaching mathematics in 'real world' contexts: It can be helpful to motivate and introduce mathematical ideas through applied problems. However, this approach should not be elevated to a general principle. If all school mathematics is taught using real world problems, then some important topics may not receive adequate attention. Teachers must use contexts with care. They need to manage the use of real-world problems or mathematical applications in ways that focus students' attention on the mathematical ideas that the problems are intended to develop."
Checkley, K. April 2006, 2	"Helping students hone problem-solving skills is a second major focus of an innovative math curriculum."
McEwan, 2000, 77	"Students become skilled problem solvers in the same way that students become good readers—by doing a lot of it. But like reading a lot, solving a lot must be done at an ever-increasing level of difficulty and with a relentless constancy."

Table 53 includes research on the effects of reading disabilities, including dyslexia, on reading and solving word problems in mathematics.

Table 53: Effects of Reading Disabilities on Reading and Solving Word Problems

Researcher(s)	Findings/Conclusions
Barton & Heidema, 2002, 2	"A second reason students need to learn how to read mathematics is that reading mathematics requires unique knowledge and skills not taught in other content areas."
Barton & Heidema, 2002, 2	". . . mathematics texts contain more concepts per word, per sentence, and per paragraph than any other kind of text (Brennan & Dunlap, 1985; Culyer, 1988; Thomas, 1988). In addition, these concepts are often abstract, so it is difficult for readers to visualize their meaning."
Barton & Heidema, 2002, 2	". . . authors of mathematics texts generally write in a very terse or compact style. Each sentence contains a lot of information, and there is little redundancy."
Barton & Heidema, 2002, 2	"Mathematics also requires students to be proficient at decoding not only words but also numeric and nonnumeric symbols. Consequently, the reader must shift from 'sounding out' words such as plus or minus to instantly recognizing their symbolic counterparts, + and -."
E. Miles, 1992b, 58	". . . dyslexic children may be handicapped in reading the text of problems."
Miller & Mercer, 1997, 6	"Because math symbols represent a way to express numerical language concepts, language skills become very important to math achievement. . . . The demands of word problems increase in each grade level. Irrelevant numerical and linguistic information in word problems is especially troublesome for many students with learning disabilities. . . . Moreover, many students with learning disabilities have reading difficulties that interfere with their ability to solve word problems."

Researcher(s)	Findings/Conclusions
Lyon,1996, 68	". . . children with disabilities in reading frequently experience persistent difficulties in solving word problems in math for the obvious reason that the printed word is difficult for them to comprehend."
E. Miles, 1992b, 68	"If we take into account how in these many different ways linguistic facility is needed in the building of basic arithmetic skills, we shall apply some of the same techniques in helping dyslexics with their mathematics as we do in teaching them literacy skills; that is, we have to make quite clear what function the symbols are performing, without taking anything for granted."
T. Miles, 1992b, 84-85	"Just as in teaching literacy to a dyslexic one does not simply correct spelling mistakes as they occur but calls attention in a systematic way to the different ways in which speech sounds can be represented by letters of the alphabet, so in the case of mathematics, as I explain to the pupil, it is normally advisable to start at a very basic level in order to make sure that they fully understand how the number system works and how the different operations are symbolized."
T. Miles, 1992b, 86	". . . for dyslexics the learning of new symbols takes extra time."
T. Miles, 1992b, 86-87	". . . the principle of 'doing first—notation afterwards' is of help to dyslexics in many different contexts, since there is no problem with their ability to 'do,' only with their ability to acquire and reproduce symbols at speed."
Dowker, 2004, ii	"Despite such variable patterns of strengths and weaknesses, some areas of arithmetic do appear to create more problems than others for children. One of the areas most commonly found to create difficulties is *memory for arithmetical facts*. For some children, this is a specific, localized problem; for children with more severe mathematical difficulties it may be associated with exclusive reliance on cumbersome counting strategies. Other common areas of difficulty include *word problem solving*, *representation of place value* and the ability to solve *multi-step arithmetic problems.*

***MLS* Application.** In each lesson phase of *MLS*, there are four kinds of lessons in the sequence: concrete, illustrative (or semiconcrete), abstract, and assessment. In the abstract lesson are instruction on problem solving, including strategies for attacking a word problem and for the elimination of irrelevant information. *Drawing Conclusions* is a printed activity that encourages visualization and higher-order thinking skills to solve word problems. These activities complement the word problems that *MLS* uses. Students can complete the activities individually, or teachers can encourage collaborative problem solving by placing the students in groups.

Fractions

Many learners fail to master fraction concepts and operations even after several years of mathematics instruction. Table 54 includes research relating to the problems that many students have in learning fractions or rational numbers. Fractions clearly pose major problems for struggling learners. E. Miles (1992b) points out that "The symbolization of fractions is something particularly difficult to grasp, because numbers in fractions cannot be treated exactly the same way as whole numbers, but this is not always realized" (p. 63). Moss (2005) adds that "As mathematics education researchers and teachers can attest, students are often vocal in their expression of dislike of fractions and other representations of rational numbers (percents and decimals). In fact, the rational-number system poses problems not only for youngsters, but for many adults as well" (p. 309). Failure to achieve a deep understanding of fractions can not only cause serious academic problems at the level where they are introduced, but in all subsequent mathematics instruction. Klein (2005) states, for instance, that

> Mathematical reasoning is systematically undermined when prerequisites for content standards are insufficiently developed. When arithmetic, particularly fraction arithmetic, is poorly developed in the elementary grades, students have little hope of understanding algebra as anything other than a maze of complicated recipes to be memorized, as is too often the case in state standards documents (p. 22).

Wu (Summer 2001) concurs:

> . . . no matter how much "algebraic thinking" is introduced in the early grades and no matter how worthwhile such exercises might be, the failure rate in algebra will continue to be high unless we radically revamp the teaching of fractions and decimals.
>
> The proper study of fractions provides a ramp that leads students gently from arithmetic up to algebra. But when the approach to fractions is defective, that ramp collapses, and students are required to scale the wall of algebra not at a gentle slope but at a ninety degree angle. Not surprisingly, many can't (p. 1).

The implications for lack of mastery of fraction concepts and operations success are clear in other research. Especially important is the relationship of fraction knowledge to later success in learning algebra.

Table 54: Fractions

Researcher(s)	Findings/Conclusions
Dowker, 2004, 8	"Hart (1981) and her team found that secondary school pupils have many difficulties, both procedural and conceptual, with many mathematical topics, including ratio and proportion; fractions and decimals; algebra; and problems involving area and volume."
Sherman, Richardson, & Yard, 2005, 139	"Several reasons have been suggested for the difficulty students experience when learning rational number concepts and skills: 1. In terms of instructional approaches, lessons are too often focused on procedures and memorizing rules rather than on developing conceptual foundations prior to skill building. 2. Specific content difficulties occur when students confuse whole number computational procedures with those for fractions. . . . 3. Estimating rational number answers can be more challenging than with whole numbers. . . . 4. Using fractional notation can be confusing to students if they do not fully understand which numeral represents the numerator and which stands for the denominator, nor how to write mixed numbers."
Brigham, Wilson, Jones, & Moisio, 1996, 1	"Fractions are a consistent and recurring area of concern for classroom teachers of students with LD. The areas of skill deficits most consistently reported by middle school and high school teachers of students with LD are related to fractions, decimals, and percents (McLeod & Armstrong, 1982). These deficits included both terminology related to fractions and operations with fractions. Studies of the performance of students with LD on secondary competency tests also found significant skill deficits in fractions, decimals, and percents (Algozzine, O'Shea, & Stoddard, 1987)."
National Research Council, 2001, 231	"Learning about rational numbers is more complicated and difficult than learning about whole numbers."

Researcher(s)	Findings/Conclusions
Ball, Ferrini-Mundy, Kilpatrick, Milgram, Schmid, & Schaar, 2005, 3	"Understanding the number meaning of fractions is critical. Ratios, proportions, and percentages cannot be properly understood without fractions. The arithmetic of fractions is important as a foundation for algebra."
National Research Council, 2001, 232	"Students' informal notions of partitioning, sharing, and measuring provide a starting point for developing the concept of rational number."
Moss, 2005, 310	"Students cannot succeed in algebra if they do not understand rational numbers."
Committee on *How People Learn,* 2005, 310	"We know from extensive research that many people—adults, students, even teachers—find the rational-number system to be very difficult. . . . The culprit appears to be the continued use of whole-number reasoning in situations where it does not apply."
Committee on *How People Learn,* 2005, 310	"Students cannot succeed in algebra if they do not understand rational numbers. But rational numbers also pervade our daily lives. We need to be able to understand them to follow recipes, calculate discounts and miles per gallon, exchange money, assess the most economical size of products, read maps, interpret scale drawings, prepare budgets, invest our savings, read financial statements, and examine campaign promises."
Committee on *How People Learn,* 2005, 319	"We know . . . that most middle school students do not create appropriate meanings for fractions, decimals, and percents; rather, they rely on memorized rules for symbol manipulation."
Klein, 2005, 21-22	"The logical development of fractions and decimals deserves special attention, rarely given in state documents. In many cases, students are inappropriately expected to multiply and divide decimal numbers a year in advance of multiplying and dividing fractions. This is problematic. What does it *mean* to multiply or divide decimal numbers, if those operations for fractions have not been introduced? How are these operations defined? All too often, we found no indication that students should understand multiplication and division of rational numbers except as procedures."
Klein & Milgram, n.d., 13	"The long division algorithm is the essential tool in establishing that any rational number has a repeating block of digits in its decimal representation. The converse, that any decimal with a repeating block is equal to a rational number, requires a different argument."
Siegler, 2003, 222	"A similar misunderstanding of the relation of symbols to magnitudes is evident in children's attempts to deal with decimal fractions. Consider how they judge the relative size of two numbers such as 2.86 and 2.357. The most common approach of fourth and fifth graders on such problems is to say that the larger number is the one with more digits to the right of the decimal point (Resnick, et al., 1989). Thus, they would judge 2.357 larger than 2.86."
Klein & Milgram, n.d., 9	"The conversion of (finite) decimals to fractions with denominators equal to a power of 10 is straightforward. It involves nothing more than the very definition of a decimal expression. But converting from a fraction to a decimal is more elaborate and involves the division algorithm in an essential way. The justification for this process is more subtle than is often recognized."
Balfanz, McPartland, & Shaw, 2002, 10	"A recent synthesis of existing research on mathematical learning by the National Research Council, as well as interviews with high school teachers indicate that operating with rational numbers (fractions, decimals, and percents) and integers (positive and negative numbers) are the two intermediate skill areas where entering high school students are most in need of extra help. . . . These two domains are conceptually challenging, procedurally complex, and vital to success in standards-based high school math."
Balfanz, McPartland, & Shaw, 2002, 10-11	"Traditionally, operations with rational numbers, and, to a lesser extent, integers, are the primary focus on instruction in upper elementary and middle school grades. However, both the TIMSS [Third International Mathematics and Science Study] study and research conducted in high poverty middle schools indicate that not all middle school students receive sufficient and effective instruction in these topics."

Researcher(s)	Findings/Conclusions
Caldwell, Feb. 3, 2006, 1	Interview with William Schmidt, a professor at Michigan State University and executive director of its Third International Math and Science Study Research Center: "Fractions . . . are very difficult for students. Instead of introducing the concept clearly enough so that they understand fractions as numbers on the number line, we oftentimes try to move too quickly to other parts of fractions, such as the operations, before they really have a clear understanding of what fractions are and how they fit into the broader number system. So, kids are trying to learn how to operate on these things, and at the same time they really don't understand what they are, so things get very muddled in their minds."
Sherman, Richardson, & Yard, 2005, 137	"Rational numbers are sometimes quite difficult for students who were successful with whole numbers in early grades. However, understanding and becoming proficient in learning about numbers that represent parts, that have infinite number of names, and that do not always follow whole numbers patterns can be daunting. Although most students can eventually operate with specific algorithms, general conceptual knowledge often remains deficient."
Sherman, Richardson, & Yard, 2005, 138	"Rational numbers are 'abstract mathematical ideas' (NCTM, 2000, 10), as are counting numbers. They can be made to correspond to points on a number line. . . Fractions, also termed rational numbers, can be expressed as 'a/b = c,' if and only if a = bc (Crouch & Baldwin, 1964)."
Sherman, Richardson, & Yard, 2005, 166	"Throughout the rational number lessons and activities, it is essential that conceptual understanding be established as a foundation for mastering algorithms and procedures. Concrete, hands-on materials and drawings, to which symbols can be connected in the same lesson, are critical components for lessons in which students achieve both fractional number sense and then computational fluency. Direct connections between materials and numerals and between mathematical examples and real life situations are keys to recognizing patterns and successful computation."
Bottge, 2002, 1	". . . raising the math achievement of students with disabilities has proved difficult (Cawley, Kahn, & Tedesco, 1989; Cawley, Parmar, Yan, & Miller, 1998), especially in fractions computation (Behr, Wachsmuth, & Post, 1985) and word problems (Jitendra, Hoff, & Beck, 1999; Xin & Jitendra, 1999)."
Kroesbergen & Van Luit, 2003, 105	"The first conclusion is that the majority of the studies describe an intervention in the domain of basic skills. The interventions in this domain also show the highest effect-sizes. The domain of basic math skills is very large and constitutes an important aspect of elementary math teaching. For this reason, it is not surprising that many of the studies are concerned with this domain. And it appears to be a domain in which interventions are effective. It may be easier to teach basic skills to the students with special needs, than to teach problem solving skills."
Mathematics Standards Study Group, 2004, 3	"After the near total focus on whole number arithmetic and the place value system in early elementary grades, the second half of elementary school mathematics ought to focus on arithmetic with fractions and decimals as well as the properties of these number systems. These number systems need to be understood in multiple ways. Students need to understand how to locate rational numbers on the (real) number line and to extend the number line to coordinates in the plane. Simple problems with proportions can be integrated into early calculations with fractions."
Klein, 2005, 16	"The long division algorithm has applications that go far beyond elementary school arithmetic. At the middle school level, it can be used to explain why rational numbers have repeating decimals. This leads to an understanding of irrational, and therefore real numbers. Division is also central to the Eulidean Algorithm for the calculation of the greatest common divisor of two integers. In high school algebra, the long division algorithm, in slightly modified form, is used for division of polynomials."

Researcher(s)	Findings/Conclusions
Brigham, Wilson, Jones, & Moisio, 1996, 3	"Many students bring a great deal of informal understanding of fractions to their instruction in mathematics; however, it is often difficult for students to integrate formal instruction with their informal knowledge (Mack, 1990). Among the problems that Mack noted were a tendency to consider fractions as whole numbers rather than proportions or rational numbers, and the inability to solve problems expressed symbolically even when the students were able to solve similar problems expressed in the context of real-world situations. Additional problems in representation of fractional numbers include lack of understanding that fractions can represent a part of a set as well as a part of a whole unit, and that fractions represent a certain number of equal sized parts (Baroody & Hume, 1991). Teachers should also bear in mind that representation of fractions can be a very abstract and difficult task for students that is sometimes beyond the ability of even their teachers (Ball, 1990)."
Brigham, Wilson, Jones, & Moisio, 1996, 5	"Comparison of fractions is sometimes difficult for students who regard fractions as discrete whole numbers rather than as proportions (Mack, 1990). Baroody and Hume (1991) suggested that students often compare whole numbers by using a strategy which indicates that the number which comes later in a counting series is the larger. When applying this strategy to fractions such as 1/3 and ¼, students might compare the denominators and erroneously conclude that the fourth is larger than the third because four comes after three in the counting series. Students committing this type of error are probably applying knowledge of whole numbers to fractions."
Brigham, Wilson, Jones, & Moisio, 1996, 6	"Many students find learning decimal numbers to be an easier task than mastering fractions (Bley & Thornton, 1995)."
Miller, Kelly, & Zhou, 2005, 172	"The concept of a fraction is a difficult one for a variety of reasons. Sophian, Garyantes, and Chang (1997) argued that the fact that dividing into more pieces or increasing the denominator means that one has less in each piece is counterintuitive. Gelman and Meck (1992) argued that preschoolers view all mathematical tasks as 'opportunities to count,' which is likely to lead one astray when it comes to thinking about rational numbers."
National Research Council, 2001, 235	"Research has verified what many teachers have observed, that students continue to use properties they learned from operating with whole numbers even though many whole number properties do not apply to rational numbers. With common fractions, for example, students may reason that 1/8 is larger than 1/7 because 8 is larger than 7. . . . Such inappropriate extensions of whole number relationships, many based on addition, can be a continuing source of trouble when students are learning to work with fractions and their multiplicative relationships."
National Research Council, 2001, 239	"An example of a common error and one that also can be traced to previous experience with whole numbers is that 'multiplying makes larger' and 'dividing makes smaller.' These generalizations are not true for the full set of rational numbers."
Fuchs & Fuchs, 2002, 2	"With a cross-sectional sample ranging across Grades 3 through 8, Parmar et al. demonstrated that features contributing to contextual realism (i.e., irrelevant information, the addition of an extra step, or the use of indirect language) increase math problem difficulty differentially for students with and without disabilities. For example, among third graders with and without learning disabilities or behavior disorders, irrelevant information added to addition problems produced a similar drop inaccuracy (38% and 34%, respectively). By Grade 8, this drop had decreased for both types of students; however, the remaining drop was more than twice as large for students with disabilities as for students without disabilities."

***MLS* Application.** *MLS* devotes two of its five units to fractions—one on understanding fractions and the other to fraction operations. Levels include fraction identification, equivalent fractions, comparing fractions, simplifying fractions, converting fractions, and the four operations: addition, subtraction, multiplication, and division of fractions. This pairing of units is analogous to the pairing of Understanding Numbers and Number Operations. By dividing the two, careful emphasis is first given to the composition and manipulation of fractions. Once the fraction concept itself is understood, care is taken to provide adequate practice in using appropriate algorithms for fraction operations.

Difficulties in Learning and Retrieving Math Facts

Closely related to the effects on working memory of disabilities in the language system are the effects on students' ability to learn their mathematics facts and to be able to retrieve them rapidly and accurately for application. The evidence that fact fluency is a major problem for students with disabilities is almost overwhelming. Landerl, Began, and Butterworth (2004) state that "The most generally agreed upon feature of children with dyscalculia is difficulty in learning and remembering arithmetic facts (Geary, 1993; Geary & Hoard, 2001; Ginsburg, 1997; Jordan, Hanich, & Kaplan, 2003b; Jordan & Montani, 1997; Kirby & Becker, 1988; Russell & Ginsburg, 1984; Shalev & Gross-Tsur, 2001)" (p. 100). The tremendous body of research relating to the fact retrieval difficulties that students with reading disabilities have is presented in Table 55. Other evidence of this problem was discussed in Chapters II and III on the manifestations of mathematics difficulties and disabilites.

Table 55: Learning and Retrieving Math Facts

Researcher(s)	Findings/Conclusions
Wu, 2001, 7	"Fluency in computation is very important for the learning of algebra. . . ."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 367	". . . solvers use phonological codes to keep operands activated. Similarly, maintenance of interim results during complex calculations seems to be mediated by a phonological code. . . . Such findings are consistent with a role for the phonological loop in short-term maintenance of verbal information. They do not preclude the possibility, however, that problem operands or other components of mathematical tasks may be temporarily stored in visual or spatial representations."
Chinn & Ashcroft, 1992, 98	"Two of the factors which hinder a dyslexic's progress in mathematics are poor immediate memory (Steeves, 1983) and difficulty in learning the basic number facts, particularly the times tables (Miles, 1983). In our experience of teaching dyslexics we have observed another handicapping factor, a poor ability to generalize and classify facts and rules in mathematics."
Chinn & Ashcroft, 1992, 98	". . . dyslexics need help in extending generalizations from limited areas to these interrelationships and cross-generalizations."
Dehaene, Piazza, Pinel, & Cohen, 2005, 449	"When faced with the simple addition problems, nondyslexics tend to use fact retrieval much more often than do dyslexics, who instead use finger-counting strategies. This is consistent with the hypothesis that an impairment of rote verbal memory is partially responsible for dyscalculia in children with dyslexia."
T. Miles, 1992a, 5	". . . despite their high potential, they were handicapped at mathematics by those parts of the subject which call for memorizing ability. [Steeves] does not suggest in detail what part is played by this memory limitation. There is evidence from other research, however, that, as far as mathematics is concerned, a weakness at immediate recall of number facts may be one of the limitations."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 370	"Many children with math disabilities have particular difficulty memorizing arithmetic tasks. . . , and these difficulties persist over time. . . . One speculation is that working memory limitations are a potential source of this difficulty."
Geary & Hoard, 2005, 281	"It seems that some MD [mathematics disabilities] children with fact-retrieval deficits do have a language-representation deficit (Geary et al., 2000), but others may not (Jordan et al., 2003a)."

Researcher(s)	Findings/Conclusions
T. Miles, 1992a, 11	"There may be an inability to visualize numbers, to memorize the multiplication tables, or to retain a series of digits in the memory for a sufficient time."
Dehaene, Piazza, Pinel, & Cohen, 2005, 443	". . . multiplication requires the integrity of language-based representations of numbers, because multiplication facts are typically learned by rote verbal memorization. Subtraction, on the other hand, is typically not learned by rote."
Dehaene, Piazza, Pinel, & Cohen, 2005, 443	"It is not rare for a patient to be much more severely impaired in multiplication than subtraction. . . ., while other patients are much more impaired in subtraction than in multiplication." . . . Our views suggest that dissociations between operations reflect a single, basic distinction between over-learned arithmetic facts, such as the multiplication table, which are stored in rote verbal memory, and the genuine understanding of number meaning that underlies nontable operations such as subtraction."
Dehaene, Piazza, Pinel, & Cohen, 2005, 443	". . . patients in whom multiplication is more impaired than subtraction typically have associated aphasia. . . . Furthermore, the lesions often spare the intraprietal cortex and can affect multiple regions known to be engaged in language processing. . . . The evidence clearly shows that multiple sites, not just the left AG, contribute to a distributed network supporting rote verbal knowledge and may cause multiplication impairments when lesioned."
Pennington, 1991, 68	"The key symptoms in dyslexia are difficulty in learning to read and spell, often with relatively better performance in arithmetic. . . . Parents or teachers may also report slow reading or writing speed, letter and number reversals, problems memorizing basic math facts, and unusual reading and spelling errors."
Pennington, 1991, 112	"The math problems found in dyslexics are of a different sort than those found in children without reading and spelling problems. Briefly, dyslexics have trouble memorizing math facts, and understanding 'word' problems because of their reading problem. Sometimes they missequence numbers they write, but usually do not have basic conceptual problems with mathematical understanding. In contrast, nondyslexic children with poor math performance appear to have fundamental conceptual problems in understanding mathematics."
Pennington, 1991, 122-123	"Dyslexic children may make mistakes because they reverse numbers or do not know basic math facts and have to rely on finger tallies. However, they rarely attempt problems that they know are too hard for them or make errors that are not even approximately correct. In contrast, children with specific math problems make a number of different kinds of errors that reveal a deficient conceptual understanding of (a) the problem they are undertaking, (b) the subroutines needed to solve it, and (c) what a reasonable answer would be. For instance, they attempt problems that are too hard, produce wildly incorrect answers, misalign columns of numbers, and make other errors that reveal a deficient sense of place value. They also misuse computational algorithms."
T. Miles, 1992a, 6	"On the assumption that all or most of the 'reading disabled' children were in fact dyslexic, these findings suggest that dyslexics may tend to have fewer number facts available for immediate use, or, in the author's words, have not yet achieved automatization."
Dowker, 2004, ii	"Despite such variable patterns of strengths and weaknesses, some areas of arithmetic do appear to create more problems than others for children. One of the areas most commonly found to create difficulties is *memory for arithmetical facts*. For some children, this is a specific, localized problem; for children with more severe mathematical difficulties it may be associated with exclusive reliance on cumbersome counting strategies. Other common areas of difficulty include *word problem solving*, *representation of place value* and the ability to solve *multi-step arithmetic problems"*.
T. Miles, 1992a, 6	". . . there is at least evidence, in the author's words—that 'the LD [learning disabled] children in this study were not so proficient in basic fact calculation as their nondisabled peers.'"
T. Miles, 1992a, 6	"Almost without exception the dyslexics had fewer number facts available than the controls. . . ."
T. Miles, 1992a, 7	"These findings suggest that some dyslexics remain weak at subtraction and that the great majority have distinctive problems with [multiplication] tables."

Researcher(s)	Findings/Conclusions
T. Miles, 1992a, 14	"Mention has already been made of the tendency on the part of dyslexics to lose the place when reciting [multiplication] tables."
T. Miles, 1992a, 13	"What is of particular interest is that dyslexics are clearly vulnerable when they recite their tables, and it seems likely that the slips and corrections arise because they find themselves under pressure."
Chinn & Ashcroft, 1992, 99	". . . most dyslexics have great difficulty in learning the times tables."
Mazzocco & McCloskey, 2005, 273	". . . researchers have hypothesized that general long-term memory deficits may lead to co-occurring math and reading disabilities by affecting learning and retrieval of arithmetic facts, words, and letter-phoneme associations (Gear, Hamson, & Hoard, 2000). Deficient phonological processing has also been suggested as an underlying cause of co-occurring reading and math disability (Geary, Hamson, & Hoard, 2000; Hanich, Jordan, Kaplan, & Dick, 2001; Russell & Ginsburg, 1984). Similarly, working memory deficits could affect math performance in a variety of ways, and executive function deficits might lead to impairments in executing math procedures, by interfering with planning, attention, or inhibitory functions."
Fayol & Seron, 2005, 12	"Some activities, such as the comparison or estimation of numbers, are performed on an analog format and are thus not language-independent, whereas others, such as arithmetic-fact retrieval, are stored in auditory-phonological representations and are thus language dependent. Such a model is thus compatible with the idea that language has either a central (i.e., affecting the form of the representation) or peripheral (i.e., affecting the modes of access to the representation) impact on arithmetical cognition."
T. Miles, 1992a, 17-18	"From the evidence cited the following are conclusions which can be accepted with a reasonable degree of confidence: • All or most dyslexics have mathematical difficulties of some kind, but these can be overcome to varying degrees and in some cases dyslexics can become extremely successful mathematicians. • They are likely to have problems in their immediate memory for 'number facts,' and where it is necessary they may resort to compensatory strategies such as counting on their fingers or putting marks on paper. • They have difficulty in learning their tables and, in reciting them, may lose the place or become confused. • They may also lose the place in adding up columns of numbers. • Their difficulties over 'left' and 'right' may affect their calculations. • They are helped if the basic concepts (addition, and so on) are introduced with concrete examples (adding and taking away blocks, for instance); otherwise the notation is far harder to understand."
Geary & Hoard, 2005, 261	"If a general deficit in the ability to retrieve information from long-term memory contributes to arithmetic fact retrieval deficits of children with MD, then these children should also show deficits on measures that assess skill at accessing other types of semantic information, such as words, from long-term memory (Geary, 1993). Geary argued that the comorbidity of MD and RD is related, in part, to difficulties in accessing both words and arithmetic facts from semantic memory, although the data on this are mixed."
Dowker, 2004, 11	"Although there is no clear association between relative verbal strengths and particular types of mathematical difficulty, there is no doubt that mathematical difficulties often co-occur with dyslexia and other forms of language difficulty. "People with dyslexia usually experience at least some difficulty in learning number facts such as multiplication tables. Miles (1993) found that 96% of a sample of 80 nine-to-twelve-year-old dyslexics were unable to recite the 6x, 7x, and 8x tables without stumbling."

Researcher(s)	Findings/Conclusions
Butterworth, 2005, 460	"Geary (1993) notes that DD [developmental dyscalculics] children have two basic functional, or phenotypic, numerical deficits: (1) the use of developmentally immature arithmetical procedures and a high frequency of procedural errors; (2) difficulty in the representation and retrieval of arithmetic facts from long-term semantic memory."
Landerl, Bevan, & Butterworth, 2004, 102	"Working memory difficulties have also been associated with developmental dyscalculia. Geary (1993) suggests that poor working memory resources not only lead to difficulty in executing calculation procedures, but may also affect learning of arithmetic facts. In general the aspect of working memory that has been focused on is the phonological loop (Baddeley, Lewis, & Vallar, 1984), normally assessed by the number of spoken items (generally digits) which can be remembered in the correct sequence."
Geary & Hoard, 2005, 256	"The most consistent finding in this literature is that children with MD/RD and MD only differ from their normal peers in the ability to use retrieval-based processes to solve simple arithmetic and simple word problems."
Kroesbergen, 2002, 4	"The group of students with difficulties learning math is very heterogeneous. . . . First, students who have difficulties learning math often show memory deficits (Rivera, 1997) and particularly problems with the storage of information in long-term memory and the retrieval of such information (Geary, Brown, & Samaranayake, 1991). These same students show greater difficulties than their peers with the automatized mastery of such basic facts as addition up to 20 or the multiplication tables . . . ."
Butterworth, 2005, 459	"It is generally agreed that children with dyscalculia have difficulty in learning and remembering arithmetic facts . . . and in executing calculation procedures."
Butterworth, 2005, 459	"Landerl, Bevan, and Butterworth (2004), in a study of ten 9-year-old DDs [developmental dyscalculics] and 18 matched controls, found that the DDs were less accurate in single-digit subtraction and multiplication than controls and also significantly slower on addition, subtraction, and multiplication."
Geary, n.d., 4	". . . it appears that many MD children have difficulties getting basic facts into long-term memory and difficulties remembering, or accessing, the facts that are eventually stored in long-term memory. It appears that these difficulties are very similar to word finding difficulties that are common in some children with RD."
Geary, n.d., 4	". . . it appears that many MD children can get facts into and out of long-term memory without too much difficulty but have trouble inhibiting other facts when they try to remember the answers to specific problems, such as 2+3."
Garnett, 1998, 1-2	". . . children manifest different types of disabilities in math. . . . Many learning disabled students have persistent trouble 'memorizing' basic number facts in all four operations . . ., despite adequate understanding and great effort expended trying to do so. Instead of readily knowing that 5 + 7 = 12, or that 4 x 6 = 24, these children continue laboriously over years to count fingers, pencil marks or scribbled circles and seem unable to develop efficient memory strategies on their own."
Geary, 2003a, 458	"Our results compliment those of Jordan et al. and suggest that children with low mathematics achievement scores but average reading achievement scores have difficulty holding information in working memory while counting and show the retrieval-inhibition deficit . . ."

Researcher(s)	Findings/Conclusions
Bryant, Hartman, & Kim, 2003, 151	"Students with mathematics learning disabilities (LD) exhibit difficulties with retrieval and cognitive skills that impede their ability to perform basic mathematical skills. Instruction in mathematical procedures (i.e., procedural knowledge) is necessary to help students learn and apply skills such as basic facts and whole-number computation. . . . Reviews of research have revealed that students with LD benefit from a combined model of academic instruction that includes both explicit and strategic instructional procedures."
Spear-Swerling, n.d., 2	"By grade 5, automatic recall of number facts is well-developed in most normally-achieving youngsters. However, youngsters with math disabilities often continue to struggle with math skills far below grade expectations, including not only automatic recall, but also many computational algorithms and math concepts."
Mazzocco & McCloskey, 2005, 271	"Examples of math-specific skills are counting, cardinality, arithmetic fact retrieval, and calculation procedure skills; these may be differentially spared or deficient in persons with different MD subtypes."
Sousa, 2001, 141	"Learning deficits can include difficulties in mastering basic number concepts, counting skills, and processing arithmetic operations as well as procedural, retrieval, and visual-spatial deficits (Geary, 2000). As with any learning disability, each of these deficits can range from mild to severe."
T. Miles, 1992a, 7	". . . Ashcraft and Fierman (1982) have distinguished 'counting' from 'memory retrieval.' When children aged 9 and upwards (grades 3, 4, and 6) were presented with addition sums and had to say if the answer given was 'true' or 'false,' there appeared to be differences in their processing procedures at different ages. Reaction time patterns suggested that third grade is a transitional stage with respect to memory structure for addition—half of these children seemed to be counting and half retrieving from memory. It may be surmised that this finding that dyslexics, if they make the transition at all, do not do so until a somewhat later date."
Geary & Hoard, 2005, 259	"There are at least two potential sources of these retrieval difficulties: a deficit in the ability to represent phonetic/semantic information in long-term memory or a deficit in the ability to inhibit irrelevant associations from entering working memory during problem solving."
Butterworth, 2005, 460	"Geary (1993) notes that DD [developmental dyscalculics] children who two basic functional, or phenotypic, numerical deficits: (1) the use of developmentally immature arithmetical procedures and a high frequency of procedural errors; (2) difficulty in the representation and retrieval of arithmetic facts from long-term semantic memory."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 363	"Processes that have been attributed to the central executive include inhibition of irrelevant information, task switching, information updating, goal management, and strategic retrieval from long-term memory. . . ."
Ontario Ministry of Education, 2005, 40	"Many children with special needs have difficulties with various aspects of memory. Some children, for instance, have problems retaining what they have learned. Children can vary in their ability to efficiently encode and/or retrieve information from long-term memory (Cutting, Koth, Mahone, & Denckla, 2003).
Geary & Hoard, 2005, 263	". . . the retrieval deficit may result from response competition during the retrieval process. . . . As an example, presentation of the problem 4 x 5 not only prompts retrieval of 20, but it also promotes retrieval of related, but irrelevant to this problem, numbers, such as 9 (4 + 5) and 25 (5 x 5). . . . There is now strong evidence that individuals with poor working memory/central executive resources have difficulties inhibiting these irrelevant associations. . . . For these individuals, poor information retrieval is more strongly related to the central executive than to the language system per se. There is evidence that some children with MD do not inhibit irrelevant associations during fact retrieval."

Researcher(s)	Findings/Conclusions
Landerl, Bevan, & Butterworth, 2004, 100	"The most generally agreed upon feature of children with dyscalculia is difficulty in learning and remembering arithmetic facts (Geary, 1993; Geary & Hoard, 2001; Ginsburg, 1997; Jordan, Hanich, & Kaplan, 2003b; Jordan & Montani, 1997; Kirby & Becker, 1988; Russell & Ginsburg, 1984; Shalev & Gross-Tsur, 2001). A second feature of children with dyscalculia is difficulty in executing calculation procedures, with immature problem-solving strategies, long solution times and high error rates (Geary, 1993)."
Jordan, Hanich, & Kaplan, May/June 2003, 847	"In conclusion, the results of the present study, together with earlier findings, suggest that deficiencies in fact retrieval, and by extension calculation fluency, are a defining feature of mathematics difficulties, specific or otherwise."
Jordan, Hanich, & Kaplan, May/June 2003, 847	"Although it is tempting to suggest that children with MD only—who have a greater range of competencies than do children with MD-RD—bypass their relatively circumscribed deficiencies in number fact mastery with calculators or other aids, it may be wiser to provide instruction aimed at fostering calculation fluency, in addition to methods that emphasize problem solving. Studies on the maintenance of mathematical competencies in adults indicate that the degree of extended rehearsal and practice provided during school years is the best predictor of performance levels in adulthood (Bahrick & Hall, 1991)."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 371	". . . general processing speed accounts for some variance in math performance, perhaps because speed enables efficient activation of representations in long-term memory, such as retrieval of arithmetic facts."
Dowker, 2004, 6	"One of the areas most commonly found to create difficulties is memory for arithmetic facts. . . . Studies of children with mathematical difficulties show them to be more consistently weak at retrieving arithmetical facts from memory than at other aspects of arithmetic. They often rely on counting strategies in arithmetic at ages when their age-mates are relying much more on fact retrieval (Russell and Ginsburg, 1984; Siegler, 1988; Geary and Brown, 1991; Ostad, 1997, 1998; Cumming and Elkins, 1999; Fei, 2000). . . . Difficulties in memory for arithmetic facts tend to be persistent. They appear to be independent of reading skills, and did not affect performance on other aspects of arithmetic."
Woodward & Montague, 2002, 19	"A comprehensive accounting of the difficulties these students [with learning disabilities] face in learning math facts is complicated by the structure of facts themselves. That is, addition and subtraction lend themselves to a variety of strategies (e.g., min doubling) that do not work for multiplication and division. The difficulties in learning multiplication facts are due to the way facts may typically be learned and then stored in associative memory."
Garnett, 1992, 2	"Teachers frequently note that 'not knowing basic math facts' is a common and conspicuous difficulty, an impediment to higher-level math, and a corrosive influence on the self-confidence of students with learning disabilities. Research confirms that many of these students are seriously inefficient in calculating basic number facts (Fleischner, Garnett, & Shepherd, 1982; Goldman, Pellegrino, & Mertz, 1988). For example, Fleischner and her colleagues found that 6th-grade students with learning disabilities calculated basic addition facts no better than nondisabled 3rd graders. On timed assessments, 5th graders with learning disabilities completed only one-third as many multiplication fact problems as their nondisabled counterparts. Similar results were obtained on addition and subtraction facts for 3rd and 4th graders. Interestingly, the students with learning disabilities were very much slower, but not significantly less accurate, than their nondisabled peers. Additionally, they demonstrated basic conceptual understanding of the basic math operations."
Fazio, 1999, 421	"Children with SLI [specific language impairment] appear to have difficulties with several aspects of information processing. . . . Such obstacles would inhibit the learning and recall of declarative knowledge of mathematics such as math facts as well as the procedural knowledge needed for recalling the steps needed to solve multidigit calculation problems."

Researcher(s)	Findings/Conclusions
Garnett, 1998, 6	"Math learning problems range from mild to severe and manifest themselves in a variety of ways. Most common are difficulties with efficient recall of basic arithmetic facts and reliability in written computation."
Root, 1994, 2	"Classroom behaviors associated with word-retrieval difficulties: . . . an inordinate amount of difficulty with arithmetic calculations (rapid response to flash cards, swift adding of columned numerals).
Geary, n.d., 5	"It appears that many—perhaps more than ½—children with MD also have difficulties learning how to read and that many children with RD also have difficulties learning basic arithmetic. In particular, children and adults with RD often have difficulties retrieving basic arithmetic facts from long-term memory."
Kroesbergen, 2002, 5.1.1	"Research shows . . . that students with math difficulties must often calculate basic facts while other students simply know the facts directly (Pellegrino & Goldman, 1987). The development of long-term memory representations also proceeds more slowly or differently for children with math difficulties when compared to their peers (Geary, Brown, & Samaranayake, 1991). This leads to difficulties in fact retrieval. In addition, these students continue to make more mistakes on basic skills than their peers."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 372	"As children acquire factual knowledge about numbers, working memory demands may decrease because more inefficient strategies such as counting all items for addition are replaced by more efficient strategies, such as counting up or fact retrieval. . . ."
Wu, 2001, 7	"We have not dealt with decimals thus far, but the problems there are entirely parallel to those in fractions. Students are generally not told, forcefully and clearly, that (finite) decimals are merely a shorthand notation for a special type of fractions, namely, those whose denominators are 10, 100, 1000, or more generally, a power of 10. . . . Incidentally, notice how the understanding of decimals is founded on an understanding of fractions."
Wu, 2001, 7	"With the proper infusion of precise definitions, clear explanations, and symbolic computations, the teaching of fractions can eventually hope to contribute to mathematics learning in general and the learning of algebra in particular."
Wu, 2001, 7	"It remains to supplement these curricular considerations of mathematics in grades five through seven with two observations. One is the glaring omission thus far of the basic reason why fractions are critical for understanding algebra: The study of linear functions, which is the dominant topic in beginning algebra, requires a good command of fractions. The slope of the graph of a linear function is by definition a fraction. . . ."

In Table 56 is other scientific evidence of the importance of fact fluency (automaticity) in becoming proficient in mathematics.

Table 56: Importance of Fact Fluency

Researcher(s)	Findings/Conclusions
Lochy, Domahs, & Delazer, 2005, 476	"This overview shows that arithmetic disorders are sensitive to intervention, even under unfavorable circumstances. . . . In studies focusing on accuracy, patients improved by at least 40% or to a normal level of performance. Quantitative improvements were often paralleled by a qualitative change of the error pattern, from implausible to more plausible errors. Finally, significantly faster responses were reached in all reported studies focusing on fluency."
T. Miles, 1992a, 14	"In the case of both literacy and numeracy it is, of course, a great advantage in the long run if a large amount of automaticity can be achieved, but it is important in both cases that alternative procedures should be available for use where necessary."
Butterworth, 2005, 459	"The majority of DD [developmental dyscalculic] children have problems with both knowledge of facts and knowledge of arithmetical procedures."

Researcher(s)	Findings/Conclusions
Mercer & Mercer, 2005, 139	". . . generalization to new situations occurs when a student demonstrates proficiency in math facts and continues to respond quickly and accurately when these facts are embedded in calculation problems."
Derbyshire & Highfield, 2004, 1	"In the late 1990s, Dr. Steel studied 241 children aged between seven and 12 to find out how they tackled simple sums. Around a third recalled the answers from their long-term memory while a third counted the answers on their fingers or used mental number lines. 'We found that retrieval was the fastest and most accurate and counting was the least accurate,' Dr. Steel told the science festival."
Kroesbergen, 2002, 5	"One of the major problems confronting these students [with learning difficulties or disabilities in math] is attaining automaticity. Special attention should therefore be devoted to the automatization of basic facts. Automaticity can be attained by practicing the skill in question. This means that such students will need extra time and possibilities to practice. In addition, such students must learn how to proceed when they do not know an answer directly or, in other words, to apply backup strategies (Lemaire & Siegler, 1995)."
Fazio, 1999, 427	"Several theorists have postulated that typically developing children may have an advantage over children with SLI [specific language impairment] information processing abilities. This advantage is not attributed to 'larger' overall working memory capacity. Rather, information processing differences are a result of a functional increase in processing efficiency gained by increased automaticity (e.g. Lahey & Bloom, 1994).
Lochy, Domahs, & Delazer, 2005, 472	"The majority of attempts to regain the skill of simple calculation have relied on drill (i.e., extensive repetition). In most cases, problems were frequently presented to the patient who was asked to answer them, getting instant feedback about his results. The rationale for this method relies on associative-learning principles, shared by most current models of arithmetical facts. . ., and supposes that traces of arithmetical facts in declarative memory have different levels of activation that determine their rate and probability of being retrieved."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 372	"The evidence reviewed here suggests that experience with numbers and math leads to domain-specific changes in the working memory demands of arithmetic. Children recognize and activate numbers more quickly with experience. This faster access to representations in long-term memory can alleviate working memory constraints because there is less time for items being held in working memory to decay. Working memory demands also decrease as procedures for manipulating number representations become more automated through practice. For example, counting becomes less effortful and the count-on strategy for addition is replaced by efficient retrieval. Thus, there is clear theoretical support for the view that working memory changes will be important in understanding how mathematical knowledge and skill develop."
Lochy, Domahs, & Delazer, 2005, 477	". . . patients with limited working memory capacities and fact-retrieval deficits should particularly profit from a training aiming at direct retrieval of arithmetic facts."
Lochy, Domahs, & Delazer, 2005, 477	". . . the importance of self-monitoring processes or strategies should not be neglected."
Garnett, 1992, 4	"In summary, cognitive psychology demonstrates that learning number facts is far more complex than just practicing them until they stick; learning them includes developing and employing a number of strategies for navigating the number system. Knowing number facts is not simple, one-step remembering; knowing them entails a sufficient assortment of associations easily retrievable from memory, a well-developed network of number relationships, easily activated counting and linking strategies, and well-practiced navigational rules for when to apply which maneuver. Indeed, this often taken-for-granted skill represents no small feat, requiring several years of frequent and varied number experiences and practice before children normally attain fluency."

Researcher(s)	Findings/Conclusions
Lochy, Domahs, & Delazer, 2005, 474	". . . is there an ideal number of facts to be trained at once? Actually, data diverge. Although learning rate was the same whatever the number of facts to be learned (e.g., 6, 12, or 18) when assessed per presentation rate of each item in Logan and Klapp's study (1991), other authors found that the acquisition of skill was faster the fewer the number of different addends given in one session (Haider & Kluwe, 1994)."
Lochy, Domahs, & Delazer, 2005, 474	". . . the order in which problems were learned influenced greatly their retrieval times, even overriding the classical size effect."
Lochy, Domahs, & Delazer, 2005, 474	"They [Campbell and Graham, 1985] suggest to divide the facts into sets of maximum six items, to train the most difficult problems first, and to adopt a strict performance criterion before moving to any new set."
Fazio, 1999, 428	"The slow, inaccurate performance on timed arithmetic tasks suggests that facilitating increased efficiency and automaticity of number facts may effectively increase performance in arithmetic calculation. Practice serves to strengthen declarative knowledge. It is the basic mechanism used to explain expertise in addition and subtraction (Ashcraft, 1985). One avenue of intervention would therefore involve extensive practice in number facts. Several computer-based interventions have been designed to provide practice in number facts."
Mercer & Mercer, 2005, 428	"The inability to acquire and maintain math facts at fluency levels sufficient for acquiring higher-level math skills is common among students with learning problems, and unfamiliarity with basic number facts plays a major role in the math difficulties of students with math learning problems. It is apparent that many students with learning disabilities lack proficiency in basic number facts and are unable to retrieve answers to math facts efficiently."
National Research Council, 2001, 182-183	"A large body of research now exists about how children in many countries actually learn single-digit operations with whole numbers. Although some educators once believed that children memorize their 'basic facts' as conditioned responses, research shows that children do not move from knowing nothing about the sums and differences of numbers to having the basic number combinations memorized. Instead, they move through a series of progressively more advanced and abstract methods for working out the answers to simple arithmetic problems. Furthermore, as children get older, they use the procedures more and more efficiently."
Brigham, Wilson, Jones, & Moisio, 1996, 5	"Perhaps the most important recommendations for teaching fraction computation are: (a) Ensure that numerical computation (e.g., the addition of fractions) is always preceded by student understanding of the meaning of the arithmetic operation, and (b) Ensure that students can describe a representation of the computational problem before they are required to master the mechanics of computation. . . . (c) Provide adequate guided practice to ensure that students do not invent error patterns to reach solutions, and (d) provide sufficient practice opportunities to ensure mastery and fluency."
Sherman, Richardson, & Yard, 2005, 104	"Once an error has been corrected by the learner, have her practice ample examples to extinguish the incorrect procedure."
McEwan, 2000, 76	"Students should be expected to memorize math facts at some agreed-upon point in their school careers. If they are not expected to do so, they won't! This might be a possible timetable (California State Board of Education, 1999, 4, 8, 11): • Addition facts (sums to 20) and the corresponding subtraction facts memorized by the end of first grade. • Multiplication tables of 2s, 5s, and 10s (to 10 x 10) memorized by the end of second grade. • The remainder of the multiplication tables memorized (all numbers between 1 and 10) by the end of third grade."

Researcher(s)	Findings/Conclusions
Garnett, 1992, 3	"Fluency with basic number facts, like fluency in reading, implies sufficient automaticity of subskills such that attentional resources need be diverted towards them only minimally for smooth coordination within complex operations. As with swift word recognition and fluency in reading text, development of number-fact fluency normally occurs with sufficient practice over a considerable time period."
Garnett, 1992, 3	"Knowing math facts is like spelling in that it is a highly visible, public aspect of performance. Everyone seems to notice when you're not good at it."
Wu, 1999, 1-2	"Sometimes a simple skill is absolutely indispensable for the understanding of more sophisticated processes. For example, the familiar long division of one number by another provides the key ingredients to understanding why fractions are repeating decimals. Or, the fact that the arithmetic of ordinary fractions (adding, multiplying, reducing to lowest terms, etc.) develops the necessary pattern for understanding rational algebraic expressions. At other times, it is the fluency in executing a basic skill that is essential for further progress in the course of one's mathematical education. The automaticity in putting a skill to use frees up mental energy to focus on the more rigorous demands of a complicated problem. Such is the case with the need to know the multiplication table (for single-digit numbers) before attempting to tackle the standard multiplication algorithm. . . . Finally, when a skill is bypassed in favor of a conceptual approach, the resulting conceptual understanding often is too superficial. This happens with almost all current attempts at facilitating the teaching of fractions."
Ainsworth & Christinson, 2000, 69	"Just as learning to count is a prerequisite skill that very young children need before they can begin exploring and manipulating number concepts, mastery of basic number facts is a necessary building block for students as they move up through the first formal years of schooling. By the time students leave elementary school, they should have these facts firmly committed to memory."
Cawley, Parmar, Foley, Salmon, & Roy, 2001, 324	"Data for rate of completion of single-digit computational items indicate that students with disabilities have lower rates than other students. One of the reasons for increasing speed of response and ultimately automatization is to enable the student to more fluently utilize fact knowledge when completing multidigit items."
National Research Council, 2001, 121	"Students need to be efficient and accurate in performing basic computations with whole numbers (6 + 7, 17 - 9, 8 x 4, and so on) without always having to refer to tables or other aides."
Akin, 2001, 1	"Consider such practices as cooking, carpentry, playing a musical instrument, horseback riding and other sports. Each builds upon a foundation of physical skills and in each case mastery consists of performing with automatic facility. . . . The skill is gradually incorporated into muscle memory."
Akin, 2001, 2	"Success at learning the alphabet . . . consists in recognizing the letters instantly without conscious effort. A dyslexic can pause and work out the difference between a 'd' and a 'b.' What is lacking is the automatic recognition response which easy facility in reading requires."
Akin, 2001, 4	"My real defense of all this symbolic manipulation is that it is easy. I hasten to add that when I speak of solving a system of two simultaneous linear equations in two unknowns as easy, I am using the word 'easy' as a term of art. None of this stuff is easy when you start learning it. But these routines all have the capacity to become easy given disciplined practice. They are easy after they have become automatic."
Akin, 2001, 5	". . . mathematics is cumulative and there are a great many skills that you have to be unthinkingly familiar with."
Ocken, 2001, 5	"Any student taking a first year calculus exam . . . must perform hundreds of small operations automatically and accurately. Indeed, a fundamental difficulty that bedevils many calculus students is that they have not learned to perform lower level mathematical operations automatically, accurately, and . . . without thinking about what they are doing. Only by submerging a concern with irrelevant detail can students choose, develop, and execute an appropriate global strategy for solving a complicated problem."

Researcher(s)	Findings/Conclusions
Ocken, 2001, 5	"How do students acquire the ability to perform lower level operations automatically? Numerical and algebraic symbol manipulation skills are not inborn. They must be learned, and for most students the process is not easy. Children need to begin slowly, with a few carefully chosen examples, in order to gain an understanding of how an algebraic process works. After that initial stage, practice for the sake of practice, i.e. drill for skill, is the path whereby the vast majority of students can reach the level of fluency and accuracy that is needed for formal mathematical competency."
Willingham, 2004, 3	"Our ability to think would be limited indeed if there were not ways to overcome the space constraint of working memory. One of the more important mechanisms is the development of automaticity. When cognitive processes . . . become automatic, they demand very little space in working memory, they occur rapidly, and they often occur without conscious effort."
Willingham, 2004, 3	"Automaticity is vital in education because it allows us to become more skillful in mental tasks. . . . The effective mathematician invokes important math facts and procedures automatically. . . . In each field, certain procedures are used again and again. Those procedures must be learned to the point of automaticity so that they no longer consume working memory space. Only then will the student be able to bypass the bottleneck imposed by working memory and move on to higher levels of competence."
National Research Council, 2001, 121	"Connected with procedural fluency is knowledge of ways to estimate the result of a procedure."
Woodward & Montague, 2002, 18	"Information processing approaches to math instruction for students with learning disabilities emphasize the importance of fluency in fact retrieval. The argument is that quick and efficient math fact recall or automaticity enables students to devote more of their cognitive resources to the procedural knowledge associated with learning algorithms (Gerber, Semmel, & Semmel, 1994; Peggegrino & Goldman, 1987)."
National Research Council, 2001, 138	"One conclusion that can be drawn is that by age 13 many students have not fully developed procedural fluency. Although most can compute well with whole numbers in simple contexts, many still have difficulties computing with rational numbers."
National Research Council, 2001, 139	"An [other] example is a multiple choice problem in which students were asked to estimate 12/13 + 7/8. The choices were 1, 2, 19, and 21. Fifty-five percent of the 13-year-olds chose either 19 or 21 as the correct response. Even modest levels of reasoning should have prevented these errors. Simply observing that 12/13 and 7/8 are numbers less than one and that the sum of two numbers less than one is less than two would have made it apparent that 19 and 21 were unreasonable answers."
National Research Council, 2001, 121	"It is important for computational procedures to be efficient, to be used accurately, and to result in correct answers. Both accuracy and efficiency can be improved with practice, which can also help students maintain fluency."
Committee on *How People Learn,* 2005, 1	"To develop competence in an area of inquiry, students must (a) have a deep foundation of factual knowledge, (b) understand facts and ideas in the context of a conceptual framework, and (c) organize knowledge in ways that facilitate retrieval and application."
Committee on *How People Learn,* 2005, 7	"Using concepts to organize information stored in memory allows for much more effective retrieval and application. Thus, the issue is not whether to emphasize facts of 'big ideas' (conceptual knowledge); both are needed. Memory of factual knowledge is enhanced by conceptual knowledge, and conceptual knowledge is clarified as it is used to help organize constellations of important details."
Committee on *How People Learn,* 2005, 243	"Time for consolidation of learning, with feedback loops should errors arise, is vital for mathematical fluency."
Mercer & Mercer, 2005, 405	"Failure to acquire mastery of math facts and to understand basic concepts in beginning math instruction contributes heavily to later learning problems because fluent recall of basic math facts makes it easier to solve more complex problems in which these basic operations are embedded. Unfortunately, students with learning problems often fail to grasp basic math facts or to develop fluency in these initial skills."

Researcher(s)	Findings/Conclusions
Berliner & Casanova, 1993, 15	". . . there is more to learning than just memory. It is true that memory is an important component of learning, but there is a difference between 'learning' and 'remembering.' We certainly want students to memorize facts (for example, multiplication facts). But mere memory is not our only goal. Students must also learn how to use facts—how to apply them to problems."
Klein, 2005, 15	"Research in cognitive psychology points to the value of automatic recall of the basic facts. Students who do not memorize the basic number facts will flounder as more complex operations are required of them, and their progress in mathematics will likely grind to a halt by the end of elementary school."

***MLS* Application.** One of the two major components of *MLS* addresses fact fluency. Ten of the 25 *MLS* tasks are devoted to fact fluency, as well as the fluency game, *Digit's Widgets*, on the webpage. Additionally, before the lessons on fluency are introduced, the program is designed to ensure that the student has a clear understanding of the concepts underlying the algorithms and operations. As an example, students develop to mastery the concept of the base-10 system, place value, and addition before being exposed to fluency practice on addition facts.

Fact Match is a set of activities that provide students with practice of basic mathematical facts and operations. Practice leads to fluency, which enables students to learn more complex mathematical processes. *MLS' Flash Cards* help students become more fluent on addition, subtraction, multiplication, and division math facts. Using the cards can also help teachers determine where students should begin in the Fluency Stage of *MLS*.

Barriers in Mathematics for English-Language Learners

Much of the literature concerning English-language learners and mathematics has to do with their difficulties in learning mathematics and English concurrently (see Chapter II). Biancarosa and Snow (2004) point out that "the problems faced by struggling readers are exacerbated when they do not speak English as their first language, are recent immigrants, or have learning disabilities. Indeed, a struggling reader may fit all three of these descriptions, making intervention a truly complicated proposition" (p. 8). Table 57 displays research findings and recommendations for practice relating to teaching mathematics to students with limited-English proficiency (see also Table 11 in Chapter II).

Table 57: Strategies for Teaching Mathematics to ELLs

Researcher(s)	Findings/Conclusions
Gray & Fleischman, 2004/2005, 84	"A review of effective instructional strategies for linguistically and culturally diverse students reveals that many of these strategies are simply extensions of approaches that work well with all students."
American Educational Research Association, 2004, 4	"Although [LEP] students can learn basic English reading skills in two years, their chances of failing later in school are still greater than native English speaking children. Even if excellent oral language support is provided in the primary grades, it takes far longer than two years for English language learners to become as fluent as native speakers and to acquire the broad vocabulary and reading comprehension skills needed for sustained academic achievement. Successful English learning requires targeted and continuing intervention."

Researcher(s)	Findings/Conclusions
Kamil, n.d., 29-30	"English-language learners face additional, unique challenges. Policies that guide instruction need to reflect the research that examines the transfer from first language to second language with ESL teaching strategies."
Zwiers, 2004/2005, 60	"Many English language learners need to learn English at accelerated rates to perform on grade level. Fluency in social language is not enough to help close the achievement gaps that are often created by a lack of academic language. We must train our students to hear, harness, and own the academic language that they need for success."
Gersten & Baker, 2006, 105	"Vocabulary learning should play a major role in successful programs for English language learners. . . . Criteria for selecting words should be considered carefully, so that words are selected that convey key concepts, are of high quality, are relevant to the bulk of the content being learned, and have meaning in the lives of students."
Gersten & Baker, 2006, 106	"The double demands of learning content and a second language are significant and the difficulty should not be underestimated. Because the spoken word is fleeting, visual aids such as graphic organizers, concept and story maps, and word banks give students a concrete system to process, reflect on, and integrate information."
Gersten & Baker, 2006, 106	"Intervention studies and several observational studies have noted that the effective use of visuals during instruction can lead to increased learning."
Bielenberg & Fillmore, 2004/2005, 46	"What English language learners need—and what teachers should provide—is training in the academic English skills that are foundational to literacy, mastery of subject matter, and superior test performance."
Bielenberg & Fillmore, 2004/2005, 47	"Mastering academic English—and thus surviving high-stakes tests—requires instructional activities that actively promote language development in the context of learning intellectually challenging content."
Abrams & Ferguson, 2004/2005, 64	"Language learners at all levels of ability need structured language lessons with extensive feedback to promote their skill development. They need specific scaffolding to build vocabulary and knowledge of language structures. In addition, such students need advocates for their general progress."
Short & Echevarria, 2004/2005, 10	"Until recently, no explicit model for effectively delivering sheltered lessons existed, and researchers had conducted few empirical investigations measuring what constitutes an effective sheltered lesson. Many educators agree on the important sheltered instruction techniques that help students comprehend content—for example, slower speech, clear enunciation, use of visuals and demonstrations, targeted vocabulary development, connections to student experiences, and use of supplementary materials."
Short & Echevarria, 2004/2005, 10	"Without systematic language development, many students never gain the academic literacy skills needed to succeed in mainstream classes, to meet content standards, and to pass standardized assessments."
Gray & Fleischman, 2004/2005, 84	"To provide meaning, scaffolding uses contextual supports—simplified language, teacher modeling, visuals and graphics, and cooperative and hands-on learning."
Sousa, 2001, 160	"The strategy of concept attainment has proven very effective for ESL students."
Sousa, 2001, 161	"The language of mathematics offers students the opportunity to deal with precise vocabulary, sequence, and syntax that can be helpful in acquiring both their native and a second language."
Sousa, 2001, 159	"ESL students may have high abilities in mathematics but have problems expressing these because of difficulties with the English language."
Sousa, 2001, 160	"Tying concrete models with verbal descriptions (in English) to mathematical concepts is a valuable way of helping ESL students bypass language barriers. This verbal labeling can demonstrate the language sense of a mathematical concept through context."

***MLS* Application.** CEI has seen schools have great success in using *MLS* as one component of the mathematics curriculum for English-language learners. *MLS* is predictably effective with this population due to its incorporation of the following research-based features:

- Emphasis on concepts, using consistent and academic vocabulary for mathematical terms.
- Use of manipulatives in teaching concepts.
- Use of modeling at the semiconcrete level and in problem-solving lessons.
- Auditory and visual instruction at the same time.
- Modeling of English pronunciation of mathematical terms.
- Use of visuals to illustrate meaning.
- Explicit teaching of algorithms/procedures.
- Adequate and varied practice to develop mastery and to develop fluency/automaticity.
- Instruction designed to accelerate learning dramatically.

Summary

Chapter IV focuses on mathematics content. The chapter began with the research on mathematical cognition, including the evidence of mathematical understandings even in infants. It then moved to definitions of mathematics from several sources. The next section includes research on who struggles in mathematics, echoing some of the research provided in Chapter I about the status of mathematics achievement in the United States. Manifestations of mathematical difficulties and disabilities were then discussed in general (with references back to Chapters II and III), noting that the manifestations of problems are similar, regardless of whether the student has a disability or not.

A description of *MLS* content was then provided, including an outline of the units, levels, and phases of the concept development scope and sequence and a description of the strand on the development of fact fluency. Following the description, the research on the most common problem areas of mathematics was provided, along with a description of the ways in which *MLS* addresses the problem. The two major areas of concept development and fact fluency not only are the emphasis in *MLS* design, but also, clearly, the subject of much research due to their importance not only in teaching general education students, but especially in teaching students who struggle with mathematics.

A final section was added on the challenges experienced by English-language learners, along with documentation of ways that *MLS* can meet their unique needs (see also Chapter II).

Chapter V will provide the research evidence that grounded CEI's decisions relating to lesson design and structure, the rationale for the concrete-semiconcrete-abstract lesson sequence and the use of manipulatives, and the use of computer-assisted instruction, including findings relating to appropriate screen design for struggling learners.

Chapter V: Research Findings that Ground *MLS'* Lesson Design

"To expect students who have a history of problems with fluency, metacognition strategies, attention, generalization, and motivation to engage in efficient learning (i.e., self-discovery learning) is not plausible"(Mercer & Mercer, 2005, p. 437).

Overview

In Chapters II and III the research on the wide variety of mathematics difficulties and disabilities was discussed. Chapter IV documented the research underlying the design decisions for the content of *MLS*—emphases on concept development and fact fluency.

Chapter V continues the discussion of research that grounds *MLS* design decisions—in this case the structure and sequence of lessons. The chapter begins with the general research on "best practices," especially those found to be effective in teaching struggling learners. The research evidence documenting the soundness of direct instruction, mastery learning, and one-on-one tutoring in structuring lessons is then discussed, followed by an explanation of *MLS'* employment of the components of these models in its various tasks.

The research on the concrete—semiconcrete—abstract sequence of lessons, including the use of manipulatives, especially as it relates to teaching struggling learners, is then presented, followed by a description of *MLS'* application of these findings. The chapter concludes with the research on the efficacy of computer-assisted instruction in teaching mathematics to struggling learners, along with the research on what works in terms of the graphic designs used on computer screens.

Deconstructing *MLS*

In order to document the scientifically-based evidence that grounds *MLS*, it was necessary to "deconstruct" it. That is, the authors of the study sat with CEI staff to identify and code all the component parts—how overall lessons are designed, what the various tasks are and how they are individually designed, the content emphases, the instructional strategies used, the assessment strategies used, and the list of program features that support implementation. The challenge, then, became how to make it clear that each of these topics does not operate in isolation from the others, but rather almost everything is happening dynamically in any student's individual lesson—just as it does in a lesson delivered solely by a teacher in a group setting.

The topics have been categorized and discussed in a logical order, but it is important for the reader to be aware at all times of the overlapping, intertwined, cross-cutting, reiterative, spiraling, interrelated juxtaposition of the various component parts, one with the other—all constantly mediated and scaffolded by the computer and the lab's teacher/facilitator. Such an approach is similar to studying separately the sheet music for each orchestra instrument. Much can be learned by doing so, but it is the symphony in performance led by an inspired and inspiring conductor that makes the music. Effective instruction, therefore, provides the desired result—the music.

Lesson Stages

The Alliance for Curriculum Reform (1999), led by Gordon Cawelti, included three "phases of teaching learning strategies" (p. 16) in their research synthesis on how to improve student achievement: instructional, practice (guided or independent), and assessment. Mercer and Mercer (2005) provided a similar set of "systematic teaching steps" (p. 133) for lesson stages for students with disabilities:

1. opening the lesson
2. conducting an interactive presentation
3. closing the lesson
4. using continuous teaching components.

The Mercer's explanations for these four steps would suggest that "opening the lesson" equates to Cawelti's instructional stage; "conducting an interactive presentation" provides the guided practice; "closing the lesson" is both review and the provision of independent practice; and "using continuous teaching components" is ongoing assessment, both on how to improve the lesson and to guide next steps for students.

The cognitive theory behind these stages is explained by Sternberg (2003), who reports on studies by Anderson on the acquisition of procedural knowledge. Anderson hypothesized that "knowledge representation of procedural skills occurs in three stages: cognitive, associative, and autonomous" (p. 270). Sternberg's explanations follow:

> During the cognitive stage, we think about explicit rules for implementing the procedure. During the associative stage, we practice using the explicit rules extensively, usually in a highly consistent manner. Finally, during the autonomous stage, we use these rules automatically and implicitly, with a high degree of integration and coordination, as well as speed and accuracy (p. 270).

Table 58 displays additional research on lesson stages or phases, as they are sometimes called:

Table 58: Lesson Stages/Phases

Researcher(s)	Findings/Conclusions
Alliance for Curriculum Reform, 1999, 16	"Three possible phases of teaching about learning strategies include: 1) modeling, in which the teacher exhibits the desired behavior; 2) guided practice, in which students perform with help from the teacher; and 3) application, in which students act independently of the teacher."

Researcher(s)	Findings/Conclusions
Jones, Wilson, & Bhojwani, 1997, 157	"Phases of structured academic presentations: • Opening: gain the students' attention; review pertinent achievements from previous instruction; state the goal of the lesson. • Body: model performance of the skill; prompt the students to perform the skill along with you; check the students' acquisition as they perform the skill independently. • Close: review the accomplishments of the lesson; preview the goals for the next lessons; assign independent work."
McEwan, 2000, 46	"Dixon and his colleagues (1998a) report that the lesson models for effective interventions most frequently followed a three-phase pattern. In the first phase, teachers not only demonstrated, but also explained, asked many questions, checked for understanding, or conducted discussions. In sharp contrast to the conventional model, students were almost always quite actively involved in the instruction during the initial phase. The second phase that was found in lessons that produced high achievement is an intermediate stage between learning something new and being proficient enough to apply that new knowledge independently. This is the help phase of the lesson, in which students gradually make a transition from teacher regulation to self-regulation. The specifics of this second phase vary considerably, from students helping one another collaboratively to high levels of teacher help with feedback and frequent correctives (additional explanation when students falter). In many cases, this second phase of instruction took up the majority of lesson time. Dixon and his colleagues observed a third phase in effective lessons, one of individual accountability."
Karp & Howell, Oct. 2004, 122	"Depending on the mathematics content and the student, a mathematics teacher may use direct modeling of a new task, guide the student's thought processes through the use of open-ended questions, or provide insight when necessary after a period of student-led inquiry. No one approach fits all students."
Ontario Ministry of Education, 2005, 63	"Modeling can take on a number of forms. Teachers can 'think aloud.' They can overtly verbalize the thought processes used to complete a particular activity. . . . Teachers can model learning strategies. . . . Teachers can demonstrate the task. The teacher may, for example, demonstrate all the steps in completing a graphic organizer or show the steps that students need to take to solve a specific type of math problem (e.g., Fuchs et al., 2003b)."
Ontario Ministry of Education, 2005, 18	"Explicit instruction requires teachers to frequently model appropriate learning strategies."
Ontario Ministry of Education, 2005, 62	"Students with special needs require guided practice to help them bridge the gap between what they know and don't know, and they need to receive explicit instruction in how to apply learned information in new situations."
Ontario Ministry of Education, 2005, 64	"The teacher can provide students with support and guidance as they initially learn new information or tasks, and then gradually phase out this support as the students become more proficient. Guided practice is critically important to many effective instructional programs, including those targeting mathematical problem solving (Fuchs et al., 2003b). . . ."
Ontario Ministry of Education, 2005, 64	"Guided practice is an important way to prevent students from forming misconceptions (Rosenshine, 1997). Some students may come to the task lacking in prior knowledge and may be overwhelmed by the complexity or amount of new information. Other students may have limited working memory capacity or poor language skills and thus will also struggle to process the information that is presented. Guided practice helps students understand and clarify task expectations and facilitates their ability to link new knowledge with existing concepts."

***MLS* Application.** The following table displays a list of each of the *MLS* tasks, with coding to indicate the specific lesson stages or phases for each task. The first set of tasks are those in the concept development strand. The second set, beginning with "Look, Listen, See, and Say" are those in the fact fluency component.

Codes are as follows:

I	=	Instruction	SA	=	Self-Assessment
P	=	Practice	M	=	Mastery
A	=	Assessment			

Table 59: *MLS'* Incorporation of Lesson Stages/Phases

MLS Task	Task Phase
Concept Building Introduction	I
Learn	I, SA
Solve	P, (I), A M
Help	I
Solve Intervention	I
Let's Review	P
Word Problems Learn	I
Word Problems Solve	P, (I), A M
Word Problems Let's Review	P
Math Game	P
Printed Activities (7,8,9)	P, A
Math Magic	P, A
Drawing Conclusions	P, A
Flash Cards	P, A
Look, Listen See and Say	I
See, Hear and Respond	I, P, A, M
Hear and Respond	I, P, A, M
See and Respond	A, M
Echo	P, A
Blank Out	P, A
Number Search	P, A
Quick Pick	P, A
Quick Answer	P, A

Lesson Models

The three lesson models—direct instruction, mastery learning, and one-to-one tutoring—that are utilized in *MLS* are clearly related. All three are goal-focused; all three include explicit and systematic strategies; all three emphasize assessment and corrective feedback; all three involve ongoing assessment to determine progress; and all three are proven methods for improving student learning. One major difference is that direct instruction models typically involve the whole class moving through instruction together; mastery learning separates students for instruction into groups, based upon their individual needs. And tutoring is one-to-one.

Direct Instruction. The Alliance for Curriculum Reform and the Educational Research Council (1999) included a definition of direct instruction in their synthesis of research on improving student achievement:

Six phased functions of direct teaching work well:
1) daily review, homework check, and, if necessary, re-teaching;
2) presentation of new content and skills in small steps;
3) guided student practice with close teacher monitoring;
4) corrective feedback and instructional reinforcement;
5) independent practice in seatwork and homework with a high (more than 90 percent) success rate; and
6) weekly and monthly reviews (p. 14).

Others define the model similarly. For instance, according to the Mercers (2005), Simmons, Fuchs, and Fuchs (1991) created a similar "instructional template to help teachers include explicit teaching steps within their lessons":

1) present an advance organizer
2) demonstrate the skill
3) provide guided practice
4) offer corrective feedback
5) set up independent practice
6) monitor practice
7) review (p. 149).

The efficacy of the direct instruction model is well studied and documented. Representative findings are provided in Table 60.

Table 60: Direct Instruction

Researcher(s)	Findings/Conclusions
Walberg & Paik, n.d., 12	"Many studies show that direct teaching can be effective in promoting student learnings. The process emphasizes systematic sequencing of lessons, a presentation of new content and skills, guided student practice, the use of feedback, and independent practice by students."
Brigham, Wilson, Jones, & Moisio, 1996, 5	"Several practices were incorporated into a direct instruction program by Perkins and Cullinan (1985) who found that student errors decreased and task mastery increased directly as a function of an instructional program that included proven mastery of prerequisite skills, daily probes, extensive and periodic review, guided practice, verbal prompts, and corrective feedback."
National Research Council, 2001, 19	". . . direct school-based instruction may play a larger part in most children's mathematical experience than it does in their reading experience."
Ellis & Fouts, 1997, 224	"We recommend that districts interested in a research-tested curriculum of basic skills for young learners and at-risk children should seriously consider D.I. It is, after all, one of a minority of educational innovations that has evidence on its side."
Whitehurst, n.d., 7	"We know that direct instruction can help students learn computational skills and understand math principles. As a corollary, we know that children don't have to discover math principles on their own or work with authentic open-ended problems in order to understand mathematical concepts."

Researcher(s)	Findings/Conclusions
Whitehurst, n.d., 1	". . . a number of studies have demonstrated that conceptual understanding can be produced through a variety of pedagogical techniques, including sequence direct instruction on the underlying principles, practice on a wide variety of problem types, and exposure to worked examples. In other words, the type of knowledge that allows a learner to solve many types of problems doesn't need to be discovered by the learner to be effective. . . . A subsequent study showed that simply telling children to 'notice the first digit' before they solve problems substantially enhanced their performance compared to basic discovery learning with the number line problems. In other words, providing direct instruction on what to attend to created conceptual understanding."
Mercer & Mercer, 2005, 131	"The findings of mathematics research indicate that students can benefit from instruction that includes both explicit and implicit methods (Mercer, Jordan, & Miller, 1994). The literature supports explicit methods such as description of procedures, modeling of skills . . ., use of cues and prompts, direct questioning of students to ensure understanding, and practice to mastery."
International Dyslexic Association, 2002, 2	"Use explicit teaching procedures. Many commercial materials do not cue teachers to use explicit teaching procedures; thus, the teacher often must adapt materials to include these procedures. Teachers can include explicit teaching steps within their lessons (i.e., present an advanced organizer, demonstrate the skill, provide guided practice, offer corrective feedback, set up independent practice, monitor practice, and review)."
International Dyslexic Association, 2002, 3	"Use step-by-step instruction. New or difficult information can be presented in small sequential steps. This helps learners with limited prior knowledge who need explicit or part-to-whole instruction."
Becker & Engelmann, n.d., 1	"The program directors . . . attribute its (direct instruction model) success to the technological details, the highly specific teacher training, and careful monitoring of student progress."
Kroesbergen & Van Luit, 2003, 106	". . . in general, self-instruction is most effective. However, for the learning of basic skills direct instruction appears to be the most effective."
Klahr & Nigam, 2004, 1	"We found that many more children learned from direct instruction than from discovery learning, but also that when asked to make broader, richer scientific judgments the (many) children who learned about experimental design from direct instruction performed as well as those (few) children who discovered the method on their own. These results challenge predictions derived from the presumed superiority of discovery approaches to teaching young children basic procedures for early scientific investigation."
Smey-Richman, 1988, 19-20	"Research on the learning of skills offers some suggestions for teaching thinking skills directly. According to Beyer, any skill is learned best when the learners are: • consciously aware of what they are doing, and how they are doing it. . . [I would add also why are they doing it!] • not distracted by other inputs competing for attention • seeing the skill modeled • engaging in frequent, intermittent practice of the skill • using feedback they received during this practice to correct their own performance of the skill • talking about what they did as they engaged in the skill • receiving guidance on how to use a skill at a time when they need the skill to accomplish a content-related goal • receiving guided opportunities to practice the skill in contexts other than the one in which the skill was originally introduced."
Butler, Miller, Lee, & Pierce, 2001, 20	". . . the focus of instruction has shifted from basic skills instruction to computation and problem-solving instruction. Techniques such as constant-time delay, peer tutoring, time trials, and direct instruction proved beneficial in improving mathematics skills. Further, students with mental retardation learned to employ cognitive strategies successfully when these techniques were included."

Researcher(s)	Findings/Conclusions
US Dept. of Ed., 1986, 35	"The basic components of direct instruction are: • Setting clear goals for students and making sure they understand those goals. • Presenting a sequence of well-organized assignments. • Giving students clear, concise explanations and illustrations of the subject matter. • Asking frequent questions to see if children understand the work, and • Giving students frequent opportunities to practice what they have learned."
Vaughn, Gersten, & Chard, 2000, 7	"Making instruction visible and explicit is an essential feature of effective interventions for students with LD (Elbaum et al., 1999; Gersten & Baker, in press; Swanson, 1999). . . . students with disabilities benefit when the elements of what they are learning are identified and demonstrated with examples. The benefit to making instruction explicit and overt is twofold. First, a teacher offers students an opportunity to learn how to think about a learning situation in a way that they would likely not discover on their own. Second, by making instruction overt, teachers and peers can provide students with LD with formative feedback to guide and correct the application of their learning."
Baker, Gersten, & Lee, 2002, 2	"The purpose of this study was to synthesize research on the effects of interventions to improve the mathematics achievement of students considered low achieving or at risk for failure. . . . Results indicated that different types of interventions led to improvements in the mathematics achievement of students experiencing mathematics difficulty, including the following: (a) providing teachers and students with data on student performance; (b) using peers as tutors or instructional guides; (c) providing clear, specific feedback to parents on their children's mathematics successes; and (d) using principles of explicit instruction in teaching math concepts and procedures."
Ortiz, 2001, 4	"Clinical teaching is carefully sequenced. First, teachers teach skills, subjects, or concepts; then they reteach using different strategies or approaches for the benefit of students who fail to meet expected performance levels after initial instruction; finally, they use informal assessment strategies to identify the possible causes of failure. Teachers conduct curriculum-based assessments to monitor student progress and use the data from these assessments to plan and modify instruction."
US Dept. of Ed., 1986, 35	"Direct instruction has been particularly effective in teaching basic skills to young and disadvantaged children."
Sousa, 2001, 22	"An analysis of almost 30 years of research indicates that the following interventions are most effective with learning disabled students: The most effective form of teaching was one that combined direct instruction (e.g., teacher-directed lecture, discussion, and learning from textbooks) with teaching students the strategies of learning (e.g., memorization techniques, study skills)."
US Dept. of Ed., 1986, 35	"When teachers explain exactly what students are expected to learn, and demonstrate the steps needed to accomplish a particular academic task, students learn more."
Schmoker, 1999, 73	"Wesley gets these results by using direct instructional methods: clear, sequenced instruction and feedback provided on an organized schedule."
Alliance for Curriculum Reform, 1999, 14	"Six phased functions of direct teaching work well: 1) daily review, homework check, and, if necessary, re-teaching; 2) presentation of new content and skills in small steps; 3) guided student practice with close teacher monitoring; 4) corrective feedback and instructional reinforcement; 5) independent practice in seatwork and homework with a high (more than 90 percent) success rate; and 6) weekly and monthly reviews."

Researcher(s)	Findings/Conclusions
Kroesbergen, 2002, 3	"Constructivists do not agree on the nature of teacher-student interactions. A distinction can be made between endogenous and exogenous constructivism with a continuum of positions occurring in between. The endogenous constructivists think that instruction should be structured to help students discover new knowledge without explicit instruction. Exogenous constructivists think that teachers should engage students by providing explicit instruction via the provision of descriptions, explanations, modeling, and guided practice with feedback. An expanded interpretation of constructivism to include both explicit and implicit instruction is receiving growing acceptance, particularly for the instruction of students with learning difficulties."
Mercer & Mercer, 2005, 128	"Explicit teaching is instruction in which the teacher serves as the provider of knowledge. Explicit teaching is based on the belief that when learning is complex and difficult for learners, the teacher must provide extensive support to students and transmit knowledge that facilitates learning. Skills and concepts are presented in a clear and direct fashion that promotes student mastery. In explicit instruction, the teacher provides an explanation or model of a skill or concept, guides students through application of the skill or concept in a variety of situations, and provides many opportunities for independent application that will ensure mastery and generalization. Explicit instruction emphasizes student mastery, and its principles are compatible with behavioral theory, direct instruction, task analysis, product-oriented effective teaching research, and exogenous constructivism."
Walberg & Paik, n.d., 12	"Many studies show that direct teaching can be effective in promoting student learnings. The process emphasizes systematic sequencing of lessons, a presentation of new content and skills, guided student practice, the use of feedback and independent practice by students."
Ellis & Fouts, 1997, 224	"We recommend that districts interested in a research-tested curriculum of basic skills for young learners and at-risk children should seriously consider D. I. It is, after all, one of a minority of educational innovations that has evidence on its side."
Steele, 2005, 4	"In most explicit instruction, there is a great deal of practice and review of new learning until mastery occurs (Grobecker, 1999). Whether it is multiplication facts, geography terms involving landforms, or vocabulary related to a biology lesson on parts of the brain, direct instructional lessons provide extensive drill and practice time (Olson and Platt, 2000). The students with LD benefit from such over learning because of their memory problems and difficulty processing information."
Steele, 2005, 4	"Another example of a direct instruction strategy appropriate for students with LD is the use of fast paced lessons with monitoring and feedback. These students can learn if the lesson includes a chance for monitoring by teacher and students, provisions of feedback, and some type of reinforcement. The elements of the lesson have been shown to be effective with children, especially those with disabilities."
Mercer & Mercer, 2005, 134	" . . . explicit modeling becomes essential for immature learners (e.g., those who have limited prior knowledge or are passive learners) to acquire and use essential knowledge."
Mercer & Mercer, 2005, 134	"Guided practice primarily consists of the teacher prompting students and checking their work. Prompts enable the teacher to help students perform the task so that initial practice will be successful."
Mercer & Mercer, 2005, 136	"A recommended success level during independent practice is 90 to 100 percent."
Mercer & Mercer, 2005, 136	"Active and frequent monitoring is a key to student learning. . . . Slavin and Madden (1989) report that the most effective programs involve frequent assessment of student progress so that programs can be modified according to individual needs."
Mercer & Mercer, 2005, 137	"A significant finding . . . is that academic feedback is positively associated with student learning. . . . Porter and Brophy (1988) report that good teachers monitor students' understanding through regular appropriate feedback. Wang (1987) reports that feedback strongly promotes mastery of content and skills for further learning, ability to study and learn independently, ability to plan and monitor learning activities, motivation for continued learning, and confidence in one's ability as a learner."

Researcher(s)	Findings/Conclusions
Mercer & Mercer, 2005, 137	"Baechle and Lian (1990) found that direct feedback significantly improved the performance of students with learning problems. . . ."
Mercer & Mercer, 2005, 432	"Explicit teacher modeling of cognitive and metacognitive strategies in solving word problems has yielded encouraging results, and these preliminary findings . . . suggest that specific strategy instruction in math holds significant promise for students with learning problems."
Mercer & Mercer, 2005, 437	". . . the majority of constructivists appear to use explicit instruction when students with moderate to mild disabilities are the target population. This position is understandable when the characteristics of these learners are considered. To expect students who have a history of problems with fluency, metacognitive strategies, memory, attention, generalization, and motivation to engage in efficient learning (i.e., self-discovery learning) is not plausible. Thus, teacher-directed instruction or explicit instruction is a primary component for teaching math to students with disabilities. . . ."
Mercer & Mercer, 2005, 437-438	"In general, direct instruction appears to be most effective for the learning of basic math facts. . . ."
Lock, 1996, 6	"One suggested schedule for the class period includes a period of review of previously covered materials, teacher-directed instruction on the concept for the day, guided practice with direct teacher interaction, and independent practice with corrective feedback. During the guided and independent practice periods, teachers should ensure that students are allowed opportunities to manipulate concrete objects to aid in their conceptual understanding of the mathematical process, identify the overall process involved in the lesson, . . . and write down numerical symbols or mathematical phrases such as addition or subtraction signs."
Jones, Wilson, & Bhojwani, 1997, 155	"More explicit instruction results in more predictable, more generalizable, and more functional achievement. If we do not explicitly teach important knowledge and skills, these objectives will not be adequately learned."
Jones, Wilson, & Bhojwani, 1997, 157	". . . instruction is teacher-led and characterized by (a) explicit performance expectations, (b) systematic prompting, (c) structured practice, (d) monitoring of achievement, and (e) reinforcement and corrective feedback."
Jones, Wilson, & Bhojwani, 1997, 158	". . . there is empirical evidence in the professional literature that direct instruction procedures have been effectively used to teach math skills to older students with LD."
Jones, Wilson, & Bhojwani, 1997, 158	"Principles for designing practice activities (Carnine, 1989): 1. Avoid memory overload by assigning manageable amounts of practice work as skills are learned. 2. Build retention by providing review within a day or two of the initial learning of difficult skills, and by providing supervised practice to prevent students from practicing misconceptions and 'misrules.'" 3. Reduce interference between concepts or applications of rules and strategies by separating practice opportunities until the discriminations between them are learned. 4. Make new learning meaningful by relating practice of subskills to the performance of the whole task, and by relating what the student has learned about mathematical relationships to what the student will learn about mathematical relationships. 5. Reduce processing demands by preteaching component skills of algorithms and strategies, and by teaching easier knowledge and skills before teaching difficult knowledge and skills. 6. Require fluent responses. 7. Ensure that skills to be practiced can be completed independently with high levels of success."

Researcher(s)	Findings/Conclusions
Kroesbergen, 2002, 2.4	". . . regarding treatment components of interventions, it appears from the meta-analysis that, in general, self-instruction is most effective. However, for the learning of basic skills, direct instruction appears to be the most effective."
Kroesbergen, 2002, 3.1	"The main characteristic of direct instruction is, in fact, that it is very structured in practice. In practice, direct instruction is teacher-led, because the teacher provides systematic explicit instruction (Jones et al., 1997). New steps in the learning process are taught one at a time, and the teacher decides (guided by the instructional program) when new steps are taught. The lessons are generally built up following the same pattern (e.g., Archer & Isaacson, 1989). In the opening phase, the students' attention is gained, previous lessons are reviewed, and the goals of the lesson are stated. In the main part of the lesson, the teacher demonstrates how a particular task can be solved and then allows the students to work together on the task. When the students appear to have sufficient understanding of the task, they are given new tasks to practice independently. The teacher monitors the students during such practice and provides feedback on completed tasks. Interventions in which students receive direct structured instruction have been frequently found to be very effective (e.g., Harris, Miller, & Mercer, 1995; Jitendra & Hoff, 1996; Van Luit, 1994; Wilson, Majsterek & Simmons, 1996)."
McEwan, 2000, 42	"There are quite a number of items . . . that have substantial empirical research bases to indicate their effectiveness in raising student achievement, for example, direct instruction (Abt, 1976) and student reading of textbooks (Donahue, Voelkl, Campbell, & Masseo, 1999), particularly among children who are at risk of academic failure."
McEwan, 2000, 48-49	"Before you completely eradicate whole-class, teacher-directed instruction, consider its effectiveness in raising student achievement, especially for at-risk students. Direct instruction is often misconstrued by those who do not thoroughly understand its approach to instructional design as rote learning without meaning. The theory upon which direct instruction is based is fully cognizant that students are not empty vessels that teachers can fill at will. Clearly, if a learner is not able to make sense out of what the teacher says and does, learning will not occur. There is a major difference, however, between the way the radical constructivists believe students construct meaning and ways in which those who practice direct instruction see it occurring—the major role the teacher plays."
McEwan, 2000, 49	"The research on the effectiveness of a direct instruction approach to teaching mathematics is impressive (Abt Associates, 1976). . . . The 20th percentile was used as a common baseline because it is the average expectation for children from economically disadvantaged backgrounds. Even more significant is the fact that the direct instruction students performed almost at the national norm, an accomplishment that demonstrates the potential for all students to be successful in mathematics (Silbert, Carnine, & Stein, 1981, 483)."
Woodward, n.d., 2	"Results indicated that students in the number sense condition generally performed better on a range of post test and maintenance test measures, though direct instruction was effective in helping many students master basic multiplication facts."
Kroesbergen, 2002, 7	"Both the form and content of the lessons should be clearly structured. With regard to the form of the lessons, it is recommended that the lessons always be built up using the same pattern including an opening phase with reflection on the previous lesson, a brief presentation of the material to be learned, a practice phase with both guided and individual practice, and a closing phase (e.g., Archer et al., 1989; Veenman, 1993). The instructional principles recommended for use with low performers include the modeling of explicit strategies, cumulative introduction of information, isolation of independent pieces of information, separation of confusing elements and terminology, use of a concrete/semi-concrete/abstract sequence, use of explicit-implicit math instruction, emphasis on relations, explicit generalization instruction, building retention, and instructional completeness (e.g., Carnine, 1989; Mercer & Mercer, 1998; Ruijssenaars, 1992). The main difference between regular instruction and special instruction is that nothing is left to chance in the latter (Ruijssenaars, 1992)."

Researcher(s)	Findings/Conclusions
Kroesbergen, 2002, 2	"The behavioral and cognitive frameworks constitute the major paradigms for studying the phenomenon of human learning. "Behaviorists recognize the existence of several different stages of learning: acquisition, proficiency, maintenance, generalization, and adaptation. Given the behaviorist's emphasis on the environment as a critical factor for learning, considerable emphasis is also placed on the teacher's arrangement of the classroom for learning. . . . One of the essential components of the behavioral approach to learning is direct instruction. The key principle underlying direct instruction is that both the curriculum materials and the teacher presentation of these materials must be very clear and unambiguous. This includes an explicit step-by-step strategy, development of mastery at each step in the learning process, strategy corrections for students errors, gradual fading of teacher-directed activities and increased independent work, use of systematic practice with an adequate range of examples, and cumulative review of newly learned concepts."
Pennington, 1991, 124-125	"If the main deficit is in executive functions, the child probably experiences the greatest difficulty on complex word problems and multistep calculations (e.g., long division). Such children can benefit from explicit written step-by-step 'recipes' or algorithms to guide them through multistep problems. In addition, these children especially need 1:1 instruction from a tutor who models metacognitive functions for the child by explicitly going through the steps in a problem, including estimates, goals, subroutines, and check procedures. The tutor is in effect making 'internal speech' external so the child can hopefully learn to use this kind of internal speech to regulate his or her own problem-solving performance. Such children can also benefit from experiences that make math problems more concrete."
National Research Council, 1997, 124	"Intensive instruction refers to a broad set of instructional features that includes, but is not limited to (a) high rates of active responding at appropriate levels; (b) careful matching of instruction with students' skill levels; (c) instructional cues, prompts, and fading to support approximations to correct responding; and (d) detailed, task-focused feedback—all features that may be incorporated into group lessons."
National Research Council, 1997, 125-126	"Meta-analysis and narrative syntheses . . . show that intensive instruction can result in impressive learning for students who otherwise would fail to achieve critical benchmarks."
National Research Council, 1997, 126-127	"Research demonstrates that many students with cognitive disabilities need extensive, structured, and explicit instruction to develop the processes and understandings that other children learn more easily and naturally. Indeed, three empirical literatures question the tenability of constructivist principles of many students with disabilities."
Smey-Richman, 1988, 19	"What can be done to improve the metacognition of low-achieving students? Numerous studies and reviews have confirmed that specific learning skills can be taught directly . . . ., whereas the executive functions are more difficult to impart and must evolve gradually over time. . . . Some supporters of this viewpoint maintain that low achievers, unlike their peers, need sustained, explicit skill instruction with much opportunity for practice and feedback. . . . Brophy concurs when he writes that lower SES learners need more structuring from their teachers, more active instruction and feedback, more redundancy, and small steps with high success rates."
US Dept. of Ed., Feb. 6, 2002, 1	". . . there is some evidence that providing this degree of explicitness to kids, showing them strategies, letting them take over and showing what they know is helpful." (Russell Gersten, University of Oregon)

Researcher(s)	Findings/Conclusions
Kroesbergen, 2002, 3.1	"Van Luit and Naglieri (1999) suggest that teaching step-by-step from concrete to abstract, working with materials to mental representation and providing task-relevant examples can certainly help. Many researchers (e.g., Jones, Wilson, & Bhojwani, 1997; Wood, Frank, & Wacker, 1998) also state that the instruction for children with special needs looks different from regular instruction. Students with math learning difficulties, whether severe or mild, clearly need structured and detailed instruction, explicit task analysis, and explicit instruction for generalization and automatization. This can be realized with direct instruction."

Mastery Learning. While direct instruction is the model for a single lesson presented to a whole class, mastery learning describes a sequence of lessons, including pre-testing of students to determine which ones need which instruction. The Alliance for Curriculum Reform (1995) defines the research supportive of the mastery learning lesson model as follows:

> More than 50 studies show that careful sequencing, monitoring, and control of the learning process raises the learning rate. Pre-testing helps determine what should be studied; this allows the teacher to avoid assigning material that has already been mastered or for which the student does not yet have requisite skills. Ensuring that students achieve mastery of initial steps in the sequence helps ensure that they will make satisfactory progress in subsequently more advanced steps. Frequent assessments of progress informs teachers and students when additional time and corrective remedies are needed (p. 16).

An early researcher on the effectiveness of mastery learning was Benjamin Bloom (1984). He found that the "average student under mastery learning was about one standard deviation above the average of the control class, or above 84 percent of the students in the control class" (p. 5). Subsequent studies also attest to the power of this model, as is evident in the references in Table 61.

Table 61: Mastery Learning

Researcher(s)	Findings/Conclusions
Bloom, 1984, 7-8	". . . the mastery learning feedback-corrective approach is primarily addressed to providing students with the cognitive and affective prerequisites for each new learning task. . . . The main point is that the mastery learning students improve their processing of the instruction, although the instruction is much in the same in both types of classes."
Levin & Long, 1981, 7	"The mastery learning studies show that when students are given extra time and appropriate help, and when they are motivated to learn, 80 percent or more can finally attain the preset mastery level on each learning unit. One of the more striking and consistent results of these studies is the pattern of learning of mastery groups versus control groups. . . . Control and mastery groups start at the same achievement level. As learning progresses, it is apparent that the mean performance level of the mastery groups becomes significantly higher than that of the control groups. This is true even before the mastery students engage in the corrective process."
Levin & Long, 1981, 8	"These studies suggest several explanations. First, the students in the mastery group are provided with the cognitive prerequisites necessary for each new learning unit in the series. Bloom calls them cognitive entry behaviors. Students who acquire the necessary prerequisites are better able to understand the instruction and, as a result, become more involved in the learning."

Researcher(s)	Findings/Conclusions
Ellis & Fouts, 1997, 185	"The research literature in mastery learning is largely positive. Some of the best-known names in educational research circles have weighed in as supporters of this approach to teaching and learning. . . . Study after study indicates the superiority of mastery learning over traditional methods in raising test scores."
Mercer & Mercer, 2005, 434	". . . mastery learning refers to teaching a skill to a level of automaticity. . . . Reaching mastery on a skill provides numerous benefits, including improved retention and improved ability to compute or solve higher-level problems. Other benefits include finishing timed tests, completing homework faster, receiving higher grades, and developing positive feelings about math."
Mercer & Mercer, 2005, 434	"Independent practice is the primary instructional format used to acquire mastery. Because practice can become boring, the teacher must try to make practice interesting or fun. Instructional games, peer teaching, computer-assisted instruction, self-correcting materials, and reinforcement are helpful in planning practice-to-mastery activities.

One-on-One Tutoring. The single most powerful form of teaching, according to Benjamin Bloom (1984), is one-on-one tutoring:

> Using the standard deviation (sigma) of the control class, which was taught under conventional conditions, it was found that the average student under tutoring was about two standard deviations above the average of the control class. Put another way, the average tutored student outperformed 98 percent of the students in the control class (p. 5).

Other researchers have documented similarly powerful results. In an ideal world, the tutorial lesson model is the one all schools would and should use. The expense of such a model, however, is prohibitive—without the use of technology. Computer-assisted instruction allows one teacher to supervise the work of many students, all receiving one-on-one instruction at the same time via the computer. Gilbert and Han (1999) noted that traditional instruction is designed for one teacher to teach many students. Tutoring is designed for one-to-one. With technology, schools can have a delivery system that is many-to-one. That is, individualized lessons can be delivered to many students with all the characteristics of expert one-on-one tutoring at once, and this concept is the one on which *MLS* is based. Tutoring is a component of all *MLS* tasks.

Table 62: One-on-One Tutoring

Researcher(s)	Findings/Conclusions
Snow, Barley, Lauer, Arens, Apthorp, Englert, & Akiba, 2005, 51-52	"Central to the practice of tutoring is that the interaction is characterized by thorough and frequent diagnostic and prescriptive exchanges between tutor and tutee. This rich cycle of feedback and tailored instruction . . . allows the tutor to attend closely to the academic needs of the learner. In successful programs this exchange is recognized and encouraged."
Snow, Barley, Lauer, Arens, Apthorp, Englert, & Akiba, 2005, 52	"Successful tutoring programs also have what can be called a 'guiding purpose.' Consider a guiding purpose to be a strong theoretical backing or at least some expressed purpose that will help guide tutors in their decision making."
Snow, Barley, Lauer, Arens, Apthorp, Englert, & Akiba, 2005, 54	"The research supplies strong evidence that tutoring is an effective strategy for addressing the needs of low-performing students."

Researcher(s)	Findings/Conclusions
Gilbert & Han, n.d., 10	"The theory of learning styles states that people have different approaches to learning and studying. . . . The most commonly used instruction environments use a one-to-many or one-to-one instructor/learner relationship. We have developed an environment that utilizes technology to deliver a many-to-one instructor/learner relationship."
Alliance for Curriculum Reform, 1995, 15	"Because it gears instruction to needs, tutoring has yielded large learning effects in several dozen studies."
Alliance for Curriculum Reform, 1999, 17	"Teaching one student or a small number with the same abilities and instructional needs can be remarkably effective."
National Research Council, 1997, 126	"Just as for students with mild disabilities, research indicates that one-to-one intensive instruction helps develop the skills of students with more severe cognitive disabilities."
Mercer & Mercer, 2005, 49	"Intensive tutorial teaching frequently is used to help students with learning problems learn a new skill. In addition, one-to-one instruction is appropriate for students who are learning skills that are different from the rest of the class. One-to-one tutoring is a powerful instructional arrangement."
Bruer, 1993, 115	"Anderson and his colleagues knew that children learn better with private tutors. Generally, children who are tutored reach the same level of achievement 4 times faster than children taught in classrooms (Anderson, Boyle, and Yost, 1985). Often, tutoring helps the weakest students most and has little effect on the most able."
Vaughn, Gersten, & Chard, 2000, 6	"Teachers agree that the most effective instruction they can provide for any student is one-on-one; that is, one teacher and one student (Moody, Vaughn, & Schumm, 1997)."

Best Practice in Lesson Design. Many of the research syntheses examined for this study did not necessarily characterize best practices as either direct instruction, mastery learning, or tutoring. Rather, just as the *MLS* design has done, they have identified those elements that are powerful, particularly the ones that are effective with struggling learners and listed them without regard for any specific lesson model. They are presented in Table 63. They do, of course, echo much of the research already reviewed in this section.

Table 63: General Best Practices in Lesson Design

Researcher(s)	Findings/Conclusions
Swanson, Hoskyn, & Lee, 1999	"Effective instructional approaches: combined approach of explicit, systematic instruction and strategic instruction: • Sequencing of instructional skills: breaking down of the task, fading of prompts or cues, sequencing short activities. • Difficulty or processed demands of task controlled: tasks are sequenced from easy to difficult. • Instructional routines (e.g., presentation of subject matter, guided and independent practice). • Modeling: teacher provides a demonstration of processes or steps to solve problem or explains how to do a task, makes use of 'think aloud.' • Drill-repetition and practice review: daily testing of skills, distributed review and practice, redundant materials or text. • Teaching to criterion."

Researcher(s)	Findings/Conclusions
Milller & Mercer, 1997, 10	"Mastropieri, Scruggs, and Shiah (1991) conducted an extensive literature search and located 30 studies that validated instructional techniques for teaching mathematics to students with learning disabilities. Included among those techniques were (a) implementing demonstration, modeling, and feedback procedures; (b) providing reinforcement for fluency building; (c) using a concrete-to-abstract teaching sequence; (d) setting goals; (e) combining demonstration with permanent model; (f) using verbalization while solving problems; (g) teaching strategies for computation and problem solving; and (h) using peers, computers, and videodiscs as alternative delivery systems."
Mercer & Mercer, 2005, 429	"Fuchs and Fuchs (2001) present four principles of prevention of math difficulties: instruct at a quick pace with varied instructional activities and high levels of engagement, set challenging standards for achievement, incorporate self-verbalization methods, and present physical and visual representations of number concepts or problem-solving situations. ". . . Fuchs and Fuchs note that there should be a focus on the individual student as the unit for instructional decision-making. intensive instruction delivery, and explicit conceptualization of skills-based instruction."
Mercer & Mercer, 2005, 133	"Teachers and teacher educators have a responsibility to examine the research and apply the findings as they develop teacher practices. Greenwood, Arreaga-Mayer, and Carta (1994) found that students in classrooms in which teachers used research-based interactive teaching practices had higher academic engagement times and achievement scores than students in classrooms in which teachers used other methods."
Griffin, 2005, 266	" . . . several developmental principles that should be considered in building learning paths and networks of knowledge . . . for the domain of whole numbers have come to light. They can be summarized as follows: • Build upon children's current knowledge. . . . • Follow the natural developmental progression when selecting new knowledge to be taught. By selecting learning objectives that are a natural next step for children . . ., the teacher will be creating a learning path that is developmentally appropriate for children, one that fits the progression of understanding as identified by researchers. • Make sure children consolidate one level of understanding before moving on to the next. For example, give them many opportunities to solve oral problems with real quantities before expecting them to use formal symbols. • Give children many opportunities to use number concepts in a broad range of contexts and to learn the language that is used in these contexts to describe quantity."
Bryant, n.d.b., 7	"What do we know about effective instructional practices? • Modeling • Examples • Opportunities to respond • Correction procedures • Thinking aloud • Flexible grouping • Student progress monitoring • Scaffolded instruction • Strategy + automaticity interventions."

Researcher(s)	Findings/Conclusions
Vaughn, Gersten, & Chard, 2000, 2	"The most interesting facet of the meta-anlaysis (Swanson, Hoskyn, & Lee, 1999) was that of instructional components analysis. The authors searched for factors associated with high effects—regardless of the model of instruction used or the content of instruction. . . . Through multiple regression analyses, . . . they discerned the three most critical factors: • Control of task difficulty (i.e., sequencing examples and problems to maintain high levels of student success). • Teaching students with LD in small interactive groups of six or fewer students. • Directed response questioning. . . . These three instructional components . . . have the potential to work in concert to influence, to the largest degree possible, student learning and students' independent functioning, regardless of instructional domain. . . . these aspects of instruction play a crucial role in virtually all areas of academic learning."
Ontario Ministry of Education, 2005, 77	"Outcomes for children across ability levels and for children with specific difficulties in mathematics are improved when math problem-solving instruction is overt, systematic and clear, and scaffolded by the teacher and peers."
Garnett, 1998, 2	"Several curriculum materials offer specific methods to help teach mastering of basic arithmetic facts . . . . Suggestions from these teaching approaches include: • Interactive and intensive practice with motivational materials such as games. • Distributed practice, meaning much practice in small doses. • Small numbers of facts per group to be mastered at one time. • Emphasis is on 'reverses,' or 'turnarounds' (e.g., 4+5/5+4, 6x7/7x6). • Student self-charting of progress. • Instruction, not just practice."
Sherman, Richardson, & Yard, 2005, 209-210	"Instructional Strategies *Establishing a Context for Interest*. An important instructional technique is to embed problem solving in mathematics lessons by relating the problem to students' interests. Teaching a Variety of Heuristics. The teacher focuses on how to use a particular strategy, such as drawing a picture, using manipulatives, or finding patterns by carrying them out with students during lessons. *Grouping Similar Types of Problems that Call for Similar Types Together.* This idea helps students find patterns in solution attempts. *Starting with Simple Problems*. Solutions are more easily found and confidence is built when students experience success quickly. They are more willing to take risks after knowing they are, in fact, able to find solutions correctly. *Rewarding Students for Small Steps of Success*. Frequent words of praise and positive comments on written work for step-by-step improvement are powerful tools for encouragement. Suggestions and hints are also encouraging because they spur students from first attempts throughout the problem-solving procedure. *Compiling a Mathematics Dictionary Journal with Students*. Important mathematical terms should be found in dictionaries and also discussed in class. The words should be defined and further identified with drawings. For example, have students write a definition for *dividend* and draw an arrow to the dividend in a long division problem. *Provide Sufficient Time for Solving Problems.* *Simplifying Numbers.* *Reduce Reading Difficulties*. Reduce the number of words and/or record the problems on a tape recorder."

Researcher(s)	Findings/Conclusions
Furner & Duffy, 2002, 68	"Based on a culmination of research, Zemelman, Daniels, and Hyde (1998) have put together what is considered the best practice for teaching math: • use of manipulatives (make learning math concrete) • use of cooperative group work • use of discussion • use of questioning and making conjectures • use of justification of thinking • use of writing in math for thinking, expressing feelings, and solving problems • use of problem-solving approaches to instruction • making content integration a part of instruction • use of calculators, computers, and all technology • teachers serving as facilitators of learning • assessments of learning as a part of instruction."

***MLS* Application.** Table 64 includes a list of the "tasks" in *MLS*. Coding was added in the second column to indicate whether the task is instructional, practice (guided or independent), or assessment. The third column indicates the type of lesson model that is used—whether direct instruction, mastery learning, or tutoring--or some combination. These three models have a wealth of scientific research behind them as to their effectiveness, especially with struggling learners. There are overlapping components of these three models, of course. All also typically include an emphasis on explicit, systematic instruction.

Table 64: *MLS'* Incorporation of Lesson Models

MLS Task	Task Phase	Lesson Model
Concept Building Introduction	I	DI, T
Learn	I, SA	DI, T
Solve	P, (I), A M	DI, ML, T
Help	I	DI, ML, T
Solve Intervention	I	DI, ML, T
Let's Review	P	DI, ML, T
Word Problems Learn	I	DI, T
Word Problems Solve	P, (I), A M	DI, ML, T
Word Problems Let's Review	P	DI, ML, T
Math Game	P	T
Printed Activities (7,8,9)	P, A	ML, T
Math Magic	P, A	ML, T
Drawing Conclusions	P, A	ML, T
Flash Cards	P, A	ML, T
Look, Listen, See and Say	I	DI, ML, T
See, Hear and Respond	I, P, A, M	DI, ML, T
Hear and Respond	I, P, A, M	DI, ML, T
See and Respond	A, M	DI, ML, T
Echo	P, A	DI, ML, T
Blank Out	P, A	DI, ML, T
Number Search	P, A	DI, ML, T
Quick Pick	P, A	DI, ML, T
Quick Answer	P, A	DI, ML, T
Digit's Widgets	P, A	T

DI=Direct Instruction; ML=Mastery Learning; T=Tutoring

Table 65 provides additional detail about the structure of *MLS* lessons. To present the lessons, *MLS* uses five steps from the research-based lesson components found in direct instruction, mastery learning, and one-on-one tutoring: Introduction, Guided Practice with Modeling, Independent Practice, Feedback, and Application.

Table 65: Lesson Components in *MLS*

Lesson Component	MLS Application	Details
Introduction	Greeting	In the greeting, Digit welcomes the student and states the title of the lesson phase. In Tactile (concrete) lessons, Digit offers to demonstrate how to use the manipulatives. The student may choose to skip the demonstration.
Guided Practice	Learn Sequence	During the Learn sequences, each new screen shows a step to solving the problem. During the steps, Digit demonstrates how to use the manipulatives, work with the mouse, or abstractly solve problems. Students participate with him as they apply the steps on the working mat, follow the mouse, or write on paper. These activities enable students to move forward when they are confident that they understand the material.
Independent Practice	Solve Sequence	In this sequence, students solve 15 problems independently. Help is available, and when students click the HELP button, text balloons appear, and Digit's voice reads the hints. HELP also provides a brief review of the steps to find the answer.
Feedback	Response	If the student's answer is correct, Digit says one of several positive responses, such as "Perfect!" or "Good Answer!" The Score Bar shows a star, apple, or check mark. If the student's answer is incorrect, Digit says one of several encouraging responses, such as "Close!" or "Almost!" Digit urges the student to try again. In Illustrative (semiconcrete) lessons, Digit mentions if the representational model or the answer is incorrect. At the end of the lesson, Digit shows how many answers were correct on the first attempt. MLS displays fireworks for 100 percent correct on the first attempt.
Reteaching	"Let me help you with this one. . ."	After three incorrect answers on a problem, Digit says, "Let me help you with this one . . . ." He then models solving using the steps he shows in the Learn sequence. After Digit demonstrates the steps, the student has three more chances to answer correctly.
Reinforcement	Let's Review	In lessons 1-9, MLS offers a Let's Review sequence for any problems the student missed during Solve. The computer tallies the problems and presents them to the student again during this sequence.
Application	Word Problems and Games	After students complete the "Learn-Solve-Let's Review" sequences in Problems lessons 7-9, MLS presents a similar sequence for the Word Problems. The word problems exercise the same skill the student just learned in the abstract format.

Lesson Component	MLS Application	Details
		Students also have the opportunity to play a game that exercises the mathematics skills they have learned. The games give students a goal to work toward as they complete the lessons. Games also provide students additional evidence that mathematics is useful and fun outside the boundaries of pencils, paper, and chalkboards.

Concrete—Semiconcrete--Abstract (CSA) Lesson Sequence

Many researchers have concluded that the CSA lesson sequence is an effective way to teach mathematical concepts, operations, and applications to students with learning problems, according to Mercer and Mercer (2005, 433). In the concrete step, students use manipulatives to help them visualize the concepts or operations that are being taught. In the second step, semiconcrete or illustrative, students are exposed to illustrations or representations of the concrete in pictures or on the computer screen. Step three moves to the abstract—the number or the word, along with applications. Research on the efficacy of this sequence of lessons is presented in Table 66.

Table 66: The CSA Lesson Sequence

Researcher(s)	Findings/Conclusions
Mercer & Mercer, 2005, 433	"Many authorities believe that the concrete-semiconcrete-abstract (CSA) sequence is an excellent way to teach students with learning problems to understand math concepts, operations, and applications."
International Dyslexia Association, 1998, 2	"To assist individuals with dyslexia in making this linkage [between concepts and procedures], it is essential that teachers and academic therapists provide instruction that allows the learner to work through the following cognitive developmental stages when teaching mathematical concepts at all grade levels: concrete, pictorial, symbolic, and abstract. Individuals with dyslexia will learn best when provided with concrete manipulatives with which they can work or experiment. These help build memory as well as allowing for revisualization when memory fails. The next stage, pictorial, is one which may be brief, but is essential for beginning the transition away from the concrete. This is where individuals recognize or draw pictures to represent concrete materials without the materials themselves. Symbolics, i.e., numerals, plus signs, etc., are introduced when individuals understand the basic concept, thereby making the connection to procedural knowledge. Finally, the abstract stage is where individuals are able to think about concepts and solve problems without the presence of manipulatives, pictures, and symbols (Steeves & Tomey, 1998a)."
Kibel, 1992, 44	"When we translate the procedure into a visual form that can be manipulated and solved spatially, there is an important change of emphasis. We engage non-verbal routes to understanding and reduce the role of language. For a dyslexic, this change of approach could be a particularly helpful one."
T. Miles, 1992a, 22	"There are two points which recur throughout the book. The first is that dyslexic children need to learn initially by operating with materials; only later should they be introduced to symbols—which should then be presented as a way of recording what has been done. The second is that the different types of language used in mathematics need to be taught specifically and systematically if they are to be understood."

Researcher(s)	Findings/Conclusions
Mercer & Mercer, 2005, 433	"Several research studies . . . reveal that the CSA sequence is an effective way to teach math to students with learning problems. Results indicate that such students do not need large numbers of formal experiences at the concrete and semiconcrete levels to understand basic facts."
Mercer & Mercer, 2005, 433	"The CSA sequence seems to be especially useful in helping students who have deficits in representing or reformulating math from word problems to equations, equations to objects, pictures or drawings to equations, and vice versa."
Miller & Mercer, 1993, 1	"Over the years, educators who have examined the mathematical deficits of students have suggested a number of remediation methods. Many of these methods feature the concrete-representational-abstract (C-R-A) teaching sequence which has been found to facilitate math learning. Implicit in this method of instruction is an emphasis on teaching students to understand the concepts of mathematics prior to memorizing facts, algorithms, and operations."
Miller & Mercer, 1993, 1	"Since success in math requires the ability to solve problems at the abstract level, it is essential that students achieve mastery at this level."
Kroesbergen, 2002, 6	"Although positive results have been found with self-instruction procedures [for teaching math strategies], many theorists think that increased teacher assistance and special forms of instructional scaffolding are needed for students with difficulties learning math (Carnine, 1997). In scaffolded instruction, the teacher gradually withdraws as the student becomes more competent and confident. Such scaffolding can be combined with the Concrete-Semiconcrete-Abstract (CSA) sequence, and many researchers and teachers believe this is an excellent manner of instruction for students with problems understanding math concepts, operations, and applications (Mercer & Mercer, 1998)."
Gardner, 1985, 132	"The sequence of development outlined here—Piaget's account of the passage from sensori-motor actions to concrete to formal operations—is the best worked-out trajectory of growth in all of developmental psychology.

Manipulatives

The CSA lesson sequence depends on the use of manipulatives in the concrete step of the lesson. According to Marzano's (1998) synthesis of research studies, "manipulatives" is one of the most powerful of instructional strategies. He notes that "The overall effect size for instructional techniques in this category was .89, indicating an achievement gain of 31 percentile points" (p. 91). He also found that "The use of computer simulation as the vehicle with which students manipulate artifacts produced the highest effect size of 1.45, indicating a percentile gain of 43 points" (p. 91). This finding aligns with *MLS'* use of the computer simulation in the semiconcrete phase of each concept lesson. Other research on the efficacy of manipulatives in teaching mathematics, particularly to those who struggle to learn, is presented in the following table.

Table 67: Use of Manipulatives in Mathematics

Researcher(s)	Findings/Conclusions
Stern, 2005, 458	"In addition to mental pictures, language plays a crucial role in the formation of concepts. The best way to teach children the meaning of spoken language is to give them the opportunity to see and touch what the words describe and, thus, work out for themselves what the words mean."

Researcher(s)	Findings/Conclusions
Garnett, 1998, 3	". . . it is important to remember that structured concrete materials are beneficial at the concept development stage for math topics at all grade levels. . . . There is research evidence that students who use concrete materials actually develop more precise and more comprehensive mental representations, often show more motivation and on-task behavior, may better understand mathematical ideas, and may better apply these to life situations. . . . Structured, concrete materials have been profitably used to develop concepts and to clarify early number relations, place value, computation, fractions, decimals, measurement, geometry, money, percentage, number bases, story problems, probability and statistics. . . and even algebra. . . ."
Given, 2002, 93-94	"Because more children prefer learning new information by handling it rather than by other sensory input . . ., the time it takes teachers to translate information into hands-on and kinesthetic learning is well worth the effort for all concerned."
Marzano, Pickering, & Pollock, 2001, 82	"Kinesthetic activities are those that involve physical movement. By definition, physical movement associated with specific knowledge generates a mental image of the knowledge in the mind of the learner. . . . Most children find this both a natural and enjoyable way to express their knowledge."
Brigham, Wilson, Jones, & Moisio, 1996, 4	"Among the materials that can be used to make representation of fractions more concrete are real objects such as erasers, paper clips, pencils, and pies. However, instruction should encourage students to move from the concrete level of representation to more abstract representations."
Pennington, 1991, 124	"If the main deficit is in spatial reasoning, the child may very likely need remedial work on place value using concrete manipulables, such Cuisenaire rods. Using graph paper to keep columns aligned in written math may also help. The child should also be taught to estimate an answer in advance and to check his or her answer either with a calculator or by hand."
Stumbo & Lusi, 2005, 5	". . . research has shown that the use of concrete materials, or 'manipulatives,' increase student achievement. Manipulatives are objects such as blocks, tiles, or sticks that are used to help students physically see the workings of a mathematical formula. . . . Using manipulatives helps students understand the 'why' of mathematics, in addition to teaching them how to use symbols and formulas."
Fuson, Kalchman, & Bransford, 2005, 232	"In mathematics, such networks of knowledge often are organized as learning paths from informal concrete methods to abbreviated, more general, and more abstract methods."
Sousa, 2001, 148	"Help students develop conceptual understanding and skills. These students need time to look at concrete models, understand them, and link them to abstract numerical representations. Allow them more time for mathematics study and for completing assignments."
Brigham, Wilson, Jones, & Moisio, 1996, 3	"In general, when students demonstrate difficulty in acquisition of relevant concepts and understanding in mathematics instruction related to fractions, teachers are advised to create representations that are more concrete and meaningful (Mastropieri & Scruggs, 1994)."
National Research Council, 2001, 97	"Special blocks, called base-10 blocks, for example, can be used to develop and support an understanding of the importance of tens, hundreds, and the meaning of the various digits."
National Research Council, 2001, 99	"Fractional values are often represented with pictures, and relationships between quantities are often represented with graphs or tables."
Spear-Swerling, n.d., 1	"Most authorities would agree that manipulatives play a helpful role in teaching math, especially in the teaching of concepts. The National Council of Teachers of Mathematics (NCTM), an organization that has been highly influential in math reform efforts in the United States, strongly advocates the use of manipulatives."
Spear-Swerling, n.d., 1	"In using manipulatives to teach basic operations involving whole numbers, it is important to use objects that are uniform and that accurately represent base-ten relationships (e.g., a 'ten' should be ten times as big as a 'one,' rather than using only color to show tens vs. ones). A mat for organizing manipulatives and for children to work on is also essential. When children begin learning two-digit and three-digit numbers, the mat is organized from right to left in columns of ones, tens, and hundreds, to reflect the way that numerals are written."

Researcher(s)	Findings/Conclusions
Spear-Swerling, n.d., 1	". . . an understanding of basic number concepts, such as being able accurately to count objects, should precede learning written numerals; an understanding of the meaning of multiplication should precede memorizing multiplication tables. Focused assessments should distinguish whether children are struggling with concepts or with other math skills, such as automatic recall of facts. Conceptual understanding can be developed through the use of visual or pictorial representations as well as through concrete manipulatives. Computer-based 'virtual' manipulatives are also increasingly available."
Spear-Swerling, n.d., 2	"Struggling students, including those with learning disabilities, are consistently found to benefit from instruction that is explicit and systematic. Important math concepts and skills should be directly and clearly taught; the sequence of instruction should emphasize learning of prerequisite skills prior to higher-level skills; instruction should take into account research on mathematical development, for example, that problems and numbers involving 0 are typically more confusing to children than those not involving 0 (e.g., writing 308 is more difficult than writing 328); and sufficient opportunities for practice are needed for children to develop automaticity with new skills. If properly used and appropriately integrated with this type of instruction, manipulatives can be very helpful in concept development, as part of a broader math program for youngsters with learning disabilities."
McEwan, 2000, 44-45	"Liping Ma (1999) interviewed Chinese and American teachers about how they would teach subtraction with regrouping. All but one of the 95 teachers interviewed mentioned manipulatives as one way they would teach this topic. Bundles of sticks, beans, and base 10 blocks were commonly mentioned. Of the American teachers, more than 80% focused their use of the manipulatives on the mechanics of computing the correct answer rather than on using them to help students gain the conceptual understanding of place value that is critical to being able to learn how to regroup. Unless teachers understand and are able to make meaningful connections between manipulatives and mathematical ideas, blocks, beans, and bundles are worthless. Only teachers who have a deep understanding of a mathematical topic can make the connection for students. American teachers seemed to believe that the mere presence of manipulatives in the classroom would create understanding."
Battista, 1999, 431	". . . if students are going to progress to a meaningful understanding of the symbolic manipulation of fractions, that understanding must come from students' reflections on their own work with physical fractional quantities. Given appropriate experiences in mentally manipulating these quantities, students can, with proper guidance, derive strictly symbolic methods for dividing fractions."
Mercer & Mercer, 2005, 433	"Lambert (1996) notes that research supports the use of manipulative objects at all grade levels to teach math concepts."
Reys, 2001, 262	"Physical engagement is also an important aspect of the development of many mathematical ideas. Thus, many tasks incorporate manipulatives as tools to help students engage in and explore mathematics. These manipulatives provide concrete representations of ideas or models of various sorts . . . that help students understand the mathematical features of a situation or a problem."
Whitehurst, n.d., 4	"Studies have shown that providing children with practice on visual representations of decimal fractions can help children transfer their knowledge to problems on which they have not been trained."
Miles, T., 1992a, 17	". . . dyslexics should be encouraged to *do* mathematics—that is, operate with concrete objects—rather than try to commit to memory a large number of routines for dealing with symbols. . . . Dienes himself (1964, 139) has stressed that 'doing' needs to come first. What constitutes teaching practice in his view is 'the introduction and manipulation of symbolism before adequate experience has been enjoyed of that which is symbolized. Although he was not talking specifically about the needs of dyslexics, and although starting with 'doing' seems desirable in the case of teaching mathematics to any child, in the case of the dyslexic a failure to do so is likely to be disastrous.

Researcher(s)	Findings/Conclusions
T. Miles, 1992a, 17	". . . dyslexic children are specifically impaired on tasks requiring perception of verbal material, while they evidence no dramatic inability to function in an environment of concrete stimuli."
Sousa, 2001, 145	"Students with special needs who use manipulatives in their mathematics classes outperform similar students who do not. Manipulatives support the tactile and spatial reinforcement of mathematical concepts, maintain focus, and help students develop the cognitive structures necessary for understanding arithmetic relationships. In addition to physical manipulatives (e.g., Cuisenaire rods and tokens), computers and software also help these learners make connections between various types of knowledge. For example, computer software can construct and dynamically connect pictured objects to symbolic representations (such as cubes to a numeral) and thus help learners generalize and draw abstract concepts from the manipulatives."
Sousa, 2001, 161	"Address visual and kinesthetic learning strengths by incorporating visual materials, manipulatives, and opportunities for movement in the classroom throughout each lesson. Demonstrations, models, and simulations are also helpful, although they are more appropriate for learning mathematics skills than mathematics concepts. They also aid in maintaining interest and student motivation."
Chinn, 1992, 35	"There is a range of concrete teaching materials available which may be of help (Dienes blocks, Cuisenaire rods, money, and so on). It is important, however, to bear in mind the warning given by Hart (1989) that the pupil must make the link between the concrete objects and the number symbol. Thus each lesson should use both actual materials and the digits which they represent."
T. Miles, 1992b, 83	"We owe to Dienes in particular (and also to Montessori, Cuisenaire and Stern) the recognition that these mathematical insights are most likely to arise if the pupil is encouraged to use structured materials—rods, blocks, and so on."
Cawelti, 1995, 105	"Long term use of concrete materials is positively related to increases in student mathematics achievement and improved attitudes toward mathematics."
Cawelti, 1995, 105	"In a comprehensive review of activity-based learning in mathematics in grades K-8, Suydam and Higgins in 1977 concluded that using manipulative materials produced greater achievement gains than not using them."
Cawelti, 1995, 105	"In a more recent report of a meta-analysis of 60 studies (grades K through post-secondary) comparing the effects of using concrete materials with the effects of more abstract instruction, Sowell concluded that the long-term use of concrete instructional materials by teachers knowledgeable in their use increased student achievement and improved student attitudes toward mathematics."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 150	"As the name implies, physical representations are models or concrete representations of the knowledge students are learning. Mathematics and science teachers commonly refer to concrete representations as 'manipulatives.' Young students might use math manipulatives to learn the processes of adding and subtracting."
Lovin, Kyger, & Allsopp, 2004, 162	"Students should have access to materials such as counters, base-10 materials, ten frames, and number lines and should be familiar with their usage."
Hall, 2004, 1	"The most successful instructional units—especially in mathematics—are those that begin with concrete, hands-on experiences for students and gradually move toward abstract applications."
Hall, 2004, 1	"The nature of most learning disabilities makes hands-on learning and a carefully prescribed progression from concrete experiences to abstract application crucial to student success. We cannot assume that these students will make conceptual leaps on their own or at the same rate as other students. We should design small increments of instruction and link them sequentially in order to allow these students time to assimilate new ideas. We should build slowly upon the foundations of the students' own episodic memory. When this foundation is missing or unstable we must provide the experiences that will create or reinforce it."

Researcher(s)	Findings/Conclusions
Hall, 2004, 1-2	"Middle school students are particularly vulnerable in our mathematics classes. Many have not yet transitioned from Piaget's concrete operational stage to the formal operational stage of development. Compound this with a learning disability and the increased emphasis on algebraic thinking in the standards for this age group, and we have a formula for academic disaster."
Hall, 2004, 3	"By nature, technology is an abstract tool. However, when used appropriately, it can facilitate student transition from the concrete operational stage to formal operations. Even though Piaget might not recognize our 21st century technology, his theories about the developmental stages of children's learning are still accurate and applicable to today's classrooms."
Mercer & Mercer, 2005, 433	"In addition to the use of the CSA sequence to teach basic math facts, research supports the effective use of manipulative objects and pictorial representations to teach fractions. . ., area and perimeter problems, and algebra. . . . Because the CSA sequence requires students to represent math concepts and operations with objects and drawings, math concepts (such as addition, place value, multiplication, fractions, and equations) are understood."
Mercer & Mercer, 2005, 442	"The use of manipulative objects requires some specific guidelines to ensure effective results: • Before abstract experiences, instruction must proceed from concrete (manipulative) experiences to semiconcrete experiences. • The main objective of manipulation aids is to help students understand and develop mental images of mathematics processes. • The activity must accurately represent the actual process. For example, a direct correlation should exist between the manipulative activities and the paper-and-pencil activities. • More than one manipulative object should be used in teaching a concept. • The aids should be used individually by each student. • The manipulative experience must involve the moving of objects. The learning occurs from the student's physical actions on the objects rather than from the object themselves. • The teacher should continuously ask students questions about their actions as they manipulate objects and should encourage students to verbalize their thinking. • The teacher should have students write the problem being solved through the use of objects and have students use objects to check answers."
Ontario Ministry of Education, 2005, 74	"The use of concrete materials, and pictures and diagrams, can be particularly helpful in teaching word problem solving to children with special needs in mathematics (Jitendra & Hoff, 1996). Concrete materials provide a context for learning mathematical concepts. As students manipulate, talk, and think, they are able to make connections, see relationships, and test, revise, and confirm their reasoning."
McEwan, 2000, 70	"Emphasize both the concrete and the symbolic in early instruction. They are equally important, and having just one, irrespective of which one it is, will put a child at a serious disadvantage. Find ways to help children learn how to skillfully translate in either direction by providing excellent instruction and constant opportunities for practice."
Sherman, Richardson, & Yard, 2005, 17	"Research indicates that students' experience using physical models to represent hundreds, tens, and ones can be effective in dealing with place value issues early in the curriculum. The materials should 'help them [students] think about how to combine quantities and eventually how this process connects with written procedure' (Kilpatrick et al., 2001, 198). However, 'merely having manipulatives available does not insure that students will think about how to group the quantities and express them symbolically' (NCTM, 2000, 80). Rather, students must construct meaning for themselves. . . ."

Researcher(s)	Findings/Conclusions
Sherman, Richardson, & Yard, 2005, 19	"If students' errors are conceptual in nature, remediation begins with using manipulative materials. These might include place value blocks, counters of any type, and place value charts. When errors are more procedural in nature, students forget rules and algorithmic steps but do understand how the system works. Remediation activities do not necessarily have to involve manipulative materials in those cases. Lessons are focused on drawing and/or representing objects and then connecting numerals to those figures or making notations as reminders."
Ma, 1999, 5	"A good vehicle . . . does not guarantee the right destination. The direction that students go with manipulatives depends largely on the steering of their teacher."
Ma, 1999, 6	"Scholars have noted that in order to promote mathematical understanding, it is necessary that teachers help to make connections between manipulatives and mathematical ideas explicit (Ball, 1992; Driscoll, 1981; Hiebert, 1984; Resnick, 1982; Schram, Nemser, & Ball, 1989). In fact, not every teacher is able to make such a connection. It seems that only the teachers who have a clear understanding of the mathematical ideas included in the topic might be able to play this role."
Hartshorn, 1990, 1-2	"Research suggests that manipulatives are particularly useful in helping children move from the concrete to the abstract level. Teachers, however, must choose activities and manipulatives carefully to support the introduction of abstract symbols."
Hartshorn, 1990, 2	"Suydam and Higgins (1977), in a review of activity-based mathematics learning in grades K-8, determined that mathematics achievement increased when manipulatives were used."
National Research Council, 2001, 197-198	". . . research has shown that students can learn well from a variety of different instructional approaches, including those that use physical materials to represent hundreds, tens, and ones, those that emphasize special counting activities (e.g., count by tens beginning with any number), and those that focus on developing mental computational methods."
Raborn, 1995, 5	"For students with kinesthetic strengths, manipulatives with tactile markings help students identify boundaries."
National Research Council, 2001, 198	". . . research on symbolic learning argues that, to be helpful, manipulatives or other physical models used in teaching must be represented by a learner both as the objects that they are and as symbols that stand for something else."
National Research Council, 2001, 198	"In order to support understanding, . . . the physical models need to show tens to be collections of ten ones and to show hundreds to be simultaneously 10 tens and 100 ones. For example, base-10 blocks have that quality, but chips all of the same size but with different colors for hundreds, tens, and ones do not."
Zemelman, S., Daniels, H., & Hyde, A., 1998, 95	". . . research on cognition has made it quite clear that abstract symbols, with all their power and generalization, are best used when the concepts underlying the symbols are truly understood. This understanding requires many varied experiences with particular situations and concrete referents (such as physical models and manipulatives)."
Allen, 2003, 3, 6	"To reach all students, teachers must learn how to use manipulatives and visual aids, which can 'embed the learning' of math concepts in students' minds."
Heaton, 2000, 6	"Learning basic skills and how to reason through activities and problems with concrete materials and language as tools are integrated goals for students in the current reforms."
Sherman, Richardson, & Yard, 2005, 5-6	Structuring Lessons for Success: Step 1. Learners connect new concepts to those with which they are familiar and are actively engaged at a concrete level of understanding. Objects such as counters and base ten blocks are manipulated to solve questions that represent authentic and interesting problems. . . . Step 2. Students represent their understanding with pictures or diagrams. . . . Step 3. Students attach numerals and number sentences to the drawings. . . . Step 4. Students practice skills and algorithmic procedures through a variety of activities and reinforcement lessons. The teacher provides continuous and targeted feedback at each learning step so that errors of misunderstanding or procedure can be corrected quickly and effectively."

Researcher(s)	Findings/Conclusions
Sherman, Richardson, & Yard, 2005, 43	"Understanding of addition begins at the concrete level (working with manipulatives), progresses to working at the semiabstract level (working with pictures), and moves to working abstractly (with written symbols)."
Anderson, Reder, & Simon, 2000, 8	"Most modern information-processing theories are 'learning-by-doing' theories which imply that learning would occur best with a combination of abstract instruction and concrete illustrations of the lessons of this instruction. Numerous experiments show combining abstract instruction with specific concrete examples (e.g., Cheng, Holyoak, Nisbett, & Oliver, 1986; Fong, Krantz, & Nisbett, 1986; Reed & Actor, 1991) is better than either one alone."
Klein, 2005, 18-19	"Manipulatives are physical objects intended to serve as teaching aids. They can be helpful in introducing new concepts for elementary students, but too much use runs the risk that the students will focus on the manipulatives more than the mathematics, and even come to depend on them. Ultimately, the goal of elementary school math is to get students to manipulate numbers, not objects, in order to solve problems."

***MLS* Application.** Concept building, the first stage in *MLS'* dual emphasis, concentrates on developing the concepts and ideas that provide the basis of mathematical understanding. Each phase in the *MLS* concept building stage incorporates four categories of instructional strategies: Tactile (or concrete), Illustrative (or semiconcrete), Problems (or abstract), and Assessment.

In Tactile (concrete) lessons, students use manipulatives to learn how to work with quantities. Having students hold three-dimensional objects in their hands encourages kinesthetic stimulation as well as visual stimulation. The computer models the step-by-step instructions with illustrations for the different types of problems. This category helps students understand how the quantities grow and change.

In Illustrative (semiconcrete) lessons, *MLS* shows students how to solve mathematics problems with graphic illustrations of the manipulatives they have been using. Students use the mouse to arrange objects on the screen and use the pictures to help them find the correct answers to problems. This category helps students begin to imagine how the quantities grow and change.

Problems (abstract) lessons show students how to solve problems at the abstract level by using numbers, mathematical symbols, and algorithms (steps that will produce a correct answer). *MLS* instructions clearly present the link between using manipulatives and completing the abstract steps. Students learn why they must perform each step in solving an equation. This category also helps students learn the symbolic way to represent the growth and change of a quantity.

The last lesson in the phase tests students' retention of the concept for that phase. Students must demonstrate mastery of the current phase's skill before proceeding to the next phase.

Although the lesson stages and models discussed at the beginning of this chapter are usually used to teach procedural knowledge and the CSA sequence is used to teach concepts or declarative knowledge, each can clearly have applications for both kinds of knowledge (see Table 41 in Chapter IV).

Computer-Assisted Instruction (CAI)

Computer-assisted instruction makes it possible for *MLS* to individualize instruction, to provide one-on-one tutoring in a class of many, to utilize multi-sensory processing strategies, to incorporate embedded assessments, to provide varied and adequate practice/repetition, and to manage student records. It would clearly be impossible to create *MLS* without this invaluable delivery system.

The preponderance of evidence in scientifically-based research substantiates the positive role of computer-assisted instruction in teaching the basic skills of mathematics. The studies referenced in Table 65 indicate that CAI is an effective strategy for diverse reasons:

- facilitates more student-centered classrooms
- is more effective than traditional methods
- is more effective than use of printed materials alone
- permits individualization
- serves to mediate students in their zone of proximal development
- assists students with learning disabilities to learn better
- encourages more time on task
- actively engages students
- is motivating
- develops fluency in mathematics
- facilitates multi-sensory processing
- provides opportunities for adequate and varied practice
- results in greater gains in a variety of basic skills
- facilitates learning for limited-English proficient students
- is effective with a variety of at-risk learners.

Interestingly, this synthesis of research findings reflects precisely the advantages that CEI's education consultants report from visits to *MLS* labs. Further, review of *SHARE* (CEI's newsmagazine) articles over even one year reveals an abundance of anecdotal evidence from teachers/facilitators, students, administrators, and parents that corroborates these scientific studies. (Past issues of *SHARE* are available on CEI's webpage and can be searched by keywords relating to diverse population groups and levels of schooling.)

Because of the effectiveness of computer-assisted instruction and its appeal to students, CEI developers expanded its *MLS* program recently to include a web-based activity center (WAC). WAC makes available online a popular mathematics game created by CEI called *Digit's Widget*s. This new feature makes possible even more repetition and practice on fact fluency, expands time on tasks, and is highly motivational. It can be accessed in the lab, at home, or on any Internet-accessible computer.

Mercer and Mercer (2005) are among the researchers who synthesized research findings relating to computer-assisted instruction. They note that "the computer can be used as a tool for classroom management as well as classroom instruction" (p. 67). They continue as follows:

> With computer-managed instruction, teachers can more efficiently develop individualized educational programs and keep records. Computers can store sequences of instructional objectives and student performance information, track student progress, and generate forms and required recordkeeping data (p. 67).

These are the functions, of course, that are performed in the *MLS Student Manager* software that comes with the *MLS* program.

According to the Mercers (2005), the most compelling attributes of computer-assisted instruction (which are also descriptive of *MLS*) are as follows:

- Instruction is individualized by branching students to items appropriate for them.
- Tasks are analyzed and presented in meaningful sequences.
- Progress is at the student's own rate.
- Reinforcement of individual student responses is immediate.
- Fluency programs enable the student to increase the rate of correct responses.
- Animation, sound effects, and game-playing situations make drill and practice multisensory and motivating.
- A computer is nonjudgmental and allows the student to make mistakes in a nonthreatening environment (pp. 67-69).

Pisano (2002) outlines several advantages of computer-assisted instruction with learners who have learning disabilities:

- Students often show increased interest and motivation when they get to sit at a computer station, especially students with attention issues. . . .
- Computers don't usually give negative comments, criticize, or provide straight failure feedback. They might not be as threatening to a student who is self-conscious about failure, in front of his/her peers or even adults.
- Computers can be that great equalizer, when it comes to writing that composition or doing a research project.
- Computers are a tool that can take away some of our children's weaknesses and make them more competitive in the classroom and in mastering the curriculum. . . . (p. 4)

A sampling of other scientific studies on the efficacy of computer-assisted instruction is provided in Table 68.

Table 68: Computer-Assisted Instruction

Researcher(s)	Findings/Conclusions
Silver-Pacuilla & Fleischman, 2006, 84	"Research in psychology has shown the power of simultaneous, multiple modes of input to gain and hold a person's attention and to improve memory."

Researcher(s)	Findings/Conclusions
Silver-Pacuilla & Fleischman, Feb. 2006, 85	"Accessibility features in common technology applications can help struggling students make important connections—to the content, among ideas, among their own sensory modes of learning, and between their digital competencies and the curriculum. These technologies, however, will not automatically create success straight out of the box. Educators need to strategically integrate these features into sound pedagogy to help struggling learners achieve both academic and technological success."
NAEYC, n.d., 2	"When used appropriately, technology can support and extend traditional materials in valuable ways. Research points to the positive effects of technology in children's learning and development, both cognitive and social (Clements, 1994; Haugland & Shade, 1994). In addition to actually developing children's abilities, technology provides an opportunity for assessment. Observing the child at the computer offers teachers a 'window' onto a child's thinking."
NAEYC, n.d., 5	"For children with special needs, technology has many potential benefits. Technology can be a powerful compensatory tool—it can augment sensory input or reduce distractions; it can provide support for cognitive processing or enhance memory and recall; it can serve as a personal 'on-demand' tutor and as an enabling device that supports independent functioning."
Dowker, 2004, 39	"[Computerized learning systems] have the important advantage that computers are motivating to many children; and that, with increasing availability of home computers and computer games, they may be used outside of as well as within a school context."
Pisano, 2002, 1	"The use of computers, related technologies, instructional modifications and learning strategies, in both the school and home settings, are major tools in addressing the educational needs of students with learning or attention difficulties."
Pisano, 2002, 3	"Students with learning differences in relation to all aspects of the curriculum—be they related to the input, processing, or output of information, can benefit from assistive technology."
Pisano, 2002, 3	"Students may present with physical limitations and/or learning issues that require nontraditional approaches to instruction and learning. Students with learning differences represent a wide range of problem areas—learning disabilities (reading, writing, and mathematics), memory issues, processing information and problem solving, attention/concentration deficits, organization issues, language and communication problems, sensory handicaps (vision and hearing), and motor limitations (fine and gross motor), including dysgraphia."
Irish, 2002, 1	"Technology offers one avenue for enhancing instructional options for students with disabilities. Computer-assisted instruction was introduced into the schools in the 1960s, and was developed to help students acquire and practice basic skills. Currently, powerful technologies such as multimedia software, which employs sound and video, are taking the lead in classrooms. According to the National Council of Teachers of Mathematics (NCTM) in *Principles and Standards for School Mathematics* (2000), technology is an essential component of effective math instruction. Not only can technology provide a unique mathematical perspective, it allows students to represent mathematics differently, which may facilitate successful learning. Poplin (1995) suggested that computer-assisted instruction (CAI) might be used effectively as a tool to enable individualization of math content and additional practice that students may need to experience success."
Irish, 2002, 5	"The NCTM Standards also suggest that teachers require adequate resources and a curriculum rich with opportunity to practice. Such resource-rich environments must include the use of technology and provide a variety of meaning-making experiences."
Irish, 2002, 17	"Computer-assisted instruction may be a viable alternative for delivery of strategy instruction related to acquisition, storage, and retrieval of basic mathematical facts. Specifically, the data would support the use of CAI in resource classrooms to enhance student performance in basic multiplication facts."

Researcher(s)	Findings/Conclusions
Dixon, Carnine, Lee, Wallin, & Chard, 1998, 29	"One study (Ball, 1988) found CAI effective for teaching fractions to regular classroom students in grade four, when used in conjunction with fraction strips, and in comparison to conventional instruction."
Bohan, 2002, 15	"Calculators, computers, and Internet capabilities are integral tools for learning and understanding mathematics."
Mauer & Davidson, 1999, 459	"Technology adds the power of efficiency."
Whitehurst, n.d., 1	"It is difficult to provide individualized feedback to students learning math in the typical classroom of 20 to 25 students."
Sousa, 2001, 145	"Students with special needs who use manipulatives in their mathematics classes outperform similar students who do not. Manipulatives support the tactile and spatial reinforcement of mathematical concepts, maintain focus, and help students develop the cognitive structures necessary for understanding arithmetic relationships. In addition to physical manipulatives (e.g., Cuisenaire rods and tokens), computers and software also help these learners make connections between various types of knowledge. For example, computer software can construct and dynamically connect pictured objects to symbolic representations (such as cubes to a numeral) and thus help learners generalize and draw abstract concepts from the manipulatives."
T. Miles, 1992b, 85	"The structured materials need not be in bright colours as if they were toys; they can quite properly be regarded as scientific apparatus and should be presented as such."
T. Miles, 1992b, 86	"If there is any doubt in the pupil's mind about how to carry out the four basic operations, or how to write them down, the structured materials can be used to provide practice."
Tileston, 2000, 69	"Much software is available to the classroom today that incorporates visual, verbal, and kinesthetic learning . . . . Students who need visuals to learn, students who are dyslexic and need graphic representations, will be able to view the learning in a format that is comfortable and meaningful to them."
Tileston, 2000, 68	"Through the use of technology, teachers will be more effectively able to monitor and provide anytime, anywhere assistance to students."
Hay, 1997, 68	"Although there may be many benefits of using technology to adapt materials for different reading levels, one significant benefit is that children may learn at their own level without the stigma of having been placed in a certain group according to reading ability."
Cawelti, 1995, 99	"From a meta-analysis of 79 non-graphing calculator studies, Hembree and Dessart concluded that the use of hand-held calculators improved student learning. In particular, they found improvement in students' understanding of arithmetical concepts and in their problem-solving skills. Their analysis also showed that students using calculators tended to have better attitudes toward mathematics and much better self-concepts in mathematics than did their counterparts who did not use calculators."
Cawelti, 1995, 99	"Data from the National Assessment of Educational Progress demonstrated that students who frequently used calculators showed the highest mathematics achievement."
Snow, Barley, Lauer, Arens, Apthorp, Englert, & Akiba, 2005, 75	" . . . an average student in CAI [computer-assisted instruction] can be expected to score 14 percentile points higher than the average student involved in more traditional instruction as a result of careful intervention."
Snow, Barley, Lauer, Arens, Apthorp, Englert, & Akiba, 2005, 76	"The research supplies strong evidence that computer-assisted instruction is an effective strategy for meeting the needs of at-risk and low-performing students."

Researcher(s)	Findings/Conclusions
Waxman & Huang, 1999, 111	" . . . there are significant differences in classroom instruction, depending on the amount of technology used by the teacher. Instruction in classroom settings where technology was not often used tended to be whole-class approaches where students generally listened or watched the teacher. Instruction in classroom settings where technology was moderately used had much less whole-class instruction and much more independent work. These findings are quite similar to previous research that supports the notion that technology use may change teaching from the traditional teacher-centered model to a more student-centered instructional approach. Another important finding from the study was that students in classrooms where technology was moderately used were also on task significantly more than were students in settings where technology was not widely used."
Padron & Waxman, 1999, 184	"Another instructional practice that can improve the teaching and learning of ELLs in the use of technology in the classroom."
Padron & Waxman, 1999, 184	"Instructional technology has been found to be beneficial for students at risk of failure and ELLs in the following ways: (a) it is motivational, (b) it is nonjudgmental, (c) it can individualize learning and tailor the instructional sequence to meet students' needs and rate of learning, (d) it allows for more autonomy, (e) it gives prompt feedback, (f) it provides the students with a sense of personal responsibility and control, (g) it can be less intimidating to students, (h) it diminishes the authoritarian role of the teacher, and (j) it decreases situations where students could be embarrassed in class for not knowing answers . . . . Furthermore, some types of technology like multimedia are effective for ELLs and students at risk because they help students connect images, sound, and symbols . . . . In addition, multimedia technology can be especially helpful for ELLs because it can facilitate auditory skill development by integrating visual presentations with sound and animation . . . . In addition, there is some indication that Latino students are kinesthetic learners and learn better through hands-on activities and in small group and individualized instruction than through whole-class or direct-instruction approaches."
Padron & Waxman, 1999, 185	" . . . student-teacher interactions were more student-centered and individualized during computer-based teaching and learning than with traditional teaching and learning."
Padron & Waxman, 1999, 185	" . . . high access to computers enabled teachers to individualize instruction more."
Padron & Waxman, 1999, 185	" . . . students who used a computerized integrated learning system (ILS) in both laboratory and classroom settings were more actively engaged in learning tasks than were students in the non-ILS classrooms."
Alliance for Curriculum Reform, 1995, 73	"Learning in which children and young people are interactive produces far more effective growth than instruction in which they are passive."
Alliance for Curriculum Reform, 1999, 68	"Use of various forms of technology can result in improved skills in comprehending and producing a second language."
Sousa, 2001, 210	"Computers and other forms of advanced technology are useful tools for helping students with learning problems."
Dixon-Krauss, 1996, 176	"Social processes, necessary in development, can be either facilitated through or imitated by the computer and associated media devices. In other words, computers can act as the 'more competent peer' in some situations, enhancing the zone of proximal development and artificially providing a sociocultural means of mediation."
National Research Council, 1997, 129	"A discussion of effective instruction would be incomplete without mentioning the use of technology, which can produce dramatic educational benefits for many students with disabilities both as an assistive device and as an instructional tool."
National Research Council, 1997, 131	"Although much has been done in the field of assistive technology, it is in instructional technology that most of the attention has been directed, especially for students with mild disabilities . . . . These applications can help individualize instruction for students with disabilities by adjusting both the presentation mode and the time a student can spend working on any given task."

Researcher(s)	Findings/Conclusions
Gagne', E., 1985, 193	"Microcomputers seem well suited to stimulating the extensive practice needed to automate a skill."
Muter, 1996, 9	"The use of computers entails huge individual differences, but it also permits extensive individualization."
Geraci, 2002, 3-4	"Learning theories that pre-date the notion of an interconnected system of electronic information presented in sensory-rich units all point to the potential for increased learning inherent in interactive environments that stimulate multiple senses, provide visual feedback, and allow for self-paced discovery."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 143	"The more we use nonlinguistic representations while learning, the better we can think about and recall our knowledge."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 147	"Drawing pictures or pictographs (e.g., symbolic pictures) to represent knowledge is a powerful way to generate nonlinguistic representations in the mind."
Rose & Meyer, 2002, 66-67	"The flexibility of new media opens new doors to diverse learners. Digital capacity to combine and transform text, speech, and images leads to a more diversified palette for communication—one that can accommodate the varied strengths and weaknesses of each medium and every brain."
Rose & Meyer, 2002, 67	"Digital media also has the potential to transform the learning process. . . . Students with various kinds of disabilities are likely to be the earliest and most obvious beneficiaries."
Rose & Meyer, 2002, 116	". . . individualizing these techniques so that each learner binds suitable presentations and supports is nearly impossible without digital content and flexible learning tools. With such resources, teachers can provide diverse pathways to recognition learning and meet the diverse needs of their students."
Mercer & Mercer, 2005, 67	"The computer can be used as a tool for classroom management as well as classroom instruction. With computer-managed instruction, teachers can more efficiently develop individualized educational programs and keep records. Computers can store sequences of instructional objectives and student performance information, track student progress, and generate proper forms and required recordkeeping data."
Mercer & Mercer, 2005, 67, 69	"Computer-assisted instruction, or CAI, refers to software that is designed to provide instruction. The computer offers some unique advantages in instructing students with learning problems. Attributes of CAI that appear useful in helping students achieve include the following: • Instruction is individualized by branching students to items appropriate for them. • Tasks are analyzed and presented in meaningful sequences. • Progress is at the student's own rate. • Reinforcement of individual student responses is immediate. • Fluency programs enable the student to increase the rate of correct responses. • Animation, sound effects, and game-playing situations make drill and practice multisensory and motivating. . . . • A computer is nonjudgmental and allows the student to make mistakes in a nonthreatening environment. • Students need to learn how to use technology to succeed in the future."
Mercer & Mercer, 2005, 70	"Although limited information exists concerning how computers can best be applied in the classroom, it is apparent that the use of appropriate software can motivate students with learning problems through individualized instruction and needed academic practice."
Rose & Meyer, 2002, vi	"Universal Design for Learning is a research-based set of principles that together form a practical framework for using technology to maximize learning opportunities for every student. UDL principles draw on brain and media research to help educators reach all students by setting appropriate learning goals, choosing and developing effective methods and materials, and developing accurate and fair ways to assess students' progress."

Researcher(s)	Findings/Conclusions
Mercer & Mercer, 2005, 70	"In general, an increasing number of research studies indicate that computer-based instruction improves student performance in word recognition, spelling, vocabulary knowledge, math computation, and reasoning skills . . . ."
Rose & Meyer, 2002, 8	"The materials and methods teachers can use either present students with barriers to understanding or enhance their opportunities to learn. By developing and applying UDL, we can minimize barriers and realize the promise each student brings to school. The task of educators is to understand how students learn and use the technology available in this digital age to provide selected supports where they are needed and position the challenge appropriately for each learner. In this way, we can engage more students and help every one progress."
Mercer & Mercer, 2005, 461	Dessart, DeRidder, and Ellington (1999) note that research suggests that calculators should be an integral part of mathematics instruction. . . . In general, calculators should be used primarily with problems that students are capable of doing by hand. Also, students should understand the math concept involved in the computation before using a calculator to solve problems."
Mercer & Mercer, 2005, 461	"Hambree (1986) reviewed 79 studies on the use of calculators and reports the following findings: • When used appropriately, calculators promote skill acquisition. • Sustained calculator use in the fourth grade appears to interfere with skill development. • The use of calculators when taking tests results in higher achievement scores. • The use of calculators improves the attitudes of students toward mathematics."
Slavin, 2005, 1	"Research supports integrated learning systems in mathematics."
Ball, Ferrini-Mundy, Kilpatrick, Milgram, Schmid, & Schaar, 2005, 3	"Calculators can have a useful role even in the lower grades, but they must be used carefully, so as not to impede the acquisition of fluency with basic facts and computational procedures. Inappropriate use of calculators may also interfere with students' understanding of the meaning of fractions and their ability to compute with fractions."
Stumbo & Lusi, 2005, 5	". . . the thoughtful use of calculators and other educational technologies also improves mathematics achievement. Contrary to what many believe, researchers have found that using calculators as part of mathematics instruction does not diminish students' computational skills and, indeed, can enhance conceptual understanding, greater ability to choose the correct operations, and greater skill in estimation and mental arithmetic, without a loss of basic computational skills."

Computer Screen Design. When CEI's sales directors are asked about issues or objections raised by potential customers, the topic of the plainness of *MLS'* screens sometimes comes up. Educators, just as many students, have become used to seeing the busyness of the MTV screen, which has influenced even some conservative news networks to include—all at once—a "talking head," a split-screen video, and a running news summary at the bottom of the screen, plus the current weather information. Educators are also used to seeing computer-assisted instruction that has busy screens, many times including music, animation, and bright colors. The *MLS* screen, then, is in some minds "too plain Jane" for those seeking "edutainment" more than effective instruction.

The review of literature, however, on what works in the design of computer screens, especially for students with learning difficulties or disabilities, is loud and clear:

- screens should be uncluttered,
- screens should use simple illustrations that reinforce the instructional goal,
- screens should use color sparingly and consistently, and

- screens should not place too much information on the screen at once.

MLS' screen design consistently reflects this important research. Other examples follow of how research reported in Table 69 is reflected in the *MLS* screen design. *MLS* students view text in one type of font, and the program uses few icons and buttons. Important information is strategically placed with careful attention to providing sufficient black space.

MLS 3.0 contains some changes in response to customer requests. For example, Digit, the mathematics instructor, is a cartoon character that used to have a cartoon voice. He now has a grown-up voice since lab facilitators reported that the cartoon voice became a distraction for some students and was not always seen as appropriate for older students, according to David Merryweather, CEI's vice president for research and development.

Table 69 includes, to a large extent, findings from a meta-analysis conducted by Geraci (2002), but also several other individual studies.

Table 69: Effective CAI Screen Design

Researcher(s)	Findings/Conclusions
Geraci, 2002, 5	"In the context of computer-based education, visual design is said to have five primary functions: (1) focusing attention, (2) developing and maintaining interest, (3) promoting deep processing, (4) promoting engagement, and (5) facilitating navigation through the content."
Mercer & Mercer, 2005, 133	"The first task in opening a lesson is to gain the students' attention."
Pisano, 2002, 4	"Just remember that sometimes too much information coming into the senses from different modalities (for example, visual and auditory) can be counterproductive, as in a program being too stimulating and distracting in order to reach an educational goal."
Geraci, 2002, 5	". . . the visual design of computer-based instruction plays a crucial role in learner comprehension, and retention of online content . . . and is also central to the learner's motivation to engage themselves in the content."
Geraci, 2002, 65-66	"The literature in the field of screen design for instruction generally agrees that when attention is given to the visual presentation of information on the screen, there is an increase in the level to which learners understand and retain the content, and the rate at which they complete instructional units is accelerated."
International Dyslexia Association, 2002, 2	"Block out extraneous stimuli. If a student is easily distracted by visual stimuli on a full worksheet or page, a blank sheet of paper can be used to cover sections of the page not being worked on at the time."
Babbitt, 2004, 1	"Most students with learning disabilities are distracted by too much stimuli coming at them at the same time. Moreover, cluttered screens often distract from the mathematics concept or procedure being studied."
Smey-Richman, 1988, 19-20	". . . any skill is learned best when the learners are not distracted by other inputs competing for attention."
Geraci, 2002, 71	". . . there were two dominant themes in nearly all the selected literature: consistency and simplicity."
Robertson & Hix, 2002, 172	"Lack of screen clutter and a logical, open path of movement proved more important than direction of movement."
Robertson & Hix, 2002, 172	"Minimize use of icons and other screen clutter."

Researcher(s)	Findings/Conclusions
Davies, Stock, & Wehmeyer, 2002, 211	"There are a number of parameters that need to be considered when examining the utility of computer assisted training and support. Okolo, Bahr, and Rieth (1993) reviewed research on computer assisted instruction for students with limited support needs, and identified a list of features for effective software that included: • Clear, uncluttered screens • Consistent commands and features from screen to screen • Appropriate sequencing and pacing • A full range of appropriate examples • Allow students to respond at a high rate • Graphics and animation that contribute to, rather than distract from, learning • Frequent, informative feedback • Adequate number of opportunities for practice • Multiple exposures to a word or a fact."
Levin & Long, 1981, 32	". . . simplicity of pictorial presentation facilitates learning. Pictures need to draw the attention of students precisely to those aspects of learning required by the instructional goal."
Adams, 1990, 367	"In general, information that is illustrated tends to be better remembered, particularly at the level of details. In addition, illustrations appear to be an effective means of inserting information that is consistent with but supplementary to the text."
National Research Council, 1999, 112	". . . comparisons of people's memories for words with their memories for pictures of the same objects show a superiority effect for pictures. The superiority effect of pictures is also true if words and pictures are combined during learning. Obviously this finding has direct relevance for improving the long-term learning of certain kinds of information."
Muter, 1996, 2	"Much of the published research on optimization of reading has been done with paper media. Research on reading from paper media has yielded the following results: • Upper case print, italics, and right justification by inserting blanks result in slower reading. • Black characteristics on a white background produces faster reading than the reverse, and most readers prefer it. • There is no effect of margins, serifs, or typeface in general, within reasonable limits. • Effects of type size, line length, and interline spacing interact."
Muter, 1996, 4	"The tendency to overuse color (the 'fruit salad' approach) can clutter up the screen and create confusion."
Muter, 1996, 5	"Evidence suggests that a large majority of users prefer positive polarity (dark characters on a light background). In theory, positive polarity reduces optical distortion, and increases visual acuity, contrast sensitivity, speed of accommodation, and depth of field."
Geraci, 2002, 43	"Among all this conjecture into the use of color, a few pertinent points did surface in the literature with near unanimity. Chief among these is that designers should use color judiciously. Many references contend that there is diminishing return as the number of colors used in a single screen increases. The notion that color should be used in a consistent fashion also appeared throughout the literature."
Geraci, 2002, 44-45	"A good way to avoid color distraction is to use colors found in nature, particularly toward the lighter side, such as grays, blues, and yellows of sky and shadow. Nature's colors are familiar and have a widely accepted harmony."
Geraci, 2002, 49	"Color affects the coding of information in human memory. Even if the colors chosen do not contribute to the message content, color can nevertheless still facilitate the retrieval of essential learning cues. Recommendations on the appropriate number of colors to use on a single screen range from 2 to less than 10."

Researcher(s)	Findings/Conclusions
Geraci, 2002, 49	"Too much color can be distracting and has been shown to degrade performance on memory and recognition tasks."
Geraci, 2002, 56	". . . screens should be designed with attention to balance, harmony, and unity."
Geraci, 2002, 67	"The literature makes unanimous calls for a consistent use of color in computer-based instruction. Remaining true to one's use of color provides a reliable context or information that eases the learning process and lets the user focus on the information and not the construct of the interface."
Geraci, 2002, 56	". . . consistency in layout is also widely believed to have great importance in the design of screens."
Geraci, 2002, 63	"One of the most fundamental dictates of good screen design is consistency in the placement of various items, use of color, access structure, style of graphics, screen density and white space."
Geraci, 2002, 63	"Strive for consistency in menus, help screens, color, layout, capitalization, fonts, and sequence of actions."
Geraci, 2002, 69	"Spatial layout has the important role of creating a visual gestalt, or underlying pattern to the information that allows the learner to build a mental scheme for grouping and processing the lesson's content."
Geraci, 2002, 70	"Here, too, the literature was nearly unanimous: paging is preferred to scrolling."
Geraci, 2002, 70	"Most of the research into screen density is founded upon the notion that users can become overwhelmed with long, continuous presentation of information. Research on memory load generally holds that students need to receive information in smaller, more digestible chunks, which promote the formation of concept building and associations in the learner's minds."
Geraci, 2002, 53	"Select a typeface with a simple, clean style and use a few typefaces in any one screen or program."
Karp & Howell, 2004, 121	"To many children with learning disabilities, school is a place of competing stimuli. . . . In order to learn, such students need structure, eliminating the disorder."
Karp & Howell, 2004, 121	"For many students with learning disabilities, the structure of the environment determines success or failure. These students are often easily distracted by the variety of sights and sounds in the room, so the teacher should choose the area of the classroom that presents the fewest distractions and keep visual displays purposeful rather than distractingly entertaining."
Barton & Heidema, 2002, 37	". . .when less-able readers confront text that is very dense or that is written above their reading level, they have to read more slowly to make sense of the information. Short-term memory can become overburdened in the process, especially if readers must sound out unfamiliar words or attempt to construct meaning about abstract concepts. By the time readers reach the end of the sentence they are reading, they may have forgotten what they read at the beginning of the sentence."

Evaluating *MLS* as a Software Tool for Mathematics Instruction

Dr. Beatrice Babbitt (April 2004), Associate Professor of Special Education at the University of Nevada, Las Vegas, used research findings for her article entitled, "10 Tips for Software Selection for Math Instruction." CEI's *MLS* program complies with all of the qualities that Dr. Babbitt mentions in the article. Following is a list of those qualities along with descriptions of how *MLS* correlates to each one.

1. The Less Clutter On The Screen, The Better

Simple screen displays are the hallmark of *MLS*. The working mats and manipulatives provided are exactly the same as those on the screen. Also, teachers may turn off *MLS'* animated teaching assistant, Digit, if students find him too distracting. With Digit turned off, the students hear only a computer voice giving them instructions.

2. Procedures Should Match Those Being Taught In School

Computational procedures for instruction follow those recommended by the National Council of Teachers of Mathematics (NCTM). *MLS* is designed to work *with* classroom texts, not as a replacement for those texts. *MLS* teaches standard algorithms and uses research-based instructional strategies to develop procedural competence.

3. Choose Modifiable Software

Evaluation and Placement procedures, some of which are computerized, facilitate appropriate placement in the program. Once a student begins the program and teachers determine what changes are necessary to accommodate the student, *MLS* allows them to modify several aspects: instructional level, speech and response speed, number of problems, inclusion of graphic displays, computer-voiced instructions, and repetition of instructions. If the student wants to hear the instructions again, he or she can click on the text and the program will repeat them. The program also presents instruction in small "chunks," allowing students to hear the instructions and read them as well.

MLS provides a dual-support system for students who need to accelerate mathematics development. The program simultaneously teaches mathematical concepts and addresses sensory processing difficulties. The **Concept Building** stage focuses on the instruction and practice of mathematical concepts, and the **Fluency** stage encompasses a series of neurological exercises that combine the use of visual, auditory and kinesthetic activities. These tasks employ many different stimulus-recall-response modes that are designed to help students achieve automaticity in arithmetic operations. The **Fluency** stimuli are basic math facts, from zero to 12, in addition, subtraction, multiplication and division. To individualize the lessons for each student, teachers can modify the number of sets, the number of equations, the level of equations the student knows, the sensory mode (visual or auditory), the response mode (type or highlight), and the speech speed.

4. Choose Software With Small Increments Between Levels

The *MLS* **Concept Building** stage has a 10-lesson delivery system for each mathematical concept. A single concept, such as single digit addition, offers three kinesthetic lessons in *MLS'* **Tactile** (concrete) category, then three representational lessons in the **Illustrative** (semiconcrete) category, and then three abstract lessons in the **Problems** category. Each 30-minute lesson includes 1 to 5 **Learn** problems that provide Guided Practice, followed by 15 **Solve** problems that offer students Independent Practice. Once the student completes those nine lessons, he or she moves to the final category of each concept, **Assessment**, which offers 15 problems with no Guided Practice (**Learn** problems). If the student achieves 80 percent on the **Assessment** lesson, the computer moves him or her forward to the next concept. If the student does not achieve 80 percent mastery, the program sends him or her to the beginning of the cycle to relearn the concepts that were presented. The teacher can manually override the lesson setting when necessary.

5. Choose Software With Helpful Feedback

When a student answers a problem correctly, he or she will hear one of several computer-voiced responses, such as "Perfect" or "Good Answer," in addition to the corresponding visual cues. Digit, the tutor, may also say, "Your blocks are correct, but your answer is incorrect. Do you need some more practice?"

6. Choose Software That Limits The Number Of Wrong Answers For A Single Problem

When the student makes a mistake, Digit responds with encouragement, saying "Close" or "Almost," and asks the student to try again. Digit even explains why the student's response was incorrect by saying something like, "Your blocks are correct. Your answer box is incorrect. Try counting the tens and ones again." After three incorrect answers, Digit not only encourages the student but also intervenes with instruction by saying, "Almost. Let me help you with this one." He then proceeds to demonstrate how to solve the missed problem.

At the end of a lesson, Digit shows how many answers the student answered correctly on the first attempt. If the student answered 100 percent of the problems correctly on the first attempt, the program displays fireworks. If the student scores less than 100 percent, Digit says something like, "Great job! You got 14 out of 15 correct." The "Let's review" screen appears and provides the student another opportunity to work each of the problems he or she missed.

7. Choose Software With Good Record Keeping Capabilities

The *MLS Student Manager* provides several printed forms and reports to facilitate record keeping. The system allows teachers to print lesson score reports to track each student's progress. The lesson score report includes the student's name, completion date, number of sessions and placement stage. The report provides additional information, such as how many times the student observed Digit modeling the problems, the total Learn time, a list of equations the student worked on, the number of attempts on each problem, and a list of problems that the student did not solve correctly. The report also includes the minutes and seconds the student spent to solve each problem, the percentage of problems correct on the first attempt, and a list of the problems the student reviewed. Class Rosters are also available from the *MLS Student Manager*.

8. Choose Software With Built-In Instructional Aids

Instructional aids, such as the manipulatives and working mats that accompany *MLS*, are critical to its success. The manipulatives for each station include Unifix cubes, base-ten blocks, fraction strips, a money tray and money, an analog clock and number tiles. Students also have paper and pencil tasks to enhance their learning.

In addition, each *MLS* lab receives a variety of materials to strengthen and extend the students' learning experiences:

- *Math Magic*—An individual or group activity that uses higher-order thinking skills to complete six intertwined equations. Number banks help the student to solve the puzzles. The math squares require the students to use estimation, logic, and sequencing skills.

- *Drawing Conclusions*—A printed activity that encourages visualization and higher-order thinking skills to solve word problems. These activities complete the word problems that the *MLS* program uses. Students can complete the activities individually, or teachers can encourage collaborative problem-solving by placing the students in groups.
- *Fact Match*—These activities provide students with practice of basic mathematical facts and operations. Practice leads to fluency, which enables students to learn more complex mathematical processes.
- *Flash Cards*—These cards help students become more fluent on addition, subtraction, multiplication, and division math facts. Using the cards can also help teachers determine where students should begin in the Fluency Stage of *MLS*.
- *Digit's Widgets*—Schools with active service/support contracts can allow their students access to *Digit's Widgets*, an online game that reinforces the development of fluency in mathematics facts.

9. Choose Software That Simulates Real-Life Solutions

MLS provides in-depth coverage of the math skills that are necessary for daily living: money, time, addition, subtraction, multiplication, and division. The program also presents Word Problems that reflect real-life situations. In the Word Problems **Learn** phase, Digit reads the story, highlights the question and the pertinent information, gives procedural hints, shows how to find the equation and points out extraneous information. Then, in the Word Problems **Solve** phase, Digit reads the story aloud, and the student finds the equation and enters the solution.

10. Remember, Software Is A Learning Tool — Not The Total Solution!

CEI advocates combining *MLS* with direct teacher instruction to provide a well-rounded mathematics program.

Summary

Chapter V began with the research on the steps in an effective lesson and then moved to a discussion of the research on direct instruction, mastery learning, and one-on-one tutoring, followed by documentation of how these models are used in the various *MLS* tasks. The next section explored the research on the concrete—semi-concrete—abstract lesson sequence, identified by research as effective instruction for struggling learners, followed by the documentation of how *MLS'* lessons follow this sequence. The use of manipulatives in teaching mathematical concepts was also discussed, with research documentation and descriptions of how *MLS* incorporates their use. The final section of Chapter V presented the research on the efficacy of computer-assisted instruction in teaching mathematics, followed by the research on effective screen design for struggling learners. This section concluded with a correlation of a research-based tool for choosing effective mathematics software for struggling students with the features of *MLS*.

Chapter VI includes the research on *MLS'* use of a variety of scientifically-based instructional strategies, including multi-sensory processing, individualization/differentiation, practice/repetition, chunking/clustering, active engagement/time-on-task, and comprehensive assessment with feedback.

Chapter VI: Research Findings that Ground *MLS'* Instructional Strategies

"Learning is a process of building neural networks."
(Wolfe, 2001, 125)

Overview

Chapter II and III included the research on learning difficulties and learning disabilities—all those reasons why learners struggle to master mathematics, even at the foundation levels. Chapter IV was the chapter that focused on *MLS* content, especially the two major strands of concept development and fact fluency. A plethora of evidence was documented that these two areas are chief among those with which students struggle, whether due to difficulties or disabilities, in becoming proficient in mathematics. In addition to a discussion of *MLS'* scope and sequence, the chapter included documentation of how *MLS* addresses each of the identified problem areas of content.

In Chapter V the research on lesson design was presented, along with documentation of *MLS'* use of components of direct instruction, mastery learning, and one-on-one tutoring—all found to be highly effective with struggling learners. Also included were discussions of the concrete-semiconcrete-abstract lesson sequence and the use of manipulatives—both integral to the *MLS* lesson design. The chapter also included the research on the efficacy of computer-assisted instruction (CAI) and the importance of screen design in making CAI instruction effective for struggling learners.

Chapter VI continues to document the scientific-based research that grounds the design decisions for *MLS*. The discussion now turns to the research-based instructional strategies used in *MLS*. In a well-taught lesson, of course, what an observer sees appears to be seamless. The content of the mathematics lesson, which most likely weaves back and forth between concepts and procedures, is interwoven with the lesson design, and the steps of the lesson include a variety and sometimes a combination of instructional strategies all at once. Further, instruction and assessment are at many times interwoven and seamless. The power or effectiveness of the presentation in impacting student learning is a result of the synergy—everything happening at once and each component a necessary but insufficient piece in itself of the total.

Major instructional strategies that are prevalent in *MLS* lessons include multi-sensory processing, individualization/differentiation, practice/repetition, time-on-task and active engagement, and clustering/chunking. These strategies were selected because the research indicates that they are the methods by which students can overcome their difficulties and/or disabilities and become proficient in mathematics content and procedures. Since the *MLS* assessment system is intended to drive instruction, that research and documentation are also included in this chapter.

Table 70 includes in the first column a list of the *MLS* tasks and activities. The embedded instructional strategies employed for each one is coded in the second column. The tasks coded "A" indicate that the activity incorporates assessment activities; the "Learn" task utilizes self-assessment.

Table 70: *MLS* Tasks and Instructional Strategies

MLS Task	Instructional Strategy
Concept Building Introduction	MSP, ID
Learn	MSP, ID, PR, TOT, SA
Solve	MSP, ID, PR, TOT, A
Help	MSP, ID, PR, TOT
Solve Intervention	MSP, ID, TOT
Let's Review	MSP, ID, PR, TOT
Word Problems Learn	MSP, ID, PR, TOT
Word Problems Solve	MSP, ID, PR, TOT, A
Word Problems Let's Review	MSP, ID, PR, TOT
Math Game	MSP, ID, PR, TOT
Printed Activities (7,8,9)	ID, PR, TOT, A
Math Magic	ID, PR, TOT, A
Drawing Conclusions	ID, PR, TOT, A
Fact Match	ID, PR, TOT, A
Flash Cards	ID, PR, TOT, A
Look, Listen, See and Say	MSP, ID, C, PR, TOT
See, Hear and Respond	MSP, ID, C, PR, TOT, A
Hear and Respond	MSP, ID, C, PR, TOT, A
See and Respond	MSP, ID, C, PR, TOT, A
Echo	MSP, ID, C, PR, TOT, A
Blank Out	MSP, ID, C, PR, TOT, A
Number Search	MSP, ID, C, PR, TOT, A
Quick Pick	MSP, ID, C, PR, TOT, A
Quick Answer	MSP, ID, C, PR, TOT, A

MSP=multi-sensory processing; ID=individualization/differentiation; PR=practice/repetition; TOT=time-on task and active engagement; C=chunking/clustering; A assessment; SA=self-assessment

Multi-Sensory Processing

The most important, most effective—and most unique—feature of *MLS* is its informed use of multi-sensory processing in instructional activities. It is through this strategy that *MLS* gets at the root cause of most learning difficulties and disabilities—faulty sensory processing. Students with learning difficulties may have this problem as a result of weak neural pathways in the brain due to any of the issues discussed in Chapter II, most especially inadequate or inappropriate instruction. Those with learning disabilities may suffer from a lesion of some kind or some other malfunction that requires a therapeutic intervention, which *MLS* provides.

Before the term is defined, it may be important to explain what multi-sensory processing is *not*. Multi-sensory processing is not just another term for the concept of learning styles. Stanovich and Stanovich (May 2003) pointed out in a recent publication that the concept of learning styles "has never been demonstrated to work in practice" (p. 30). One of the harmful practices that has evolved from that popular concept has been the matching of auditory learners with phonics instruction and visual/kinesthetic learners with holistic instruction, they explained. They continued: "Excluding students identified as 'visual/kinesthetic' learners from effective phonics instruction is a bad instructional practice—bad because it is not only research based, it is actually contradicted by research" (p. 30). A similar statement could be made about mathematics. Excluding students identified as "auditory" learners from effective use of manipulatives in learning mathematical concepts is an equally bad instructional practice. There is ample evidence documented in Chapter V that manipulatives are critical in the developmental of conceptual (or declarative) knowledge, especially for learners who struggle due to difficulties or disabilities.

A potential outcome of the slavish practice over time of matching students solely with their preferences would be the handicapping, rather than empowerment, of a learner since over-accommodating the learner's learning preference would never strengthen weak neural pathways nor allow alternative new pathways to be built—both essential to the effective teaching of students with learning difficulties and/or disabilities. Stanovich and Stanovich referenced research that "found no consistent evidence for the idea that modality strengths and weaknesses could be identified in a reliable and valid way that warranted differential instructional prescriptions" (p. 30). Another study found likewise—that "the idea of modality preferences did not hold up to empirical scrutiny" (p. 30). These researchers are not stating that there is no such thing as learning preferences. What they are saying is that those preferences must not dictate teaching methods for mathematics instruction. CEI's programs use visual AND auditory AND kinesthetic AND tactile methods to teach both reading and mathematics.

Multi-sensory processing, as opposed to learning styles, is a term that comes out of the research of cognitive scientists, neurobiologists, linguists, and other experts who study how people learn, remember, retrieve, and apply knowledge and skills. It is, according to Mercer and Mercer (2005), "based on the premise that some students learn best when content is presented in *several* [emphasis added] modalities. Frequently, kinesthetic (movement) and tactile (touch) stimulation is *[sic]* used along with visual and auditory modalities" (p. 306). In multi-sensory processing all the relevant senses are employed for each student so that neural pathways that enable people to learn and remember—and learn mathematics—are accessed and strengthened—or built, regardless of the individual's weaknesses or strengths in learning. Multi-sensory processing uses multiple levels of processing so that learning is retained and so that it can be retrieved, regardless of the sensory modality in which it was originally encoded. Sternberg (2003) says it this way: ". . . when information is encoded in various contexts, the information also seems to be retrieved more readily in various contexts, at least when there is minimal delay between the conditioning contexts and the novel context" (p. 205). Kandel (2006) agrees. He says that long term memory is "stored in the same area of the cerebral cortex that originally processed the information" (p. 129).

The power of multi-sensory processing strategies, in conjunction with practice/repetition exercises, is that they strengthen weak neural pathways and build new ones to compensate when a neural pathway is absent or damaged. The combination of *MLS'* strategies contribute to its therapeutic nature. Kandel (2006) explains: ". . . brain circuity has a built-in redundancy. Many sensory, motor, and cognitive functions are served by more than one neural pathway—the same information processed simultaneously and in parallel in different regions of the brain. When one region or pathway is damaged, others may be able to compensate, at least partially, for the loss" (p. 129). Research on the efficacy of multi-sensory processing strategies in interventions has been building for almost a century. A 1921 article by Fernald advocated "using a technique that integrated several sensory modalities including visual, auditory, kinesthetic, and tactile" (Hallahan & Mock, 2003, p. 13).

The most salient of the scientific research findings on multi-sensory efficacy, which is overtly related to the achievement of fact fluency and to the development of memory (long-term recall of concepts and procedures embedded through effective instruction), follows in Table 71. The research includes several references to UDL or Universal Design for Learning, which Rose and Meyer (2002) define as "the intersection where all our initiatives—integrated units, multi-sensory teaching, multiple intelligences, differentiated instruction, use of computers in schools, performance-based assessment, and others—come together" (p. 7). *MLS* embraces UDL in the design of its lessons that integrate instruction for both concepts and procedures; in its emphasis on multi-sensory processing strategies and multiple intelligences; in its incorporation of individualized and differentiated instruction; in its utilization of computer-assisted instruction; and in its comprehensive assessment system, including performance-based assessment.

Table 71: Research Findings on Multi-Sensory Processing

Researcher(s)	Findings/Conclusions
Rose & Meyer, 2002, 114	"When we learn, we incorporate new knowledge into old knowledge. In neural network terms, new learning is integrated into networks that have been shaped by previous learning. Consequently, what the brain already knows can influence what it will learn from a new example or experience."
Wolfe, 2001, 78	"Everything in memory begins as a sensory input from the environment."
Wolfe & Brandt, 1998, 10	"The brain changes physiologically as a result of experience. The environment in which a brain operates determines to a large degree the functioning ability of the brain."
Wolfe, 1998, 61	"The only way to get information into the brain is through our senses."
Kandel, 2006, 257	"What is learning but a set of sensory signals from the environment, with different types of learning resulting from different types of sensory signals."
Kandel, 2006, 59	"The neuron doctrine (the cell theory as it applies to the brain) states that the nerve cell, or neuron, is the fundamental building block and elementary signaling of the brain."
Kandel, 2006, 59	"The nerve cell is not simply a marvelous piece of biology. It is the key to understanding how the brain works."
Karp & Howell, Oct. 2004, 120	"All students have a unique profile of relative strengths and weaknesses, including how they process different types of information."
Given, 2002, 81	"Listening, speaking, reading, writing, and other academic skill development depend on the cognitive system. The cognitive system depends on sensory input and the adequate functioning of the attention, information processing, and memory subsystems for the construction of knowledge and skills."
Wolfe, 2001, 128	"The more fully we process information over time, the more connections we make, the more consolidation takes place, and the better the memory will be."
Erlauer, 2003, 11	"These billions of neurons alone do not make a brain intelligent. It is when the neuron's dendrites (long tentacles that look like tree branches) reach out and connect to another neuron's dendrites that learning occurs. These connections, or synapses, are the pathways for new learning."
Erlauer, 2003, 55	"A stand-alone (brain cell) holding a tidbit of information does the brain little good. It is when that neuron connects to another neuron, and that one to another neuron, and so on, that the connections and learning take place."
Sousa, 2001, 11	"Learning occurs when the synapses make physical and chemical changes so that the influence of one neuron on another also changes. For instance, a set of neurons 'learns' to fire together. Repeated firings make successive firings easier, and, eventually, automatic under certain conditions. Thus, a memory is formed."

Researcher(s)	Findings/Conclusions
Sousa, 2001, 12	"For all practical purposes, the capacity of the brain to store information is unlimited. That is, with about 100 billion neurons, each with thousands of dendrites, the number of potential neural pathways is incomprehensible."
Dehaene, Piazza, Pinel, & Cohen, 2005, 433	". . . number processing rests on a distinct neural circuity, which can be reproducibly identified in different subjects with various neuroimaging, neuropsychological, and brain-stimulation methods."
Bruer, 1993, 102	"How we store knowledge depends on how we learn it."
Berliner & Casanova, 1993, 79	"The research implies that the more ways you can enhance imagery when teaching verbal material, the more likely it is that your students will remember what you taught. . . . if we want to help children remember the things that we deem important, we should help them whenever we can to construct visual representations and give them some multisensory experiences during learning."
Tileston, 2000, 21-22	". . . we cannot assume that students come to us with the structures already in place to learn new material. We must first establish what they know and understand and where there are no previous connections, supply them for the student."
Kujala, Karma, Ceponiene, Belitz, Turkkila, Tervaniemi, & Naatanen, 2001, 7	"It might be that learning to structure sensory input also affects the processing of faster stimulus elements than those originally used in the training."
Posner, 2004, 4	"It is also important to note that these networks have not proved to be as separate as though they were in different brains. Indeed each node in these networks communicates with other nodes of the network and with other networks. . . . Exact calculation of numerical quantity can bring in language networks. If a visual digit is spelled out making a word, it will activate left occipital areas that are also activated by nonnumerical words. These are all important examples of how real-world actions may draw on multiple neural systems and are thus related to multiple forms of intelligence."
Sprenger, 1999, 85	"Automatic memory retrieval is similar to procedural memory retrieval. I think of the information stored in the cerebellum as long strings of neurons hooked together by strong and healthy dendrites and axons. They appear like dominos. All I have to do is trigger the first neuron, and they fire in a systematic way, just as the fall of the first domino triggers the others to fall in turn."
Miller & Mercer, 1997, 5	"The information-processing model provides numerous perspectives for examining the math difficulties of students with learning disabilities. Information-processing theory focuses on which information is acquired and how. Its primary features include attention, sensation, perception, short-term memory, long-term memory, and response."
Battista, 1999, 429	". . . all current major scientific theories describing students' mathematics learning agree that mathematical ideas must be personally constructed by students as they try to make sense of situations (including, of course, communications from others and from textbooks). Support for the basic tenets of this 'constructivist' view comes from the noted psychologist Jean Piaget and, more recently, from scientists attempting to connect brain function to psychology. For instance, Nobel laureate Francis Crick has stated, 'Seeing is a constructive process, meaning that the brain does not passively record the incoming visual information. It actively seeks to interpret it.' Similarly, psychologist Robert Ornstein asserts, 'Our experiences, percepts, memories are not of the world directly but are our own creation, a dream of the world, one that evolved to produce just enough information for us to adapt to local circumstances."
Battista, 1999, 431	"Research clearly shows that such 'construction-focused' mathematics instruction produces more powerful mathematical thinkers."
Wakefield, 1999, 235	"Piaget said that children cannot see, hear, or remember that which they cannot understand. If the mental structures are not in place to support what is seen or heard, there will be no mental connection, and consequently it will not be remembered."
Whitehurst, n.d., 3	". . . there is research that suggests where some of practices and assumptions of both the constructivists and their critics may require more nuanced implementation."

Researcher(s)	Findings/Conclusions
Reys, 2001, 261	"Standards-based materials help students make sense of mathematics in several ways. Sense-making is promoted by spending substantial time on the fundamental ideas of a mathematical domain, such as rational numbers."
Campbell & Epp, 2005, 357	"Kashiwagi, Kashiwagi, and Hasegawa (1987) studied Japanese aphasics with impaired performance for simple multiplication. Despite extensive practice, the patients could not relearn multiplication with verbal presentation and responses. They did, however, learn to generate the multiplication facts given visual presentation combined with written responses. Such findings support the theory that the representations underlying multiplication facts can involve multiple codes that are differentially involved as a function of surface form."
Campbell & Epp, 2005, 357	"Our review identified a variety of types of evidence for the conclusion that retrieval processes for simple arithmetic depend to some extent on surface format."
Lochy, Domahs, & Delazer, 2005, 473	"Calculation training took place twice weekly over 8 weeks. Problems were presented visually and at the same time read aloud by the therapist. The patients were allowed to answer in their preferred modality. Instant feedback was provided and errors were discussed with the patients if necessary. Training led to long-lasting improvements, evidenced by accuracy rates of more than 90%."
Rose & Meyer, 2002, 17	"Because smoothly functioning recognition networks take advantage of both top-down and bottom-up processes, teaching to both processes rather than focusing exclusively on one or the other is the wisest choice. A positive example is the recent truce in the 'phonics wars.' Most programs have not adopted a form of reading instruction that incorporates both the top-down whole language method and bottom-up phonics. This balanced approach is consistent with the way the learning brain works."
Sousa, 2001, 149	"Use as many multisensory approaches as possible."
Chinn, 1992, 32	"It is an obvious requirement that the teaching of mathematics to dyslexics should be multisensory."
Kibel, 1992, 45	"Alex talked as he handled as he looked. It was multisensory learning."
Chinn & Ashcroft, 1992, 101	". . . the pupil needs to develop an understanding of the place values of units, tens and hundreds. (The methods to be used to teach this are not described here, but should be multisensory and involve as many manipulative materials as is possible.)"
Erlauer, 2003, 156	"Brain-compatible instructional strategies work because they are based on research, match common sense, and involve teaching the way students learn."
Rose & Meyer, 2002, 19	". . . the overt and subtle differences in how students best recognize patterns suggest that more varied means of presentation can reach more students."
Molholm, Ritter, Murray, Javitt, Schroeder, & Foxe, 2002, 115	"Integration of information from multiple senses is fundamental to perception and cognition."
Stern, 2005, 457	"A structured, multisensory approach is of special importance to children with learning disabilities. These children have difficulty with language concepts and associations and memory. They are usually struggling with a combination of these deficits and may also have difficulties with attention. To understand concepts, students with learning disabilities must learn to receive and integrate information from as many different senses as possible."
Stern, 2005, 458	"What is involved in the formation of concepts? Children seem to reason with mental pictures. Multisensory materials will have fulfilled their purpose when the children can visualize the concepts presented."
Mauer, 1999, 385	"A premise is that children whose sensory input is not organized or integrated in the brain have sensory integrative dysfunction. Such a disorder leads to disorganized, maladaptive interactions with the environment from which faulty internal sensory feedback is produced, further perpetuating difficulties and causing problems in learning, development, and behavior. Learning involves the organization and adaptation of that information to any situation. These abilities are lacking in children with sensory integrative dysfunction."

Researcher(s)	Findings/Conclusions
Snowling, 1987, 147	". . . it is good practice to encourage dyslexics to use all their senses during learning—to rely upon their strengths to compensate for and circumvent their weaknesses."
Mauer, 1999, 386	"One common symptom of children with sensory integrative dysfunction is the inability to maintain an appropriate state of alertness through ordinary activities, as well as to focus and attend to a task. This is especially evident with language comprehension tasks consisting of intense amounts of auditory information that the nervous system must process."
Mauer, 1999, 387	"Ayres defined the goal of SI [sensory integration] therapy as improving the way the brain processes and organizes sensations."
Caine & Caine, 1991, 86	"Success depends on using all of the senses and immersing the learner in a multitude of complex and interactive experiences."
Mauer, 1999, 383	"Sensory integration (SI) theory and intervention have been used for the treatment of children with a wide range of learning and developmental challenges. SI refers to the ability to organize, integrate, and use sensory information from the body and the environment . . . . SI theory is based on the belief that the integration of the sensory system is the foundation for successful development of motor abilities, organization, attention, language, and interpersonal relationships."
Tileston, 2000, 19-20	"The classroom that is enriched with teaching techniques from all three modalities, and in which new information is given in 15- to 20-minute segments for secondary and 7- to 10-minute segments for elementary students with time for processing in between, will be a place where quality learning is possible."
Bruer, 1993, 265	"We should present school subjects in a variety of ways, using multiple representations that resonate with the students' multiple intelligences. We should assess intelligence and learning in a variety of ways, also."
National Research Council, 1999, 111	". . . specific types of instruction can modify the brain, enabling it to use alternative sensory input to accomplish adaptive functions, in this case, communication."
Berliner & Cassanova, 1993, 79	"What contemporary research on long-term memory reminds us is that we never stop learning through movement, touch, and imagery, even when the verbal/symbolic learning mode becomes dominant. Thus, if we want to help children remember the things that we deem important, we should help them whenever we can to construct visual representations and give them some multisensory experiences during learning."
International Dyslexia Association, 2000, 1	"Multisensory teaching is simultaneously visual, auditory, and kinesthetic-tactile to enhance memory and learning."
International Dyslexia Association, 2000, 2	"There is a growing body of evidence supporting multisensory teaching. Current research, much of it supported by the National Institute of Child Health and Human Development (NICHD), converges on the efficacy of explicit structured language teaching for children with dyslexia. Young children in structured, sequential, multisensory intervention programs, who were also trained in phonemic awareness, made significant gains in decoding skills. These multisensory approaches used direct, explict teaching of letter-sound relationships, syllable patterns, and meaningful word parts. Studies in clinical settings showed similar results for a wide range of ages and abilities."
National Study Group, 2004, 16	"Learning is more powerful when students are prompted to take information presented in one format and 'represent' it in an alternative way. Cognitive research tells us that we process information in multiple ways—visually and through auditory-verbal channels. Students' learning and recall can be improved by integrating information from both the verbal and visual-spatial forms of representation."
Mauer, 1999, 385	". . . children whose sensory input is not organized or integrated in the brain have sensory integrative dysfunction. Such a disorder leads to disorganized, maladaptive interactions with the environment from which faulty internal sensory feedback is produced, further perpetuating difficulties and causing problems in learning, development, and behavior."
Rose & Meyer, 2002, 111	"By seeing, hearing, smelling, or touching many instances of a pattern, recognition networks can extract the critical features that define that pattern and identify new instances that share those features."

Researcher(s)	Findings/Conclusions
Rose & Meyer, 2002, 114	"Because learners' recognition networks have varying abilities to process visual, aural, olfactory, or tactile patterns, a single means of presentation doesn't work for all students. . . . Providing multiple representations of patterns through a variety of media, formats, organizations, levels of detail, and degree of depth includes more learning by offering both choice and redundancy."
Shaywitz, 2003, 84	"The brain's reliance on patterns of connectivity may have particular relevance to the teaching of reading since within these systems patterns of neural connections are continually reinforced and strengthened as a result of repeated practice and experiences."
Rose & Meyer, 2002, 114	"Research has shown that teaching in multiple modalities (a technique sometimes called transmediation) not only increases access for students with difficulties but also improves learning generally among all students (Siegil, 1995)."
Tileston, 2000, 13	"Only about 20% of students learn auditorily, the other 80% learn either visually or kinesthetically."
Sousa, 2001, 17	"Studies of sensory preferences in school children over the past 40 years have shown shifts among the percentage of students with particular preferences. . . . Note that nearly one-half of this population has a visual preference and just under one-fifth has an auditory preference. Yet, in too many secondary classrooms, talk is the main mode of instruction, often accompanied by minimal overheads or charts. Over one-third of students have a kinesthetic-tactile preference, indicating that movement helps their learning. But think of how much kids in secondary schools just sit at their desks, moving only to change classrooms."
Rose & Meyer, 2002, 6	" . . . more recent theories, such as Multiple Intelligences theory (Gardner, 1993), are consistent with what we are now discovering about the learning brain—namely that students do not have one global learning capacity, but many multifaceted learning capacities, and that a disability or challenge in one area may be countered by extraordinary ability in another."
Herrell, 2000, 144	"The use of multiple intelligences strategies supports the students' learning of new materials because it allows them to use the processing systems in which they integrate knowledge most effectively. By providing multiple ways for the students to demonstrate their understanding, their confidence in their own abilities is fostered and their anxiety is reduced."
Posner, 2004, 3	"Gardner (1983) outlined several forms of intelligence: linguistic, musical, logical-mathematical, spatial, bodily kinesthetic, and inter- and intrapersonal. Neuroimaging studies have used activation tasks that can be seen as involving all of these forms of intelligence. For example, presentation of visual and auditory words activates a largely left-sided set of areas of the anterior and posterior cortex and the cerebellum. Simple arithmetical tasks that involve processing the quantity of a visual digit activate left and right occipital and parietal areas."
Marzano, 1998, 25	". . . experiences can and frequently are encoded in memory using all three modalities. That is, experiences are stored or encoded as three dimensional 'packets.' This modularity assumption is quite consistent with current brain theory."
Kandel, 2006, 276	"Finally, the growth and maintenance of new synaptic terminals makes memory persist. Thus, if you remember anything of this book, it will be because your brain is slightly different after you have finished reading it."

***MLS* Applicaton.** Table 71 documents the utilization of multi-sensory processing in 19 of the 24 *MLS* tasks.

Individualized and Differentiated Instruction

The incredible power of one-on-one tutoring (see Chapter V), as contrasted to all other grouping options for learning, is the ideal form of instruction for any learner. When instruction is one-on-

one, the student can always be in his or her "zone of proximal development" (Dixon-Krauss, 1996, p. 14); instruction can be perfectly designed with necessary scaffolding and other methods used in differentiation; presentation can always be multi-sensory; pacing can be done in such a way that no time is ever wasted; the content can be selected to align with precise needs; assessment and feedback can be ongoing; and every learner can be highly successful. But what is a teacher to do with a room full of diverse, struggling learners? Computer-assisted instruction in the hands of a trained and caring teacher can deliver all those benefits so that even in a room full of learners at many different levels, instruction can be individualized and differentiated.

The design of *MLS* allows each learner to be assigned to the units, levels, and phases of concept development that prior assessment identified as needs. He or she is also assigned a series of exercises in the fluency component of *MLS* to ensure mastery of the whole number operations of addition, subtraction, multiplication, and division. The *MLS* lab facilitator can individualize and/or differentiate instruction, therefore, in content, pacing, amount of practice, and lesson parameters so that each student is adequately challenged and motivated to make progress. The multi-sensory processing strategies address each student's unique learning needs.

Individualization and differentiation are critically important, according to the scientific research cited in Table 72, for there is great diversity in the age, ability, and needs of the range of struggling learners described in Chapters II and III. Also, for those who know Lev Vygotsky's work, delivering instruction in what he called the "zone of proximal development" is necessary for effective learning (Dixon-Krauss, 1996, p. 14). That zone, which changes frequently, is the area in which a learner can perform with the help of an expert peer or adult mediator—or a computer. Once the learner can perform independently, he or she has moved out of the previous zone and is ready for the next challenge in a new zone.

Table 72: Research Findings on Individualization/Differentiation

Researcher(s)	Findings/Conclusions
Rose & Meyer, 2002, 17	"In anatomy, connectivity, physiology, and chemistry, each of us has a brain that is slightly different from everyone else's."
Rose & Meyer, 2002, 83	"Universal Design for Learning provides a framework for individualizing learning in a standards-based environment through flexible pedagogy and tools. It challenges teachers to incorporate flexibility into instructional methods and materials as a way to accommodate every student in the classroom."
Rose & Meyer, 2002, 6	"The challenge posed by greater diversity and greater accountability is to enable students with widely divergent needs, skills, and interests to attain the same high standards."
Rose & Meyer, 2002, 7	"Among the educational approaches UDL supports is differentiated instruction (Tomlinson, 1999), wherein teachers individualize criteria for student success, teaching methods, and means of student expression while monitoring student progress through ongoing, embedded assessment."
Rose & Meyer, 2002, 129	"Context preferences are individual. An optimal context for one student is not necessarily optimal for another."
Kroesbergen, 2002, 7	"A first step in the remediation of mathematics problems is diagnosing the problem and mapping the specific needs of the student in question."
Dowker, 2004, 22	"Effective interventions imply some form of assessment, whether formal or informal, to (a) indicate the strengths, weaknesses and educational needs of an individual or group; and (b) to evaluate the effectiveness of the intervention in improving performance."

Researcher(s)	Findings/Conclusions
Zigmond, 2003, 119	"The bedrock of special education is instruction focused on individual need."
Zigmond, 2003, 120	"Effective teaching strategies and an individualized approach are the most critical ingredients in special education. . . ."
Dowker, 2004, 15	"The componential nature of arithmetic is important in planning and formulating interventions with children who are experiencing arithmetical difficulties. Any extra help in arithmetic is likely to give some benefit. However, interventions that focus on the particular components with which an individual child has difficulty are likely to be more effective than those which assume that all children's arithmetical difficulties are similar (Weaver, 1954; Keogh, Major, Omari, Gandara, and Reid, 1980)."
Rose & Meyer, 2002, 5	"Cultural, education, and legal changes have significantly altered the mix of students in regular education classrooms. Today's typical classroom might include students whose first language is not English; students who are reading on grade level; students with behavioral, attentional, and motivational problems; students from varied cultural backgrounds; and students classified as gifted. In addition, there are students with particular needs, such as limited vision, motor disabilities, emotional difficulties, speech and language difficulties, and learning disabilities."
Levine & Schwartz, n.d., 2	"When a student is having difficulty, it is therefore important to begin the diagnostic process by posing the following questions, 'Where is the breakdown occurring?' and 'Which of the neurodevelopmental functions required to learn and apply this subskill are weak or unable to assume their share or play their vital roles?' Thus, a child may harbor a neurodevelopmental dysfunction in a particular function and/or there may exist a dysfunction at the functions between functions. In either case, the breakdown prevents the student from succeeding."
Lochy, Domahs, & Delazer, 2005, 476	"The choice of the method in rehabilitation, either conceptually based or emphasizing drill and repetition, will depend on the goals, abilities, and limitations of the patient. Conceptual training will provide better understanding and adaptive knowledge."
Rose & Meyer, 2002, 21	"As teachers, understanding the patterns of strengths and weaknesses within a learner's recognition networks can help us individualize the kind of challenge and support we provide, thus maximizing every student's opportunity to learn."
Wood, Frank, & Wacker, 1998, 336	". . . these results suggest that matching instructional strategies to student needs can be a highly effective approach to intervention and warrants further evaluation."
Karp & Howell, Oct. 2006, 119	" [students with special needs] require different learning conditions and methods than do the majority of their peers (Kauffman 1999; Levine 1993; Thurlow 2000; Ysseldyke et al. 2001)."
Stotsky, 2005, 2	"Students learn in a variety of ways. Basing most learning on student discovery is time-consuming, does not insure that students end up learning the right concepts, and can delay or prevent progression to the next level."
Rose & Meyer, 2002, 69	"The key to helping all students achieve is identifying and removing barriers from our teaching methods and curriculum materials. Drawing from brain research and using new media, the UDL framework proposes that educators strive for three kinds of flexibility: • To represent information in multiple formats and media. • To provide multiple pathways for students' action and expression. • To provide multiple ways to engage students' interest and motivation."
Sousa, 2001, 149	"Build on children's strengths. In all areas of learning, teachers can often turn a student's failure into success if they build on what the student already knows how to do. Many years of research into learning styles has demonstrated effective ways of recruiting style strengths to build up weaknesses."

Researcher(s)	Findings/Conclusions
Dowker, 2004, 32	". . . appropriate individualized instruction depends on appropriate selection of the components of arithmetic to be used in assessment and intervention. This is still an issue for debate and one which requires considerable further research. One of the main potential problems, which was more common in the past than nowadays, is to assume that the components to be addressed must necessarily correspond to specific arithmetical operations: eg. Treating 'addition,' 'subtraction,' 'multiplication,' 'division,' etc. as separate components. It is, of course, quite possible for children to have specific problems with a particular arithmetical operation. Indeed, as we have seen, it is possible for a particular arithmetical operation to be selectively impaired in adult patients following brain damage. Nevertheless, it is an over-simplification to assure that these operations are likely to be the primary components of arithmetical processing. Current classifications tend to place great emphasis on the type of procedural knowledge ('knowing how'), conceptual knowledge ('knowing what it all means'), and in some theories utilizational knowledge ('knowing when to apply it') (see, for example, Greeno, Riley, and Gelman, 1994). A potential danger of over-emphasizing the different operations as separate components is that it may encourage children, and perhaps adults, to ignore the relationships between the different operations."
Dowker, 2004, 32	"Another potential problem—again commoner in the past though still a danger nowadays—is looking at children's difficulties only in terms of procedural errors. . . . diagnosing the incorrect strategies is not always the final step. There may be a conceptual reason why the incorrect strategy is acquired and maintained or there may be unperceived conceptual strengths, which need to be noted and built on (Tilton, 1947; Ginsburg, 1977)."
Lyon, 1996, 72	"When policymakers consider 'inclusionary' models of instruction, they must consider carefully whether those models can provide the critical elements of intensity and the appropriate duration of instruction, along with teacher expertise in multiple teaching methods and in accommodating individual learning differences."
Dowker, 2004, 42	". . . individualized work with children who are falling behind in arithmetic has a significant impact on their performance. The amount of time given to such individualized work does not, in many cases, need to be very large to be effective."
Rose & Meyer, 2002, 70	"The central practical premise of UDL is that a curriculum should include alternatives to make it accessible and appropriate for individuals with different backgrounds, learning styles, abilities, and disabilities in widely varied learning contexts. The 'universal' in universal design does not imply one optimal solution for everyone. Rather it reflects an awareness of the unique nature of each learner and the need to accommodate differences, creating learning experiences that suit the learner and maximize his or her ability to progress."
Rose & Meyer, 2002, 71	"Addressing the divergent needs of special populations increases usability for everyone."
Rose & Meyer, 2002, 73	"Non-educators often make the mistake of equating access to information with access to learning. In reality, these are two separate goals. In fact, increasing access to information can actually undermine learning because it sometimes requires reducing or eliminating the challenge or resistance that is essential to learning."
Rose & Meyer, 2002, 76	"Brain research provides a basis for determining the kinds of teaching and learning alternatives most useful for a particular student in a given circumstance."

Researcher(s)	Findings/Conclusions
Rose & Meyer, 2002, 109	"To support students' diverse recognition networks: • Provide multiple examples • Highlight critical features • Provide multiple media and formats • Support background context To support students' diverse strategic networks: • Provide flexible models of skilled performance • Provide opportunities to practice with supports • Provide ongoing, relevant feedback • Offer flexible opportunities for demonstrating skill To support students' diverse affective networks: • Offer choices of content and tools • Offer adjustable levels of challenge • Office choices of rewards • Offer choices of learning context."
National Alliance for Black School Educators, 2002, 20	"Types of classroom support that the professional literature suggests are important to the success of African American students are: • Differentiated curriculum that is appropriate to all learners. • Instruction that is culturally relevant and culturally appropriate. • Adaptation of instruction for a wide variety of learning styles within each cultural or ethnic population. • Experienced and culturally-competent general education personnel. • Individualized intervention strategies that reflect students' cultural contexts. • Home-school-community collaboration."
Chen, 2004, 4	"MI [Multiple Intelligences] theory can be applied to the development of instructional techniques as well. For example, a teacher can provide multiple entry points to the study of a particular topic by using different media and encouraging students to express their understanding of the topic through diverse representational methods such as writing, three-dimensional models, or dramatizations. Such instructional approaches make it possible for students to find ways of learning that are attuned to their predispositions and therefore increase their motivation and engagement in the learning process. Use of these approaches also increases the likelihood that every student will attain some understanding of the topic at hand."
Shearer, 2004, 7	"Teachers often mistakenly think of MI [Multiple Intelligences] as being synonymous to learning styles in spite of Howard Gardner's words to the contrary (Gardner, 1999). Learning styles theories have been with us since the 1950s, and many versions are available to help teachers to describe the unique learning preferences of students. Learning style theories usually refer to personality characteristics or preferences in the process of learning, while MI emphasizes the skill of creating a product, providing a service, or problem solving."
Karp & Howell, Oct. 2006, 119	"The first myth is that students with special needs are vastly different from the regular school population and must be spoonfed information or they will not be able to learn it. The second myth is that students with special needs are just like other children in the class and 'good teaching' is good teaching for all students. Both of these myths limit the success that students with learning disabilities can attain."
Shearer, 2004, 7	". . . MI [Multiple Intelligences] can be used to get beyond a psychiatric label for better educational planning that can focus on building strengths rather than merely managing deficits. . . . The obvious challenge . . . is to figure out how to enlist these MI strengths to build academic limitations, to manage problematic behaviors, and to maximize the development of each student's unique MI gifts."

Researcher(s)	Findings/Conclusions
Kroesbergen, 2002, 5	"Students with difficulties learning math require special attention and instruction adapted to their specific needs. Given the heterogeneity of this group of students, their educational needs are likely to be quite diverse. Nevertheless, many educators argue that most of the students with mathematical difficulties (including students with learning disabilities and mild mental retardation) have more or less the same educational needs as their learning patterns do not differ qualitatively from each other (Kavale & Forness, 1992; Van Lieshout, Jaspers, & Landewe, 1994). It is thus recognized here that students may differ in their educational needs but still have a lot in common. . . . a number of general educational needs can thus be identified and seen to reflect those areas in which the students encounter the most difficulties: automaticity, strategy use, and metacognitive skills (Rivera, 1997)."
Wright, 1996, 3	"The fundamental principle in helping a child with a disability in mathematics is to work with the child to define his or her strengths. As these strengths are acknowledged, one uses them to reconfigure what is difficult."
Dowker, 2004, 2	"Some children have difficulties with many academic subjects, of which arithmetic is merely one; some have specific delays in arithmetic, which will eventually be resolved; and some have persisting, specific problems with arithmetic. The causes of such difficulties are also varied, though they tend to overlap; they include individual characteristics (e.g. unusual patterns of brain development); inadequate or inappropriate teaching; and lack of preschool home experience with mathematical activities and language. The type and extent of intervention needed to address arithmetical difficulties will depend in part on the nature and causes of these difficulties."
Levin & Long, 1981, 61	"In classes where strong support and concern for individuality exist, teachers manage to feel or diagnose individual differences and to use them as a guide in their instructional plans and decisions."
Ontario Ministry of Education, 2005, 14	"The theory behind differentiated instruction comes from the views of Vygotsky (1980). According to Vygotsky, social context and the interactions of the students within that context play a fundamental role in the acquisition of knowledge. Students in their 'zone of proximal development' can, with assistance, resolve a problem that they could not have resolved alone and move on to another level of knowledge. Teachers can help accelerate students' cognitive development (Vienneau, 2005) by supporting children in resolving problems, by questioning their conceptions, and by asking them to justify their positions (LaFortune & Deaudelin, 2001). They can also provide specific interventions, known in this context as 'scaffolding.'"
Ontario Ministry of Education, 2005, 14	"Differentiated instruction requires teachers to transform their practice from a program-based pedagogy to a student-based pedagogy. Teachers attempt to adapt pedagogical interventions to the needs of each student, acknowledging that each student differs in interests, learning profile, and level of functioning. Differentiated instruction may facilitate high levels of both student engagement and curricular achievement (Carol, 2003; Tomlinson, 2004)."
Ontario Ministry of Education, 2005, 14	"Students can develop their potential if they are provided with appropriate activities in an environment that is planned and organized to meet the needs of all students."
Ontario Ministry of Education, 2005, 14-15	"In a differentiated class, the teacher provides instruction at the level students have reached in terms of the curriculum. The learning goals must be adjusted to the abilities of each individual. Students should be observed and evaluated in the learning situation to determine what the expectations should be, using a formative approach; periodic overviews of skills should be done and decisions should be made based on progress."
Freeman & Freeman, 2002, 142	"Vygotsky (1978) has shown that students develop new concepts by working with an adult or a more capable peer who asks questions or points out aspects of a problem. Instruction that is within a student's zone of proximal development (ZPD), the area just beyond the student's current level of proficiency, serves as a scaffold to mediate learning. What students can first do with help, they can later do independently."

Researcher(s)	Findings/Conclusions
Rose & Meyer, 2002, 11	"Learners cannot be reduced to simple categories such as 'disabled' or 'bright.' They differ within and across all three brain networks [recognition, strategic, and affective], showing shades of strength and weakness that make each of them unique."
Hart & Risley, 1995, 193	"We learned from the longitudinal data that the problem of skill differences among children at the time of school entry is bigger, more intractable, and more important than we had thought."
Erlauer, 2003, 11	". . . every brain, due to its different dendrite connections, experiences, and memories, is as different as each individual's fingerprints."
Milller & Mercer, 1997, 9	". . . individualization is going to be needed to adequately address the impact of the specific math disability that emerges from each individual's unique learning characteristics."
Reigeluth, 1997, 204	"If the goal of the standards movement is to accelerate learning for all students, especially low-achieving students, then we must recognize that different students learn at different rates. Yet our current system is characterized by grade levels with classes and classrooms in which all students typically learn the same thing at the same time. By holding time constant, we force achievement to vary among students, with the consequence that the low-achieving ones gradually accumulate deficits in learning that handicap them in their future learning endeavors."
Reigeluth, 1997, 204	"We need customization to replace standardization, in order to have an education system that is focused on learning (attaining high standards) rather than on sorting."
Reigeluth, 1997, 204	". . . we should not expect all students to meet the standards within the same time frames. Further rationale for this conclusion is provided by differences in developmental rates for learners of the same age, differences in opportunities to learn outside of school, differences in prior knowledge and skills, differences in interests, and many other factors."
Elkind, 1997, 241	"Every child, to paraphrase Clyde Kluckhohn and Henry Murray, is like all other children, like some other children, and like no other child."
Darling-Hammond & Falk, 1997, 193	"Schools that successfully support the learning of diverse student populations exhibit a strong commitment to finding and implementing practices that respond to a wide range of individual differences."
Miller & Mercer, 1997, 8	"The teachers in the survey stated that the heterogeneous make-up of their classes and the large number of students they had to teach made it difficult to vary instructional procedures."
Levine, 2002, 308	"I would like teachers to become the community's front-line experts on mind development and learning in the age group(s) they work with. Whether he or she teaches honors science, business math, freshman football, or driver's education, a teacher should be knowledgeable about the high specific neurodevelopmental functions required for success in these realms and the differences in learning that teachers are likely to encounter among any cohort of students. The recent outpouring of research on brain function and learning should flow directly into classrooms. A teacher who acquires background knowledge about neurodevelopmental matters can understand the ways in which different learners have their personal ways of learning."
Levine, 2002, 310	"As teachers gain neurodevelopmental expertise, they are in a far better position to understand students who are struggling to keep up . . . . A teacher then has the option either to bypass the student's area of difficulty or intervene and seek to repair the student's breakdown—or, even better, do both."
Sousa, 2001, 208	"It is important to remember that students with learning problems can learn when teachers spend the time and use their expertise to find the appropriate ways to teach these students."
Sousa, 2001, 208	"Learn about learning. Educators in all areas need to update their knowledge base about what neuroscience is revealing about how the brain learns. These discoveries and insights can help explain problems and improve classroom skills. Teachers should draw on the knowledge of special educators and researchers to address specific problems."

Researcher(s)	Findings/Conclusions
Dixon-Krauss, 1996, 14-15	"Vygotsky believes that good instruction is aimed at the learner's zone of proximal development. He describes the zone of proximal development as encompassing the gap between the child's level of actual development determined by independent problem solving and her level of potential development determined by problem solving supported by an adult or through collaboration with more capable peers. In order for the child to be operating within her zone of proximal development (1) she must be engaged in an instructional activity that is too difficult for her to perform independently; and (2) her performance must be supported by an adult or capable peer."
Moll, 1990, 3	". . . the zone makes possible 'performance before competence.'"
Reis, Kaplan, Tomlinson, Westberg, Callahan, & Cooper, 1998, 75	" . . . recent research indicates that only a small number of teachers offer differentiation in their classroom."
Alliance for Curriculum Reform, 1995, 18	"Adaptive instruction is an integrated diagnostic-prescriptive process that combines several practices—tutoring, mastery and cooperative learning strategies—into a classroom management system that tailors instruction to individual and group needs. Strong and consistent achievement effects of adaptive programs have been demonstrated [in research studies]."
Tomlinson, 2001, 1	"At its most basic level, differentiating instruction means 'shaking up' what goes on in the classroom so that students have multiple options for taking in information, making sense of ideas, and expressing what they learn. In other words, a differentiated classroom provides different avenues to acquiring content, to processing or making sense of ideas, and to development of products so that each student can learn effectively."
Hay, 1997, 68	". . . effective practice in special education, as measured by teacher decision making about instructional modifications and student achievement in reading, math, and spelling, centers instructional decision making on the individual student . . . . This process is called individually referenced decision making."
National Research Council, 1997, 124-125	"Individually referenced decision making is perhaps the signature feature of effective special education practice, exemplifying a basic value . . . . Corroborating evidence documents how individually referenced decision making enhances learning for students with cognitive disabilities. A meta-analysis of a number of studies summarized the efficacy of individually referenced decision making for students with cognitive disabilities (with an effect size of .70 standards deviation units). . . ."
Dixon-Krauss, 1996, 14	"What the child can do in cooperation today he can do alone tomorrow. Therefore, the only good kind of instruction is that which marches ahead of development and leads it; it must be aimed not so much at the ripe as at the ripening functions." (Vygotsky quoted)
Sousa, 2001, 20	"Learning disabilities are characterized by a significant difference between a child's achievement and that individual's overall intelligence. Students with learning disabilities often exhibit a wide variety of traits including problems with spoken and written language, reading, arithmetic, reasoning ability, and organization skills. These may be accompanied by inattention, hyperactivity, impulsivity, motor disorders, perceptual impairment, and a low tolerance for frustration. Because each of these traits can run the gamut from mild to severe, it is necessary to assess each student's disabilities to determine the best approach for effective teaching."
Karp & Howell, 2004, 119-120	"Individualizing of content taught and methods used with students with special needs is one of the basic tenets of special education. *Equity and Standards for School Mathematics* (NCTM, 2000) states, 'Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students' (p. 12)."

Researcher(s)	Findings/Conclusions
Karp & Howell, 2004, 120	"Teachers must consider the following four components of individualization [for students with special needs]: • Remove specific barriers • Structure the environment • Incorporate more time and practice • Provide clarity."
Lovin, Kyger, & Allsopp, 2004, 158-160	"Although each student has individual strengths and weaknesses, most children with learning problems share some common characteristics (Miller & Mercer, 1997). Students with learning problems tend to be passive learners. . . . These students need to be actively engaged in relevant learning situations that allow them to build and expand their conceptual knowledge while giving them the support to develop necessary underlying skills. . . . "Quite often, students with learning problems also have attention problems (Mercer & Mercer, 2001; Miller and Mercer, 1997). They may indeed attend, but they attend to the irrelevant details. . . . These students benefit from a structured, consistent environment in which clear expectations are communicated for learning and doing mathematics. Communicating clear expectations does not mean that the teacher must tell students how to do a task; rather, the teacher should give students a way to understand what is expected and a way to monitor their progress through a particular task. . . . "Students with attention problems often have difficulty with time management and transitions, but at the same time they benefit from a variety of opportunities to move and be physically engaged in learning. Integrating visual organization into a lesson format and giving students opportunities to move and interact with their peers in structured situations are important to their success (Vaughn, Bos, & Schuum, 2003). "Difficulty with memory is another common characteristics of students with learning problems (Mercer, Jordan, & Miller, 1996). . . . Explicitly (that is, purposefully and clearly) linking new information to prior knowledge and experience within relevant, authentic contexts allows students to 'hook' new information to previously learned information, thereby facilitating the memory retrieval process (Mercer & Mercer, 2001). "Language problems can also interfere with the learning of many students with learning problems. . . . When students fail to see the links between concepts, mathematics becomes a rote exercise, and understanding remains at the algorithmic level. This type of learning can be detrimental, especially for those students who have difficulty with memory retrieval. "Student learning is facilitated by reviewing previous concepts and explicitly demonstrating links in relevant, problem-solving contexts. In particular, vocabulary is often a barrier to students with learning problems."

Given the power of individualization and differentiation, educational practice is apparently moving to individual education plans (IEPs) for all students, not just those in special education. Increasingly, especially for struggling students, but even for gifted/talented students, there are legislative mandates for individual plans. Arkansas is an example. Their comprehensive legislation that was designed to revise their former state accountability legislation to comply with *NCLB*, included several requirements related to the provision of a student academic improvement plan (SAIP) for all students failing a portion of the state assessments. Table 73 displays those requirements, along with ways that *ELS* and *MLS* implementation can assist a school or district in compliance, as well as, and more importantly, in effectiveness in improving academic achievement. (See end of this chapter for descriptions of CEI's assessments, e.g., *DSTR, DSTM*, and *LET II*.)

Table 73: *MLS* Correlation with Arkansas Mandate for Individualization

Arkansas Dept. of Ed. Rules	CEI's Role
7.03 By the year 2013-2014 all students are expected to perform at the proficient level or above.	**CEI** provides effective interventions for all students "with educational differences." These interventions will assist schools in preparing students to perform at the proficient level.
7.04 Beginning with the 2004-05 school year, any student failing to achieve at the proficient level on the State mandated CRT, that student shall be evaluated by school personnel, who shall jointly develop, with the student's parents, **a student Academic Improvement Plan (AIP)** to assist the student in achieving the expected standard in subject area(s) where performance is deficient.	**CEI's** *ELS* and *MLS* programs are highly recommended as the school's intervention strategy for all students failing to achieve at the proficient level in language arts and/or mathematics. The third-party assessments provided with the programs will provide additional diagnostic data to determine student strengths and weaknesses. The programs are both highly individualized and differentiated and will enable schools through one intervention to meet the diverse needs of students failing to perform well. Further, the programs enable staff to monitor student progress frequently and to make adjustments in the student's program for improved learning. Summative data will help the school predict achievement on the state benchmarks. The Student Management System will greatly facilitate record keeping for the AIP.
7.04.2 The AIP shall be developed cooperatively by appropriate teachers and/or other school personnel knowledgeable about the student's performance or responsible for the remediation in consultation with the student's parents. An **analysis of student deficiencies** based on test data and previous student records shall be available for use in developing the Plan. The Plan shall be signed by the appropriate school administrator and the parent/guardian.	In addition to the state scores on previous assessments, student grades, and other available records, **CEI** school partners will also have at their disposal the results of the *DSTR*, the *DSTM*, and the *LET II*, all of which will enable them to diagnose "student deficiencies" and then to prescribe appropriate instruction. The *ELS* and *MLS Placement Tests* will assist the committee to place the student at the appropriate level of lessons to maximize the time spent. **CEI**'s parent awareness session will enable parents of students served to understand the program and how it will benefit their child.

Arkansas Dept. of Ed. Rules	CEI's Role
7.04.3 The AIP should be flexible, should contain **multiple remediation methods and strategies, and should include an intensive instructional program** different from the previous year's regular classroom instructional program. Examples of strategies and methods include, but are not limited to, computer assisted instruction, tutorial, extended year, learning labs within the school day, Saturday school, double blocking instruction in deficient areas during the school day, etc.	**CEI**'s *ELS* and *MLS* programs are expressly designed to provide "multiple remediation methods and strategies" that are well grounded in scientifically-based evidence. Both *ELS* and *MLS* are "intensive instructional programs" that serve as interventions and are, therefore, different from regular classroom instruction. They are true therapeutic cognitive interventions that address the root causes of learning problems and failure and correct them so that students can access general education curriculum and grade-level standards. The strategies used in these programs enable schools to use the programs in a variety of ways: Computer assisted instruction Tutorials Extended year Learning labs Saturday school Before/after school Double blocking **CEI** recommends that students be engaged in the *ELS* program for at least 45 minutes each day for at least 4 days a week for maximum benefit. *MLS* students need 5 days.
7.04.4 The AIP shall **include formative assessment strategies** and shall be revised periodically based on results from the formative assessments.	Built into the management of the *ELS* and *MLS* programs is a formative assessment system that requires the teacher to daily and periodically evaluate progress and to make appropriate adjustments to the student's program of lessons. Mastery lessons are administered regularly. Students failing to master are automatically recycled.
7.04.5 The AIP shall include standards-based **supplemental/remedial strategies** aligned with the child's deficiencies.	Both *ELS* and *MLS* provide the necessary remediation to address the learning deficiencies of virtually all children "with educational differences." **CEI**'s research papers on both *ELS* and *MLS* document the scientific evidence on the efficacy of content design, lesson models, instructional strategies, and implementation features used in the programs.
7.04.6 **A highly qualified teacher** and/or a highly qualified paraprofessional under the guidance of a highly qualified teacher shall provide instructional delivery under the AIP.	*ELS* and *MLS* labs may be staffed by a highly qualified teacher or by a highly qualified paraprofessional under the guidance of a highly qualified teacher. About 60% of **CEI**'s school partners assign paraprofessionals to their labs.
7.04.7 The **AIP should be individualized**; however, similar deficiencies based on test data, may be remediated through group instruction.	Both *ELS* and *MLS* programs are totally individualized, so a diverse group of students can all be served effectively in one lab.
7.05 Retention for failure to participate in the Academic Improvement Plan 7.05.5 Any student who does not score at the proficient level on the criterion-referenced assessments in reading, writing, and mathematics shall continue to be provided with remedial or supplemental instruction until the expectations are met or the student is not subject to compulsory school attendance.	*ELS* and *MLS* programs are highly motivating since instruction is carefully scaffolded for participating students to ensure that they experience a reasonable degree of success, which encourages them to stay on task. The programs are large enough that students needing interventions more than one year will have adequate instruction.

Texas has a similar requirement in their requirements for Accelerated Reading Instruction (ARI) and Accelerated Mathematics Instruction (AMI). Table 74 displays the correlation for *MLS* with the AMI requirements.

Table 74: *MLS* Correlation with Accelerated Mathematics Requirements (as per 2005-06 Accelerated Mathematics Instruction, Texas Education Agency)

Accelerated Math	*Mathematical Learning Systems*
Identification of Struggling Students Results from math diagnostic instruments are a primary criterion used to identify students in an AMI program. A district-wide mathematics assessment is recommended for students in Grades K-2. The Texas Math Diagnostic System (TMDS) is available for use with students in Grades 3-12.	**CEI** provides two third-party assessments for schools to use in identifying students for placement in *MLS* labs and for diagnosing their individual and specific needs: 1. *Diagnostic Screening Test for Mathematics (DSTM),* published by Slosson. 2. *Learning Efficiency Test (LET-II),* published by Academic Therapy Publications. Additionally, some schools use either local or commercial supplemental assessments to inform decision-making. **CEI** also provides its own *MLS Placement Test* to provide information to teachers on the appropriate level and lesson for each individual student.
Additional assessments throughout the program should be used to measure progress and inform instruction.	Each program task includes embedded assessments, and daily lesson reports indicate student progress. Another formative assessment is based on teacher observations of performance and progress. Each *MLS* lesson phase includes three lesson steps (concrete, semiconcrete, and abstract presentation) and one assessment step. Each of the lesson steps includes a Mastery lesson. Students failing to achieve 80% mastery (based on developers' research) on their first attempt are then recycled through the lessons in each step until they have enough practice. Once they have mastered the first three lesson steps, they do the Assessment lesson. If they do not achieve 80% mastery, they recycle the whole lesson phase. The *DSTM* is administered as a pre/post-test to measure gains. Initial teacher training and follow-up/coaching focus on how to use the results of the various assessments to make initial placement decisions and then how to use them to inform instruction: adapting the lesson level and program settings to ensure challenging, yet successful progress.
Instructional Priorities All students identified as struggling in grades K-6, on each campus, should receive needed instructional mathematics intervention. In addition, students who fail one or more of the state-mandated grade 5 mathematics assessments administered in spring 2006 may receive intervention with these funds.	**CEI** was established almost 20 years ago with its niche being to design and market learning solutions for struggling learners, K-adult. **CEI** has documented evidence that *MLS* produces accelerated results for a variety of struggling learners: students who are economically disadvantaged, limited-English, dyslexic, and/or special education identified. Regardless of whether students struggle due to difficulties or disabilities, *MLS* is a proven effective intervention.

Accelerated Math	*Mathematical Learning Systems*
Program Structure Provision of AMI program instruction may reflect several program formats: during the regular school day, before/after school.	Schools own their room and station licenses once they are purchased and may use them with as many students per day as they wish, in before/after school programs, and during summer school without additional licensing fees. An annual service/support fee is charged to keep software updated, to receive testing materials and other supplies, to provide training, to have access to technical support, and to have access to educational consulting.
Intervention provided during the regular school day is strongly recommended because of its timeliness and effectiveness. It is recommended that only a portion of these funds be utilized for summer school.	**CEI** recommends that students engage in *MLS* instruction at least 45 minutes per day, five days per week—for maximal results.
Prioritization of AMI fund expenditures should focus an intervention for the students who need the most assistance first; then, provide additional assistance/funding to other students struggling in mathematics.	*MLS* serves those students at the lowest levels of performance, regardless of whether they have difficulties (including the struggle to learn mathematics and English at once) with mathematics or whether they have disabilities.
Prompt provision of math intervention program with frequent monitoring of individual student's progress is strongly recommended.	**CEI** will ship software within 48 hours of receiving a school's purchase order. Also, training for lab teachers will immediately be scheduled.
A locally-developed districtwide math diagnostic assessment is the primary indicator for student placement in a math intervention program in grades K-2. The Texas Math Diagnostic System (TMDS) is recommended for use with grades 3-6 students along with grade 3, 4, and 5 TAKS math results, locally developed assessments and teacher observations.	See description of assessments above.
Continuous monitoring of identified students with available math diagnostic tools is important.	The *DSTM* is administered twice per year—as a pre- and post-test. Other monitoring occurs daily.
Programs should focus on conceptual development in mathematics content.	See "Best Practices" in mathematics below. *MLS* should be offered at least 45 minutes per day, five days per week—for maximal results. The use of software allows one-on-one instruction which is totally individualized and differentiated for each learner—more effective and more efficient than small-group instruction. The concrete--semiconcrete—abstract lesson sequence, manipulatives, and multi-sensory processing are key strategies that make *MLS* effective.

Accelerated Math	*Mathematical Learning Systems*
Best Practices *A placement process that effectively identifies students at-risk for math difficulties, including dyslexia, and promptly triggers student placement in an intervention program. *Note: Research verifies that almost all dyslexic students also struggle with mathematics—in understanding mathematical terms, in sequencing, in solving word problems, and in learning multiplication tables (see Kibel, 1992; Miles, 1992; Henderson, 1992; Chinn & Ashcroft, 1992; Pennington, 1991; Butterworth, 2005; Miller and Mercer, 1997; etc.)*	See assessment section above for the rich variety of assessment tools provided as a part of the *MLS* program. *MLS'* emphasis, based on scientifically-based evidence of what is needed in a mathematics intervention program, is on concept development and fact fluency. It includes instruction in algorithms, and it includes many lessons on problem solving, including instruction in how to eliminate irrelevant information in a word problem. Instructional strategies are grounded in the research on the efficacy of direct instruction for struggling learners, use of manipulatives, concrete-semiconcrete-abstract sequence of lessons, computer-assisted instruction, practice/repetition, individualization/differentiation, chunking, time-on-task, and multisensory integration strategies.
*A program instructional format that is consistently informed by assessment data and classroom data, and that provides repeated opportunities for students to engage in intensive, targeted learning. *Note: TEA does not provide a list of "Best Practices" for mathematics interventions.* *CEI's research, however, identifies the following content as critical in a math intervention:* *<u>concept development</u> (Mercer & Mercer, 2005; Donovan & Bransford, 2005; Fuson, Kalchman, & Bransford, 2005; Geary & Hoard; Cawelti, 1999; Lochy, Domahs, & Delazer, 2005; Sousa, 2001; Siegler & Booth, 2005; Butterworth, 2005; etc.)* *<u>fact fluency</u> (Mercer & Mercer, 2005; Donovan & Bransford, 2005; Fuson, Kahchman, & Bransford, 2005; Lochy, Domahs, & Delazer, 2005; LeFevre, DeStefano, Coleman, & Shanahan, 2005; etc.)*	The active engagement of the *MLS* lab teacher is a signature component of the program. He/she is trained to use all available assessment data on a daily basis to ensure the most effective, challenging, yet appropriate instruction is provided to each individual student, according to his/her needs. Another important feature of *MLS* is its provision of many varied and engaging practice exercises for each lesson sequence to ensure that mastery is at least 80%. Most other remedial programs fail to provide enough practice for the students who struggle most. A major emphasis in *MLS* is "fact fluency," which research identifies as the area causing the most problems in students with mathematical difficulties or disabilities. CEI's documentation of *MLS'* correlation with scientific research on best practice in teaching mathematics to struggling learners, *Why MLS Works: Its Scientific, Theoretical, and Evaluation Research Base* is available. See also **CEI**'s correlation of the *MLS* content with the *DSTM* and with the NCTM standards. *MLS* focuses on concept development and fact fluency. To teach concepts, the student moves from concrete (with use of physical manipulatives) to illustrative (depictions of the manipulatives on the computer screen) to abstract (use of numbers in problem solving). Learning concepts also means learning math vocabulary. To develop fluency, the student is engaged in multiple, varied, and adequate practice/repetition exercises sufficient to move the learning to long-term memory for recall, retrieval, and application. *MLS* also includes mathematics games and word problem applications embedded in the number operation unit. The web-enhanced activity center (WAC) includes an engaging math game, *Digit's Widgets*, to further develop expertise in math facts and fluent recall. See assessment information above.

Accelerated Math	***Mathematical Learning Systems***
	MLS also incorporates "Frequent, Multiple Assessments," "Corrective Feedback," "Informed Instruction (Data-Driven Decision Making)," and "Self Assessment" in the program. The lab teacher can e-mail the reports of assessment results directly to the classroom teacher or can share hard copies of the reports that can be printed and available each day. Lesson Reports are available.

Even the college level is moving toward individualized and differentiated instruction. Texas adopted in 2003 the Texas Success Initiative (TSI). Its purpose is to "improve individualized programs to ensure the success of students in higher education." The requirements now mandate that publicly-funded colleges assess each entering undergraduate student, using an instrument approved by the Texas Higher Education Coordinating Board to determine "the student's readiness to enroll in freshman-level academic coursework." For each student who fails to meet the minimum passing standards, the college must:

(1) Establish a program to advise the student regarding developmental education necessary to ensure the readiness of that student in performing freshman-level academic coursework.
(2) Determine a plan, working with the student, for academic success, which shall include developmental education and may include provisions for enrollment in appropriate non-developmental coursework.

According to the rules, each academic success plan shall:

(1) Be designed on an individual basis to provide the best opportunity for each student to succeed in performing freshman-level academic coursework.
(2) Provide to the student a description of the appropriate developmental education considered necessary to ensure the readiness of that student to perform freshman-level academic coursework.
(3) Provide to the student an appropriate measure for determining readiness to perform freshman-level academic coursework. (28 *TexReg* 10753)

CEI has both *ELS* and *MLS* labs in colleges, adult education centers, literacy centers, prisons, and other settings where there are struggling adult learners.

***MLS* Application.** In summary, *MLS* utilizes individualized/differentiated instruction in every one of its tasks (Table 67); in the placement of students into the program; in the ongoing assessment of progress, with feedback; in the automatic recycling of non-mastery students; in fluency lesson assignments; and in the motivation program. *MLS* also meets requirements of various states in offering individualized instruction for struggling students, including those in special education, limited-English proficient programs, Title I and other at-risk programs, Response-to-Intervention, dyslexia programs, and even in developmental education at the college level. *MLS' Student Management* system enables the lab teacher/facilitator to produce records that track student engagement in the program, as well as individual progress toward mastery.

Practice and Repetition

CEI staff are frequently asked what they think is the most important feature of its programs—the feature most responsible for student success. The truth of the matter, of course, is that there is no one feature that would make the programs successful without the presence of others. Content is critical. Multi-sensory processing is critical. Computer-assisted instruction is critical. Individualization/differentiation is critical. Assessment with feedback is critical. The role of the teacher is critical. And so on. But one feature that almost everyone includes in his or her list is the richness of CEI's practice and repetition exercises. What is known is that there are many programs available that have sound designs and which include practice exercises. However, very few, if any outside of CEI's programs, include the variety and adequacy of practice tasks found in *MLS*—both of which are absolutely required in order for a student with learning difficulties and/or disabilities to develop mathematics proficiency.

As noted earlier, practice/repetition, along with multi-sensory processing, are the power in *MLS* as an intervention. Kandel (2006) explains why. First he defines explicit memory: "explicit (or declarative) memory is the conscious recall of people, places, objects, facts, and events" (p. 132). Explicit memory is generally the memory required for concept development in mathematics. Kandel further explains that "implicit memory often has an automatic quality. It is recalled directly through performance, without any conscious effort or even awareness that we are drawing on memory" (p. 132). Implicit memory is what *MLS* develops, for example, in its fact fluency strand. Kandel goes on to explain: "constant repetition can transform explicit memory into implicit memory" (p. 132).

Practice/repetition is included as one of the lesson steps discussed in Chapter V; it is also critical to the lesson models of direct instruction, mastery learning, and one-to-one tutoring discussed in Chapter V. Another example is the emphasis on practice or repetition in the section on fact fluency development (Tables 52 and 53) discussed in Chapter IV. Much of the research on multi-sensory processing (Table 68) involves the importance of adequate and varied practice/repetition in encoding knowledge and skills into long-term memory and in the strengthening and building of neural pathways in the brain that facilitate learning. Kandel (2006) states an old truth: "Practice does make perfect" (p. 206).

The research literature is rich in this area—and abundant. Table 75 includes a wide sampling.

Table 75: Research Findings on Practice and Repetition

Researcher(s)	Findings/Conclusions
Gardner, 1985, 81	"In this, as in every intellectual realm, practice is the *sine qua non* of eventual success."
Kandel, 2006, 204	". . . the duration of short-term memory storage depends on the length of time a synapse is weakened or strengthened."

Researcher(s)	Findings/Conclusions
Kandel, 2006, 206	"Behavior experiments suggest that short-term memory grades naturally into long-term memory and, moreover, that it does so with repetition."
Whitehurst, n.d., 5	". . . the type of practice that results in skills becoming automatic takes considerable repetition and time-on-task. This is true for hitting a tennis ball or playing the violin or decoding written text or doing mathematical calculations. Doing something over and over again until you don't have to think about it may rarely be great fun, particularly in the context of other ways that children can spend their time. By failing to acknowledge that mathematical learning involves work, the United States may be placing a ceiling on the levels of proficiency that it can expect its students to achieve."
LeFevre, DeStefano, Coleman, & Shanahan, 2005, 362	". . . practice in a domain leads to the development of domain-specific, long-term retrieval structures that interact very efficiently with conscious working memory processes."
Lochy, Domahs, & Delazer, 2005, 472	"Kashiwagi, Kashiwagi, and Hasegawa (1987) used drill to reestablish the retrieval of simple arithmetic facts in eight chronic aphasic subjects. All patients showed preserved addition and subtraction but severely impaired multiplication and division. . . . After 1 or 2 months of daily training and additional homework, all participants improved significantly in the retrieval of multiplication facts. However, this was only true for the visual-written route, targeted by intervention. The auditory-verbal route, on the other hand, virtually exclusively used in healthy Japanese subjects, did not improve. Thus, a successful reorganization of fact retrieval had taken place; the auditory-verbal route affected by aphasia was replaced by the visual-written route, relatively preserved."
Erlauer, 2003, 81	"The new concept or skill must be understood and usually related to prior knowledge or experience. The information then must be practiced or manipulated, and used or applied numerous times before it becomes ingrained in the brain's long-term memory."
Marzano, 1992, 61	"Regardless of whether the process is learned to the level of automaticity or the level of expert control, it is extended practice that gets the learner there."
Marzano, 1992, 61	"In short, it is practice—a lot of it—that enables a learner to internalize a skill or process."
Cawelti, 1999, 122	"Many successful reform-oriented programs include time for students to practice what they have learned and discovered. Students need opportunities to practice what they are learning and to experience performing the kinds of tasks in which they are expected to demonstrate competence."
Erlauer, 2003, 129	". . . requiring the students to continue working on the task until they achieve success appears to improve learning and achievement."
Marzano, Pickering, & Pollock, 2001, 67	"Mastering a skill requires a fair amount of focused practice."
Marzano, 1992, 48	"Elaboration involves making many and varied linkages between new information and old."
Marzano, 1992, 48	"Cognitive psychologists have taught us a lot about storing information in long-term memory. In fact, we know more about how information can be stored for easy retrieval than we do about almost any other aspect of learning. Unfortunately, what we know is usually not taught in the classroom. Most students use only verbal rehearsal, perhaps the weakest of all the strategies available, to help them remember what they have learned. Verbal rehearsal involves saying, reading, or writing information several times. Although verbal rehearsal works, its effectiveness is surpassed by other strategies, all of which fall under the general category of elaboration."
Marzano, 1992, 49	"Virtually all memorization techniques use some form of elaboration. One of the most powerful ways to elaborate information is to imagine mental pictures, physical sensations, and emotions associated with the information."

Researcher(s)	Findings/Conclusions
Marzano, 1992, 60	"Guided practice is a powerful instructional technique for helping students understand procedural knowledge at a conceptual level. . . . Vygotsky hypothesized that a learner needs the most guidance when working in the zone of development in which she has not yet acquired a skill but has some initial idea of it—in effect, when the learner is shaping a procedure she has been introduced to. What is now called scaffolded instruction is, at its core, guiding a learner through the shaping of a skill or process."
Marzano, Pickering, & Pollock, 2001, 69	"While practicing, students should adapt and shape what they have learned."
McGuinness, 1997, 168	"Competency stems from practice (repetition). Children willingly practice or repeat actions to obtain mastery. Just because repetition may look boring to an adult doesn't mean it's boring to a child."
Levine & Swartz, n.d., 7	"A wide range of techniques can be applied to enhance deficient subskills. These include exercise to automate (render fast and effortless) slow and labored writing. Vigorous practice with letter formation or the recall of spelling are examples."
Wolfe, 2001, 101	"There are many ways to rehearse information or a skill. One type, called rote rehearsal, consists of repeating the information or the action over and over . . . . It is easy to see why rote rehearsal is essential for forming the strong neural connections necessary to get a skill or habit to the automatic level."
National Research Council, 1999, 113	". . . classes of words, pictures, and other categories of information that involve complex cognitive processing on a repeated basis activate the brain. Activation sets into motion the events that are encoded as part of long-term memory."
National Research Council, 1999, 113	"One of the simplest rules is that practice increases learning: in the brain, there is a similar relationship between the amount of experience in a complex environment and the amount of structural change."
Sharron & Coulter, 994, 101-102	"To consolidate new thought processes into the general intellectual repertoire of a child, so that their use becomes an intrinsic need, requires a degree of over-learning which can be tiresome and which can encourage an undesirable attitude to work. The instruments have therefore been construed to try to consolidate habits through varied repetition. Particular functions and skills are represented in different ways, altering the content or using different modalities."
Sprenger, 1999, 74	"There are two ways to help students access their procedural memory lane. One is to have students perform the material often enough that it becomes a procedure. . . . When a procedure is repeated frequently, the brain stores it in the cerebellum for easy access."
Sprenger, 1999, 75	"The automatic memory lane stores multiplication tables, the alphabet, the ability to decode words, and dozens of other memories triggered by simple associations. . . . Other automatic strategies include the use of flash cards, repetition through daily oral work (in math, geography, language, vocabulary, and so on), and oral conditioning. . . . Each of these strategies has its own benefits. Students will tire of the same strategy, so provide variety."
Sprenger, 1999, 77	"Repetition is a plus; try to find a way to use it."
Levin & Long, 1981, 34	"Most of these studies seem to indicate that learning is more effective if students can practice in a variety of situations. When students are required to cope with frequent changes in the practice exercises, they learn to identify the essential elements in each learning task. They also learn how to adapt to changing circumstances and how to identify common patterns in learning situations. These behaviors are believed to be evoked by varied practices and to facilitate student learning and performance."
Gagne', R., 1985, 173	"Sheer repetition of labels or facts, in a kind of 'over rehearsal,' does not necessarily lead to better encoding or retention. . . . However, when retrieval is practiced, substantial improvement occurs in the later recall of learning information."
Gagne', R., 1985, 173	"The practice of verbal information items, when it involves retrieval on the part of learners, constitutes a review of the information being learned and stored. Such review can provide the occasion for additional and more elaborate encoding. It can also increase the variety of retrieval cues learners have at their disposal."

Researcher(s)	Findings/Conclusions
Gagne', R., 1985, 255	"Increasing amounts of practice constitute a fairly dependable factor for affecting amount of retention."
Gagne', R., 1985, 278	"Variety in the cueing function can also be provided by requiring the learner to reinstate the learned capacity at various times following initial learning. This is done in the technique of spaced review. Even a 'next day' recall and review of a learned rule or concept, for example, may greatly enhance its retention over longer periods."
International Dyslexia Association, 2002, 2	"Provide additional practice activities. Some materials do not provide enough practice activities for students with learning problems to acquire mastery on selected skills. Teachers must then supplement the material with practice activities. Recommended practice exercises include instructional games, peer teaching activities, self-correcting materials, computer software programs, and additional worksheets."
National Study Group, 2004, 16	"Research tells us that a powerful way to promote long-term retention and transfer is to allow students to practice retrieving previously taught material from long-term memory."
Herrell, 2000, 184	"Repetition and innovation strategies provide students with multiple opportunities to learn new concepts. The choice of repetitions and innovations should be based on observation of the students' understanding of the concepts being presented. Each repetition or innovation should build on the last so that the students are experiencing gradually more difficult applications of the concepts. These activities are especially supportive of English language learners because they see multiple definitions and uses of the new concepts and vocabulary they are using."
Sternberg, 2003, 73	". . . automatization occurs as a result of practice, such that highly practiced activities can be automatized and thus become highly automatic."
Sternberg, 2003, 183	". . . distribution of study (memory rehearsal) sessions over time affects the consolidation of information in long-term memory."
Sternberg, 2003, 183	". . . people tend to remember information longer when they acquire it via distributed practice (i.e., learning in which various sessions are spaced over time) rather than via massed practice (with sessions crammed together all at once). The greater the distribution of learning trials over time, the more the participants remembered over long periods."
Kandel, 2006, 204	". . . long-term memory typically requires repeated, spaced training with intervals of rest."
Sternberg, 2003, 183	"Why would distributing learning trials over days make a difference? One possibility is that information is learned in variable contexts, which helps strengthen and begin to consolidate it."
Fuson, Kalchman, & Bransford, 2005, 243	"Time for consolidation of learning, with feedback loops should errors arise, is vital for mathematical fluency."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 117	". . . when learning a skill students need a great deal of practice in order to achieve mastery. Students also need time to shape and adapt the skill so they can use it effectively."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 130	". . . it is certain that without practice, little learning occurs."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 130	". . . when students first begin practicing a skill, their learning progresses rapidly. However, students probably need at least 20 practice sessions before you can be reasonably sure they grasp the new skill enough to use it effectively on their own. Although the more students practice, the smaller the learning increment, practice always enhances learning. Only after a great deal of practice can students perform a skill with speed and accuracy."

Researcher(s)	Findings/Conclusions
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 131	"At first, practice sessions should be close together—massed practice. Over time, you can space them apart—distributed practice. . . . When students are first learning a new skill or process, they should practice it immediately and often. That is, they should engage in massed practice. . . . The teacher should gradually increase the interval of time between practice sessions. Instead of practicing every day, students should practice every other day, then every third day, and so on. Lengthening the intervals of time between practice sessions involves students in distributed practice. Over time, students will internalize the new skill."
Rose & Meyer, 2002, 25	". . . to acquire skills, students need support for both top-down and bottom-up strategic processing. They learn best when they have not only good instruction and good models, but also plenty of opportunity to practice and to receive ongoing, relevant feedback. The kinds of models and supports most suitable for individual learners depend on the student's particular strategic strengths and weaknesses."
Rose & Meyer, 2002, 25	". . . we know students who seem to learn best by doing; these are the students who achieve expertise only after lots of practice and feedback--an indication of strong bottom-up strategic processing."
Marzano, Pickering, & Pollock, 2001, 67, 69	"Mastering a skill requires a fair amount of focused practice. . . . It is only after a great deal of practice that students can perform a skill with speed and accuracy.
McEwan, 2000, 47	"Before you banish practice as nostalgia math, evaluate these findings: 'The argument that practice to automatize the development of basic cognitive skills, such as fact retrieval, is unnecessary and unwanted in mathematics education fails to appreciate the importance of basic skills for mathematical development. As noted earlier, drill and practice provide an environment in which the child can notice regularities in mathematical operations and glean basic concepts from these regularities' (Briars & Siegler, 1984)."
McEwan, 2000, 48	"No educator believes in killing motivation through rote learning that denies the importance of understanding. Delete the word drill from your instructional lexicon and insert the word practice instead. 'Nothing flies more in the face of the last 20 years of research than the assertion that practice is bad. All evidence, from the laboratory and from extensive case studies of professionals, indicates that real competence only comes with extensive practice (e.g., Ericsson, Krample, & Tesche-Romer, 1993; Hayes, 1985). In denying the critical role of practice one is denying children the very thing they need to achieve real competence. The instructional task is not to kill motivation by demanding drill, but to find tasks that provide practice while at the same time sustaining interest' (Anderson, Reder, & Simon, November 1999)."
McEwan, 2000, 48	"Even when American teachers do expect their students to practice, they ask them to practice the wrong things. Although more than half of the Japanese, Korean, Czech, and Hungarian eighth-grade mathematics teachers surveyed for the TIMSS [Third International Mathematics and Science Study] study reported having students practice writing equations in every lesson or most lessons, only a third of U.S. teachers reported practice in writing equations. American teachers reported that their eighth-grade students were still practicing computation, which should have been practiced and mastered earlier (Schmidt et al., 1999, 73)."
National Research Council, 2001, 122-123	"When students practice procedures they do not understand, there is a danger they will practice incorrect procedures, thereby making it more difficult to learn correct ones. . . . "If students have been using incorrect procedures for several years, then instruction emphasizing understanding may be less effective."
McEwan, 2000, 74	"Procedural learning requires extensive practice on a wide variety of problems on which the procedure might eventually be used (Geary, 1994, 269)."

Researcher(s)	Findings/Conclusions
McEwan, 2000, 74	"Although procedural learning has fallen on hard times since the NCTM standards were published in 1989 and is considered unnecessary by many constructivists (Cobb et al., 1992), it remains a critical component of raising mathematics achievement. The automaticity that results from quality practice has the potential to free up students' attention and working memory so they can concentrate on other features of a problem (Geary & Widaman, 1992; NCTM Research Advisory Committee, 1988)."
McEwan, 2000, 74-75	"Not only do the current mathematical reforms deemphasize memory and practice generally, they specifically recommend decreased attention to mastery of the long division algorithm, paper-and-pencil fraction computation, teaching computations out of context, and drilling on paper and pencil algorithms (NCTM, 1989, 21, 73). There are those who are even ready to completely abandon computational algorithms. They believe that 'drill and practice of computational algorithms devour an incredibly large proportion of instruction time, precluding any real chance for actually applying mathematics and developing the conceptual understanding that underlies mathematical literacy' (Leinwand, Feb. 9, 1994). "But before we end our 'obsessive love affair with pencil-and-paper computation,' as Leinwand (1994) characterizes it, ponder whether the students who struggle with computational algorithms because of lack of automaticity will really fare any better with calculators. Since 'algorithmic thinking provides the formal structure for mathematical growth and understanding' (Mingus & Grassl, 1998, 32) and it is understanding that is critical to solving the problem, these students may continue to flounder even with a calculator crutch."
McEwan, 2000, 75	"Basic math facts are a prerequisite for solving even simple word problems (Wu, 1999), and mastery of the algorithms that manipulate those facts is even more critical. Conversely, a curriculum that stresses only the memorization and drill of the facts and the algorithms without daily solving of challenging word problems is as destitute as one that omits them completely. Both are necessary. Facts and algorithms are the tools of problem solving. If a student cannot master the facts and algorithms, it is not likely that the presence of a calculator will suddenly bestow problem-solving abilities on a student. Calculators give power, but it is not a magical power."
McEwan, 2000, 75	"The purpose of learning algorithms is to facilitate fluent and accurate problem solving, not to be able to regurgitate formulas. In fact, when students are practicing simple problems using the mastered algorithm, they often develop additional insights and strategies they are unaware of using (Siegler & Stern, 1998)."
Bohan, 2002, 36-37	"No person becomes proficient at basic facts, computational algorithms, and more complex problem-solving techniques without practice. Practice is a requirement in any human endeavor if proficiency is the target."
Bohan, 2002, 37	"The development of true proficiency in mathematics demands multiple opportunities for meaningful practice in which students use the mathematics they learn in challenging and enjoyable settings."
Willingham, 2004, 1	"Intuition tells us that more practice leads to better memory. Research tells us something more precise. Memory in either the short- or long-term requires ongoing practice. Let's first consider memory in the short-term, meaning days or weeks. Suppose I am trying to learn the procedures necessary for a bill to become a federal law. I might study these facts (using any number of techniques) and periodically test myself. Suppose further that I study until I perform perfectly on my self test. Do I know these facts? Yes, I know them *now*. But what about tomorrow? In order to protect this learning from the ravages of forgetting, I need to practice beyond one perfect citation. Studying material that one already knows is called *overlearning*. Because memory is prone to forgetting, one cannot learn material to a criterion and then expect the memory to stay at that level very long."

Researcher(s)	Findings/Conclusions
Willingham, 2004, 1	"It is difficult to overstate the value of practice. For a new skill to become automatic or new knowledge to become long-lasting, sustained practice, *beyond the point of mastery*, is necessary."
Willingham, 2004, 2	"Studies show that if material is studied for one semester or one year, it will be retained adequately for perhaps a year after the last practice (Semb, Ellis, & Araujo, 1993), but most of it will be forgotten by the end of three or four years in the absence of further practice. If material is studied for three or four years, however, the learning may be retained for as long as 50 years after the last practice (Bahrick, 1984; Bahrick & Hall, 1991). There is some forgetting over the first five years, but after that, forgetting stops and the remainder will not be forgotten even if it is not practiced again. Researchers have examined a large number of variables that potentially could account for why research subjects forgot or failed to forget material, and they concluded that the key variable in very long-term memory was practice."
Willingham, 2004, 5	". . . if we want children to understand and appreciate excellence, we would do well to send the message that excellence requires sustained practice. The athletes and artists revered by many students excel not solely by virtue of their talent, but because of their hard work."
Willingham, 2004, 5	"Practice is done for the sake of improvement. Practice, therefore, requires concentration and requires feedback about whether or not progress is being made. Plainly put, practice is not easy. It requires a student's time and effort, and it is, therefore, worth considering when it is appropriate."
Willingham, 2004, 5	"The following types of material are worthy of practice: 1. The core skills and knowledge that will be used again and again. In this case, we give practice in order to ensure automaticity. . . . The student who does not have simple math facts at his or her disposal will struggle with higher math. 2. The type of knowledge that students need to know well in the short term to enable long-term retention of key concepts. In this case, short-term overlearning is merited. . . . 3. The type of knowledge we believe is important enough that students should remember it later in life. In this case, one might consider certain material so vital to an education that it is worthy of sustained practice over many years to assure that students remember it all of their life. . . ."
Willingham, 2004, 4	"Research studies indicate that experts are right, at least in that they do practice a great deal. Descriptive studies (Roe, 1953) of eminent scientists indicate that the most important factor predicting their success is not innate talent or intelligence, but the willingness to work hard for extended periods of time."
Whitehurst, n.d., 5	"Cognitive psychologists have discovered that humans have fixed limits on the attention and memory that can be used to solve problems. One way around these limits is to have certain components of a task become so routine and over-learned that they become automatic. . . . Although teaching children to understand the principles of multiplication by having them double a cookie recipe may seem like a good idea, if the child doesn't know the times table, the cognitive requirements of working with cookie dough and the cognitive requirements of multiplication will be too much to handle and will detract from learning. A clear instructional implication is that discovery activities should come later in a sequence of instruction, after children have acquired the requisite background knowledge to handle open-ended, real life problems. I am not saying that discovery activities should wait until graduate school. They can occur at any grade. However, the child should be prepared for the activity so that he can focus on what is important to the instructional goal. This is a basic principle of instructional design that is often ignored in approaches that rely on discovery activities."
T. Miles, 1992a, 14	"In general, dyslexics need more exposures before their responses become automatic."

Researcher(s)	Findings/Conclusions
T. Miles, 1992a, 14	"In the case of both literacy and numeracy it is, of course, a great advantage in the long run if a large amount of automaticity can be achieved, but it is important in both cases that alternative procedures should be available for use where necessary."
Fayol & Seron, 2005, 17	"These findings strongly suggest that the automatization in number processing is achieved gradually as numerical skills progress and that six-year-old children do not automatically access the analog representation of quantity when confronted with digits. Rubinsten, Henik, Berger, and Shahar-Shalev (2002) have confirmed the key elements of these conclusions. The Arabic code becomes established rapidly as a structure—easy to learn and use, at least for small quantities—and is relatively slow in acquiring the capacity to activate the precise quantities with which it is associated quickly."
Marzano, 1992, 61	"The final stage of learning a skill or a process is to internalize the knowledge: to practice it to the point where you can perform it with relative ease. Actually, it is most accurate to think of skills and processes as being located on a continuum of skill levels ranging from controlled processing to automaticity."
Sternberg, 2003, 270	"J. R. Anderson (1980) has hypothesized that knowledge representation of procedural skills occurs in three stages: cognitive, associative, and autonomous. During the cognitive stage, we think about explicit rules for implementing the procedure. During the associative stage, we practice using the explicit rules extensively, usually in a highly consistent manner. Finally, during the autonomous stage, we use these rules automatically and implicitly, with a high degree of integration and coordination, as well as speed and accuracy."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 328	"The last aspect of learning a new skill or process is internalizing it. For some skills and processes, internalizing means learning them to the point where you can use them without much conscious thought. This level of proficiency is called automaticity because you use the skill or process automatically. In fact, you must learn many skills and processes to the level of automaticity if they are to be truly useful."
Rose & Meyer, 2002, 120	"To achieve complex strategic goals like playing tennis, driving a car, or writing a research paper, a learner must automatize, or over-learn, the individual steps in the process until each is automatic. Only when the subcomponents come automatically can a tennis player concentrate on game strategy, a driver concentrate on destination and route, and a student concentrate on the style and clarity of the research paper. This requires extensive practice."
Rose & Meyer, 2002, 120	"Electronic media are ideal for providing scaffolds in the context of learning."
Marzano, Pickering, & Pollock, 2001, 140	"One highly generalizable research finding relative to skill learning is that skills must be learned at a level at which they require little or no conscious thought. Technically, this is referred to as learning a skill to the level of automaticity. . . . To do this, students must engage in practice that gradually becomes distributed, as opposed to massed."
Karp & Howell, 2004, 122	"The key to successful practice is neither the amount of time spent on the skill in one sitting nor the use of time-pressured tests. Successful practice depends on repeated interactions with mathematics content, in small doses, throughout the day and week as the opportunity arises. Students with memory-related difficulties must continue to practice a new skill beyond the point of just achieving correct responses. The skill should be repeated periodically after some time passes to help lock information into long-term memory."
Karp & Howell, 2004, 122	"Over time, fluency-building practice with concepts helps students have the facility they need to solve problems and answer mathematical questions (Johnson and Layng, 1994)."

Researcher(s)	Findings/Conclusions
Ontario Ministry of Education, 2005, 61	"Students with special needs benefit from cumulative review of important concepts and skills. Cumulative review of previously mastered content promotes retention. Early in the learning of a new skill, children are error-prone, not very fluent, and inconsistent in their application of skills to new situations. Children with special needs, in particular, can be more error-prone and less fluent or consistent for longer periods of time than their classmates. Hence, these children may need more opportunities to practice their skills and to review prior learning."
McEwan, 2000, 74	"Overlearning and repeated exposure leads to automaticity, just as practice with phrases and patterns leads to automaticity in reading words (Hall & Moats, 1998, 140). The same principle applies in mathematics learning. Automaticity and fluency free up working memory so that students can concentrate on interpretation and metacognition, whether the task is playing a sonata, reading a book, or solving a problem."
National Research Council, 2001, 193	"Practicing single-digit calculations is essential for developing fluency with them. This practice can occur in many different contexts, including solving word problems. Drill alone does not develop mastery of single-digit combinations. Practice that follows substantial initial experiences that support understanding and emphasize 'thinking strategies' has been shown to improve student achievement with single-digit calculations."
National Research Council, 2001, 193	"It is helpful for some practice to be targeted at recent learning. After students discuss a new procedure, they can benefit from practicing it. . . . It is also helpful for some practice to be cumulative, occurring well after initial learning and reviewing the more advanced procedures that have been learned."
National Research Council, 2001, 351	"The role of practice in mathematics, as in sports or music, is to be able to execute procedures automatically without conscious thought. That is, a procedure is practiced over and over until so-called *automaticity* is attained."
National Research Council, 2001, 351	"The availability of calculators and computers raises the question of which mathematical procedures today need to be practiced to the point of automatization. Single-digit whole number addition, subtraction, multiplication, and division certainly need to be automatic, since they are used in almost all other numerical procedures."

***MLS* Application.** *MLS* includes practice/repetition exercises in 22 of its 24 tasks (see Table 70). This strategy is necessary for the embedding of concepts (including vocabulary) into long-term memory, including the development of proficient problem-solving at the abstract level of concept development. Practice/repetition is also a dominant strategy in developing automaticity and accurate, rapid fact fluency.

Chunking/Clustering

Chunking or clustering bits of information into some meaningful pattern is a useful procedure to allow a person to hold more information in short-term memory than is ordinarily possible. Chunking/clustering is a useful strategy in all of *MLS's* fact fluency tasks. The National Research Council (1999) noted the following:

> Perhaps the most pervasive strategy used to improve memory performance is clustering: organizing disparate pieces of information into meaningful units. Clustering is a strategy that depends on organizing knowledge (p. 84).

There is a significant body of cognitive psychology research verifying the efficacy of this strategy. A sampling of findings is provided in Table 76.

Table 76: Research Findings on Chunking/Clustering

Researcher(s)	Findings/Conclusions
Erlauer, 2003, 56	"Knowing how the brain chunks and categorizes information is useful to teachers in helping students connect new information to prior knowledge. For instance, demonstrating how the new skill of multiplication is related to the previously learned concept of addition can make it easier for the students' brains to make connections and learn the new concept. An important thing for teachers to keep in mind is that one student's brain may chunk or categorize information differently from another student's brain."
National Research Council, 1999, 84-85	"Known as the chunking effect, this memory strategy improves the performance of children, as well as adults."
McGuinness, 1997, 251	"The human brain is particularly adept at storing recurring patterns, and very inefficient at remembering randomness."
Marzano, Pickering, & Pollock, 2001, 15	"Presenting students with explicit guidance in identifying similarities and differences enhances students' understanding of and ability to use knowledge. Probably the most straightforward way to help students identify similarities and differences between topics is to simply present these similarities and differences to them. In fact, a great deal of research attests to the effectiveness of this rather direct approach."
Wolfe, 2001, 99	"Working memory is indeed limited. Still, before we become too discouraged with its space limitations, we need to realize that these limitations can be circumvented somewhat by the ability to 'chunk' information. In discussing the number of items that one can hold in immediate memory, Miller noted that the items did not have to be single bits but could be chunks of information. A chunk is defined as any meaningful unit of information."
Bruer, 1993, 63	"Clustering helps us remember things by exploiting the schema structure of long-term memory; we remember the words by associating them with the appropriate schema."
McGilly, 1995, 5	". . . knowledge can be organized in large, interconnected bodies, where pieces of knowledge are conceptually linked to other pieces . . . . The critical difference is not the amount of information, but how the information is organized."
Sharron & Coulter, 1994, 140	". . . comparison is one of the basic building blocks of cognition and of abstract systematic thought."
Marzano, Pickering, & Pollock, 2001, 15	"Presenting students with explicit guidance in identifying similarities and differences enhances students' understanding of and ability to use knowledge."
Caine & Caine, 1991, 7	"The brain is designed to be a pattern detector."

***MLS* Application.** Chunkling/clustering is used in all the fact fluency tasks in *MLS*. (See Table 70.)

Engaged Time-on-Task

The scientific research on the importance of student engagement and time-on-task is abundant. The Alliance for Curriculum Reform (1995) documented more than 130 studies that "support the obvious idea that the more students study, other things being equal, the more they learn." They added that "It is one of the most consistent findings in educational research, if not all psychological and social research." But there is a caution in interpreting the findings, they said, "Time alone, however, does not suffice. Learning activities should reflect educational goals" (p. 11). Mercer and Mercer (2005) stated in their research synthesis the following:

> The finding that academic learning time is related positively to more student learning is consistent in the research for both general education students and students with learning problems. To foster a positive and productive learning environment, students should spend as much time as possible engaged in meaningful academic tasks (p. 34).

Gettinger (1991) found (as quoted in Mercer and Mercer, 2005) that

> . . . students with learning disabilities required significantly more time to achieve mastery on a reading comprehension task than students without learning disabilities. In essence, students with learning problems need ample time for learning, high rates of success, and strategies on how to learn and retain relevant information (p. 34).

These findings emphasize time-on-task and active engagement as being critical for students with learning difficulties/disabilities. Interestingly, this conclusion is linked with the importance of two other research-based strategies in *MLS*: high rates of success (see Motivation in Chapter VII) and strategies to learn and retain relevant information (see Multi-sensory Processing and other strategies discussed in this chapter and in Chapter V). Additional research findings on this topic are provided in Table 77.

Table 77: Research Findings on Engaged Time-on-Task

Researcher(s)	Findings/Conclusions
Cawelti, 1995, 102	"As might be expected, there is also a positive relationship between total time allocated to mathematics and general mathematics achievement."
Cawelti, 1999, 118	"The strong relationship between OTL [opportunity to learn] and student performance in mathematics has been documented in many research studies."
Snow, Burns, & Griffin, 129	"Classroom practices in ineffective schools (regardless of community SES) were characterized by significantly lower rates of student time on task, less teacher presentation of new material, lower rates of teacher communication of high academic expectations, fewer instances of positive reinforcement, more classroom interruptions, more discipline problems, and a classroom ambiance generally rated as less friendly."
Snow, Burns, & Griffin, 129	"Time on task is a good predictor of achievement gains."
US Dept. of Ed., 1986, 34	"How much time students are actively engaged in learning contributes strongly to their achievement. The amount of time available for learning is determined by the instructional and management skills of the teacher and the priorities set by the school administration."
Taylor, Pearson, Clark, & Walpole, 2000, 158	"As has been found in the research on effective teachers. . ., the most accomplished teachers in this study managed, on average, to engage virtually all of their students in the work of the classroom."
Alliance for Curriculum Reform, 1995, 11	"More than 130 studies support the obvious idea that the more students study, other things being equal, the more they learn. It is one of the most consistent findings in educational research, if not all psychological and social research. Time alone, however, does not suffice. Learning activities should reflect educational goals. This alignment or coordination of means with goals can be called 'curricular focus.'"
Alvermann, 2001, 7	". . . the level of student engagement (including its sustainability over time) is the mediating factor, or avenue, through which classroom instruction influences student outcomes."

Researcher(s)	Findings/Conclusions
Levin & Long, 1981, 2	". . . the 1978 report of the National Academy of Education stressed that 'the answer to the question of how schools can improve educational attainment lies in spending more time on those attainments we value. There is a striking convergence of evidence that points to the role of time-on-task—engaged time—in improving performance in school subject matters."
Levin & Long, 1981, 2	"Studies . . . generally demonstrate that, within a classroom, students who are more involved in their learning have higher achievement than students who are less involved in classroom learning activities."
Levin & Long, 1981, 5	"In this study, direct interaction with the learning materials and the teacher produced higher levels of achievement than merely listening to or watching the interaction."
Levin & Long, 1981, 6	"All the studies share one underlying principle. If instructional processes and procedures elicit student behavior relevant to the learning task, student involvement is likely to increase."
Gagne', R., 1985, 256	"The amount of time devoted to learning may be expected to affect the amount of learning. As a number of empirical studies have shown, the time students spend in actual learning (time on task) is a particularly potent variable in the determination of what is learned, as indicated by student proficiency in school subjects."
Mercer & Mercer, 2005, 34	"The finding that academic learning time is related positively to more student learning is consistent in the research for both general education students and students with learning problems (A. Reynolds, 1992). To foster a positive and productive learning environment, students should spend as much time as possible engaged in meaningful academic tasks."
Lock, 1996, 6	"Providing enough time for instruction is crucial."
Karp & Howell, Oct. 2004, 122	"To achieve deep learning, students with special needs require extended time per topic for adequate practice and application."
Ontario Ministry of Education, 2005, 61	"Instruction must be of sufficient duration and intensity to produce adequate learning and application to new situations. Students with special needs may require interventions of longer duration and intensity than other students in order to achieve mastery of both foundational and higher-level skills (e.g., Blachman et al., 2004). Research has shown that students with special needs may need more learning opportunities distributed over a longer time to make sufficient gains."
Smith, 2002, 127	"Learning can't be forced. Mathematics is not something that can be learned in a hurry, especially if the learner finds it difficult or confusing. It is absurd to expect everyone to learn mathematics at the same rate, yet the constraints of today's approaches to education often put students and teachers in a time-bind. Students who 'fall behind' can rarely catch up, and their task is always harder for them."

***MLS* Application.** CEI recommends that targeted students be assigned a minimum of 45 minutes a day in the *MLS* lab, five days per week, for as many weeks as are necessary for the student to achieve proficiency in the foundational concepts and in fact fluency. Students can extend class time by engaging in the web-accessible activity, *Digit's Widgets*. Many schools further extend instructional time through before- and/or after-school programs, inter-sessions, Saturday tutorials, and/or summer school. Engagement is promoted by interesting, hands-on activities; ongoing written and auditory feedback; high levels of success; and other motivational activities (see Chapter VII).

Assessment and Feedback

CEI, recognizing the integral role of assessment in improving student performance, provides a comprehensive assessment system for its *MLS* program. Mercer and Mercer (2005) advise practitioners as follows:

> To aid instructional programming, assessment should provide information in two areas. First, information is needed to help the teacher select *what* to teach the individual student. Second, information is needed to help the teacher determine *how* to teach the student (p. 84).

The intent of CEI in designing its assessment program is to do just that, and that is why *MLS* includes a variety of assessments. The various types and uses of *MLS* assessment instruments are delineated below:

Diagnostic Assessments

The *Learning Efficiency Test II (LET-II)*, developed by Dr. Raymond Webster (1998), is a third-party diagnostic assessment that provides information to the teacher/facilitator on the student's learning strengths and weaknesses and on learning preferences. Information is also provided on immediate, short-term, and long-term recall. Use of this information allows the teacher better to determine each individual student's learning needs and provides guidance in setting lesson parameters for maximal effectiveness. These data are, therefore, useful, in helping teachers determine *how* to teach each individual student.

Another third-party assessment, *Diagnostic Screening Test: Mathematics (DSTM)* by Gnagey and Gnagey (1982), provides teachers with data to assist them in determining *what* to teach each individual student. The *DSTM* is a diagnostic instrument in that it identifies content/skill strengths and weaknesses.

A third instrument that assists teachers in diagnosing student needs is the *MLS Placement Test*, a criterion-referenced assessment designed by CEI that helps the teacher determine which unit, lesson, and phase to place the student in for beginning instruction.

These three assessments satisfy what Sherman, Richardson, and Yard (2005) say is required as a first step in planning for instruction needs: "to conduct a current status assessment." They continue: "The value of assessment, in general, is that it leads to an overall perception of the functional abilities of a learner's strengths and areas of concern. Data collected for a Data Analysis Sheet informs instruction and prescribes a more accessible environment to influence future learning" (p. 9). Sousa (2001) outlines diagnostic assessment similarly: "The teacher's first task is to determine the students' level of cognitive awareness and the strategies each brings to the mathematics task. . . . Knowing the levels of the students' cognitive awareness and prerequisite skills will give the teacher valuable information for selecting and introducing new concepts and skills" (p. 151).

Dynamic (Formative) Assessments
A number of both the concept development and the fact fluency "tasks" designed for the *MLS* program (such as Math Magic; Fact Match; See, Hear, and Respond, and Echo) are, in actuality, embedded dynamic assessments—assessments that help the teacher/facilitator decide where the student is in learning the lesson's content or skill.

The ongoing engagement of the teacher/facilitator in monitoring student performance and progress is a signature component of the *MLS* program design. The teacher/facilitator combines his or her informal observations, as well as observations during student recitation, with the objective data provided in progress reports to determine next steps for instruction for each individual student.

Daily computer-generated reports also provide a record of each student's performance on the day's lessons for additional information and analysis. Teachers/facilitators use the data to make determinations about the next appropriate lesson and the parameters that should be set.

Mastery Assessments
Mastery lessons are also built into the *MLS* program so that the teacher/facilitator can determine whether the student achieved a high level of mastery (at least 80%). If not, the student is automatically assigned to a lesson recycle until an acceptable level is achieved. The last lesson of each lesson phase tests students' retention of the concept for that phase. Mastery lessons in the Concept Development strand are included in the following tasks: Solve; Word Problems Solve; See, Hear, and Respond; Hear and Respond; and See and Respond.

Summative Assessment
The post-test that is available in the *Diagnostic Screening Test for Mathematics (DSTM)* serves as a summative assessment for *MLS*. It provides grade-equivalent scores for both the pre- and post-test so that "value-added" can be calculated—that is, the growth achieved by an individual student, a class, or all the students served in the lab. Post-test scores are good indicators, as well, of student performance on the state criterion-referenced tests mandated under *NCLB*.

Relevant research on the importance of various kinds of assessments are cited in Table 78:

Table 78: Research Findings on Assessment

Researcher(s)	Findings/Conclusions
Ontario Ministry of Education, 2005, 75	"Assessment should be planned to focus on important conceptual and procedural understandings and should be linked with instruction. . . . A variety of assessment strategies will allow teachers to gain insight into what all students know and can do."
Alliance for Curriculum Reform, 1995, 83	"Assessment that focuses on what is being taught in a school's curriculum and on the modes of instruction used in the curriculum promotes learners' growth toward curricular goals."
Mercer & Mercer, 2005, 84	"Information for determining what and how to teach an individual is gathered by both formal and informal evaluation procedures."

Researcher(s)	Findings/Conclusions
Chinn, 1992, 24	"It is of importance for teachers to recognize that a test does not always test what it sets out to test. A failure on a mathematics problem may be due to a cause which does not have anything to do with actual mathematical skill."
Wolfe, 2005, 1	"Teaching without assessing is like driving with your eyes closed. Knowing when to stop and when to proceed, noticing warning signs, and avoiding obstacles are all key components in successful teaching and safe driving. Everyone is aware of the importance of real time feedback while driving, but not everyone understands the importance of real-time assessment in instruction."
Dixon-Krauss, 1996, 138	"An underlying premise of this movement of empowerment is that not only should diagnosis be a blueprint for instruction based on looking for strengths, it should also involve a shift toward looking for the cause of the problem in the social and educational context, not within the student. In other words, examiners are not just asking what is wrong with the child, but also what is wrong with the child's instruction."
Sherman, Richardson, & Yard, 2005, 1	"The child must be the focus of any pedagogical decision being made because a learner's cognitive, emotional, and physical needs vary widely and have great impact on achievement. For learners to succeed, teachers must assess students' individual abilities and characteristics and choose appropriate and effective instructional strategies accordingly."
Wolfe, 2005, 1	"Research shows that the use of diagnostic and formative assessments—assessments occurring before and during instruction—has a positive effect on student achievement. This positive effect is documented by externally mandated assessments, as well as other measures of student achievement. Not only is achievement improved overall, but the difference in achievement between high and low achievers is narrowed because formative assessment helps low achievers even more than other students."
Whitehurst, n.d., 7	"We know that at the classroom level, frequent assessment is useful, particularly when teachers are given help on what they should do for children who aren't performing well."
Levine, 2002, 210	"The great baseball catcher Yogi Berra has been quoted as saying, 'You can observe a lot by watching.' Teachers have nearly exclusive access to what I call the observable phenomena, the windows that offer an unobstructed view into a child's learning mind."
Levine, 2002, 311	"Observable phenomena provide insights that are unavailable on the standardized achievement or diagnostic tests commonly used in schools and clinics . . . . A sizable number of the dysfunctions described in this book are not detectable on any test. But we know they are there because we can see them."
Dowker, 2004, 19	"Observation by the teacher provides the opportunity to discover individual strengths and working patterns during school mathematics lessons, and to ask the children questions about their written work which may lead to reflection and reconsideration."
Erlauer, 2003, 117	"Informal assessment is as simple as watching and listening to students. Day-to-day classroom observations and conversations with students yield a tremendous amount of information about what individuals are understanding, to what level they are applying new concepts, how the class is performing as a whole, what interests are held by the students, and how students work with others in the class."
US Dept. of Ed., 1986, 43	"Frequent and systematic monitoring of students' progress helps students, parents, teachers, administrators, and policymakers identify strengths and weaknesses in learning and instruction."
Levin-Epstein, n.d., 2	". . . even more useful than taking a single, end-of-year snapshot of student achievement is the practice of continually assessing students throughout the school year, a practice made easier by the latest generation of software."
Donovan & Bransford, 2005, 16	"Formative assessments—ongoing assessments designed to make students' thinking visible to both teachers and students—are essential. Assessments are a central feature of both a learner-centered and a knowledge-centered classroom."

Researcher(s)	Findings/Conclusions
Dixon-Krauss, 1996, 125	"The key feature of the dynamic approach [to assessment] is that it links assessment with instruction because it occurs during instruction rather than after the fact. Dynamic assessment provides the teacher with different types of information than static assessment, and it requires different methods for obtaining and analyzing this information."
Dixon-Krauss, 1996, 126	"The most important feature of dynamic assessment is that the type of information it provides can be used by teachers to address problems, issues, and concerns in classroom instruction."
Dixon-Krauss, 1996, 129	". . . dynamic assessment provides information on the amount and type of help students need to perform the tasks."
McEwan, 2000, 56	"Obviously, if a teacher wants to make on-the-spot changes in instructional methodologies or determine if reteaching is necessary, formative testing is essential. Summative testing can tell you precisely what is wrong, but only formative, curriculum-based assessment can tell you precisely what is wrong."
Sherman, Richardson, & Yard, 2005, 11	"Mathematics instruction should focus on all factors that affect learning, while building on students' mathematical strengths and recognizing students' error patterns."
Chinn, 1992, 25	"It is another limitation of criterion-referenced tests that they do not look at the possible existence of error patterns. . . . It is important to examine the child's wrong answers, since this is where the teacher can determine if concepts have been misunderstood or algorithms not mastered; and it is then possible to start on the process of correction."
Chinn, 1992, 25	"In general, if after giving a pupil a test the teacher considers only the score and not the types of errors she is losing valuable information. Tests may be failed for many different reasons; and if two pupils both fail a particular item, it by no means follows that they are at the same level of understanding."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 133	"It's easy for errors to creep into a skill when students are first learning it. Consequently, one aspect of shaping skills or processes—procedural knowledge—is to point out errors and pitfalls to students."
Dixon-Krauss, 1996, 125	"Vygotsky believed that educational assessment . . . include measuring students' potential development or what they are in the process of learning. He described the zone of proximal development as encompassing the discrepancy between a student's actual level of development and the higher level she can reach when her performance is supported by assistance during collaboration with an adult or capable peer."
Levine & Swartz, 6	"Multiple forms and sources of assessment information should be gathered. Evidence should derive from direct observations by teachers and parents, interviews with the child, careful analyses of work samples, as well as formal testing procedures."
Rose & Meyer, 2002, 83	"Good pedagogy also includes effective and ongoing assessment, not only to measure a student's progress, but also to adjust instruction and to evaluate the effectiveness of methods and materials. Ongoing assessment enables teachers to ensure that the goals they have set and the methods and materials they are using continue to support students' progress."
Jones, Wilson, & Bhojwani, 1997, 158	"Unless instructional assessments are conducted frequently and with reference to the students' performance on specific tasks, it will not be possible to use the information to make rational decisions for improving instruction. To an increasing extent, educators have come to the conclusion that traditional standardized achievement testing does not provide adequate information for solving instructional problems, and that a greater emphasis should be placed on data from functional or curriculum based measurements. . . ."
US Dept. of Ed., Feb. 6, 2002, 1	". . . when kids and/or their teachers get ongoing information, every two weeks, every four weeks, of where they are in math in terms of either the state standards or some framework, it invariably enhances performance." [Russell Gersten, University of Oregon]

Researcher(s)	Findings/Conclusions
Ontario Ministry of Education, 2005, 75	"Research indicates that making assessment an integral part of classroom practice is associated with improved student learning. Black and William (1998) reviewed about 250 research studies and concluded that the learning of students, including low achievers, is generally enhanced in classrooms where teachers include attention to formative assessment in making judgments about teaching and learning (National Council of Teachers of Mathematics, 2000, 1)."
Dowker, 2004, 19	"Standardized tests can be used to provide guidance about a pupil's mathematical achievement level relative to others. However, they do not describe individual strengths and weaknesses. . . . The problem is reduced, but not totally eliminated, by using tests which differentiate between different components of mathematics. . . ."
Stumbo & Lusi, 2005, 7	"While summative assessments can be helpful as education leaders make decisions about curriculum, school organizations, and staffing assignments, more helpful are assessments designed to be 'formative' in nature. Formative assessments are diagnostic tests that give teachers rapid feedback on the individual progress of students and immediately inform instruction. Formative assessments tend to be given at the classroom level and are key tools that teachers use to make decisions about their day-to-day lesson plans."
Wolfe, 2005, 1	"Summative assessments, occurring at the end of instruction, are less helpful. . . . summative assessments occur too late to assist teachers in making real-time adjustments to instruction or help students make adjustments in their learning strategies."

Corrective Feedback

Feedback to students on their progress is an important feature of *MLS* instruction. Marzano (1998) found in his research synthesis of multiple studies that the overall effect size for corrective feedback strategies was .74, "indicating a percentile gain of 33 points" (p. 94). Students receive auditory feedback from the computer tutor, Digit, as they work through the lessons. Further, this feedback is differentiated on each item when they struggle. They receive teacher feedback as a part of various practice and assessment activities, as well as a part of the teacher's observations of their progress. They provide their own feedback through various self-assessment exercises. Daily progress printouts provide written and verbal feedback for the student. Results of more formal assessments are also provided as feedback, along with interpretations. Table 79 includes the research findings on the critical importance of corrective feedback, especially for struggling students. Such feedback serves to make practice perfect, as well as to motivate students to continue their efforts toward mastery.

Table 79: Research Findings on Assessment Feedback

Researcher(s)	Findings/Conclusions
Ontario Ministry of Education, 2005, 116	"Immediate feedback, such as congratulations for the correct answer or response, increases student learning and a sense of competence."
Lochy, Domahs, & Delazer, 2005, 472	"The principle of errorless learning. . ., applied in rehabilitation by providing instant feedback or by learning problems together with their answers in order to prevent erroneous associations from being established or strengthened, converges with associative learning theories."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 185	"Some education researchers believe providing feedback is the most powerful thing that a classroom teacher can do to enhance student achievement. After considering the findings from almost 8,000 studies, researcher John Hattie (1992) commented: 'The most powerful single modification that enhances achievement is feedback. The simplest prescription for improving education must be 'dollops of feedback.'"

Researcher(s)	Findings/Conclusions
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 187	"Research suggests that certain practices render feedback on classroom assessments most effective: • Give timely feedback. Stated negatively, if too much time (e.g., one week or more) elapses from the time students take a test until they receive feedback on it, what they learn from that assessment will be minimal. • Explain what was correct and what was incorrect on an assessment. . . . An assessment is much more likely to have a positive influence on students' learning if time is set aside to make sure students understand what they did well and what they did not do well."
Cotton, 2000, 24	"Some investigations have found instructional reinforcement to have the most powerful positive effect on student achievement of all indicators of instructional quality. And research in general supports the practice of letting students know how they are doing and corroborating their accurate responses—in classroom recitations, on homework assignments, as part of instructional software programs, and so forth."
Gagne', R., 1985, 254	"Following the performance which shows that learning has occurred, there must be a communication to the learner about the correctness and the degree of correctness of the performance. This event may be carried out in a number of different ways . . . . Some valuable kinds of feedback can be provided in computer-aided instruction."
Mercer & Mercer, 2005, 433	"Monitoring the math progress of students with learning problems and giving feedback . . . have yielded excellent results regarding student achievement. Gersten, Carnine, and Woodward (1987) report that teachers who provide immediate corrective feedback on errors produce high student achievement. Moreover, Robinson, DePascale, and Roberts (1989) found that feedback helped students with learning disabilities complete more problems and improved accuracy from 73 percent to 94 percent. They stress the importance of feedback."
Marzano, Pickering, & Pollock, 2001, 96	"One of the most generalizable strategies a teacher can use is to provide students with feedback relative to how well they are doing. In fact, feedback seems to work well in so many situations that it led researcher John Hattie (1992) to make the following comment after analyzing almost 8,000 studies: 'The most powerful single modification that enhances achievement is feedback. The simplest prescription for improving education must be dollops of feedback."
Marzano, Pickering, & Pollock, 2001, 96	"Feedback should be 'corrective' in nature. . . . This means that it provides students with an explanation of what they are doing that is correct and what they are doing that is not correct."
Marzano, Pickering, & Pollock, 2001, 97	"Feedback should be timely. . . . Feedback given immediately after a test-like situation is best. In general, the more delay that occurs in giving feedback, the less improvement there is in achievement."
Marzano, Pickering, & Pollock, 2001, 98	"Feedback should be specific to a criterion. . . . A different way of saying this is that feedback should be criterion-referenced, as opposed to norm-referenced."
Levin & Long, 1981, 18	"This study demonstrates that students in the feedback and corrective group learned more than the students who were deprived of feedback and correction."
Levin & Long, 1981, 19	"According to Bloom, under more ideal conditions of feedback and correctives, as many as 90 percent of the students can achieve the same performance level reached by the top 20 percent of the students who are deprived of feedback and corrective opportunities."
Levin & Long, 1981, 19	"Feedback and corrective procedures related to an appropriate standard help most students, regardless of intelligence or aptitude, to attain the desired educational goals."
Marzano, Pickering, & Pollock, 2001, 96	"Feedback should be 'corrective' in nature."
Marzano, Pickering, & Pollock, 2001, 97	"Feedback should be timely."
Marzano, Pickering, & Pollock, 2001, 98	"Feedback should be specific to a criterion."

Researcher(s)	Findings/Conclusions
Levin & Long 1981, 18	"At the end of the eight learning units, students in both groups took a final achievement test and a retention test. Wentling's results, when measured on the final achievement test and the retention test, indicated significant advantages in favor of students who were provided with feedback and corrective procedures in relation to a standard."
Erlauer, 2003, 126	"In the classroom, it is far easier for a student to learn of his or her mistake, immediately learn how to correct it, practice it more, and reap the rewards of success on the next trial. Without immediate feedback, that student may have continued to practice incorrectly, imbedding incorrect knowledge into the brain. It is demoralizing for a student to learn, after weeks of practicing a skill, that he or she has been doing it incorrectly all that time. Not only does the student need to start over from scratch in relearning the skill, but first the brain has to work hard to start over from scratch in relearning the skill."
Erlauer, 2003, 126	"Eric Jensen (1998) states that, as a general rule of thumb, students should receive some form of feedback at least once every half-hour during lessons. Jensen (2000) also contends that the most effective feedback is prompt, specific, multimodal, and comes from differing people including oneself."
Erlauer, 2003, 128	"Feedback must be very specific to assist the learner in knowing exactly what to keep doing and what to change."
Whitehurst, n.d., 2	"A number of studies have examined the value of feedback into the classroom that results from frequent assessment of students. One approach that has been studied provided teachers with weekly performance graphs on individual students. Children in classrooms in which students received this feedback performed at significantly higher levels than students in classrooms in which the performance graphs were not available. Other studies have shown that student performance is enhanced still further when instructional materials tailored to each student's progress are provided to teachers along with performance graphs."
Erlauer, 2003, 123	"Through feedback, we learn cause and effect."
Erlauer, 2003, 123	"When prompt feedback is received, the learner can either make a quick correction and move on or proceed with the confidence that he or she is on the right path. One of the most difficult things for a brain to do is to unlearn deeply embedded knowledge or skills."
Marzano, 1998, 95	"The techniques that activated beliefs about self attributes had an effect size of .74, indicating a percentile gain of 27 points. These techniques primarily utilized praise as the vehicle for enhancing students' beliefs about themselves relative to accomplishing specific academic tasks. It is important to note that the use of praise as an instructional technique was both focused and accurate. That is, teachers praised students on their accomplishments relative to specific academic tasks as opposed to providing students with generalized praise."
Marzano, 1998, 96	"Indeed these findings indicate that praise, when effectively used, can generate a percentile gain of 27 points."
Marzano, 1998, 127	"The simple act of setting clear instructional goals, then, produces significant gains in student learning. Added to this, providing feedback to students regarding the strategies they have selected to complete a task and the effectiveness with which they are utilizing those strategies produces an overall effect size of .72, indicating a percentile gain of 26 points."
Marzano, 1998, 128	". . . the most powerful single moderator that enhances achievement is feedback. The simplest prescription for improving education must be 'dollops of feedback' (Hattie, 1992, p. 9)."

Corrective feedback in the *MLS* program is one component of the motivation emphasis that is discussed in some detail in Chapter VII, and it also responds to some of the motivational issues outlined in Chapters II-III, especially those relating to self-efficacy.

Informed Instruction (Data-driven Decision-making)

MLS teachers/facilitators are trained to make use of all available assessment data to make initial placement decisions of students into the program and then to adjust and adapt instruction based on the data from observations, ongoing assessments, performance on mastery lessons, and the reports that are generated. This process results in what is termed "informed instruction" or "data-driven decision-making" as it pertains to instruction, and it is found through scientific studies to be highly effective in teaching struggling learners. One of the major roles of the *MLS* teacher/facilitator is to continually monitor individual student progress and to make appropriate adjustments to the assignments so that every student masters the content and skills necessary.

Samway and McKeon (1999) outlined the following uses of assessment:

- Determine what students can do;
- Establish students' learning strategies, skills, and processes;
- Make instructional decisions; and
- Decide how to flexibly group students for instruction (p. 62).

A "learner-centered assessment program," again according to Samway and McKeon (1999), has the following features:

- Is ongoing and continuous
- Determines what students can do linguistically and academically
- Identifies students' learning strategies, skills, and processes
- Facilitates sound instructional decision making
- Assists in grouping students for instruction
- Addresses all language modalities (listening, speaking, reading, and writing)
- Incorporates student self-assessment
- Invites parent assessment of students (p. 62).

In an *MLS* lab, the teacher/facilitator sets lesson parameters and perhaps adjusts the level of instruction for individualized/differentiated instruction instead of using data for grouping students. One-to-one tutoring is the optimal grouping for students, as was discussed in Chapter V and under "Individualization/Differentiation" in this chapter. *MLS'* diagnostic, formative (or dynamic), and summative assessments create the kind of "learner-centered assessment program" that researchers advocate. Table 80 includes compelling research findings related to the assessment process and use of data to inform instruction.

Table 80: Research Findings on Informed Instruction (Data-driven Decision-making)

Researcher(s)	Findings/Conclusions
Berliner & Casanova, 1993, 90	"Some teachers have been found to possess the kind of extraordinary knowledge of their students that allows them to predict quite accurately which items on a test each of the students can do and which they cannot. In one of my own research studies we found a strong positive correlation between teachers' ability to predict their students' scores and the actual achievement of those students. The ones who knew more about their students' abilities were the ones whose students achieved more."

Researcher(s)	Findings/Conclusions
Berliner & Casanova, 1993, 92	"Regular assessment will help teachers to understand what their students really do know, so that those teachers can adapt instruction accordingly."
Walberg & Paik, n.d., 17	"More than fifty studies show that careful sequencing, monitoring, and control of the learning process raise the learning rate. Pre-testing helps determine what should be studied; this allows the teacher to avoid assigning material that has already been mastered or for which the student does not yet have the prerequisite skills. Ensuring that students achieve mastery of initial steps in the sequence helps ensure that they will make satisfactory progress in subsequent, more advanced steps. Frequent assessment of progress informs teachers and students when additional time and corrective remedies are needed."
Erlauer, 2003, 117	"To make these informal observations and conversations valuable for the teacher in driving instruction, he or she must take the time to reflect on what is seen and heard, contemplate what it all means, and take the next steps based on his or her judgments. To make informal assessment valuable for the students, teachers must talk over their observations and resulting insights with the children."
Levin-Epstein, n.d., 1	"Today's software solutions have the capability to provide curriculum tailored to every student's strengths and weaknesses, allow teachers to monitor student performance in real time, administer assessments and adjust instruction in line with the results, interface with gradebooks, send reports to parents—and more."
Safer & Fleischman, 2005, 81	"Research has demonstrated that when teachers use student progress monitoring, students learn more, teacher decision-making improves, and students become more aware of their own performance. A significant body of research conducted over the past 30 years has shown this method to be a reliable and valid predictor of subsequent performance on a variety of outcome measures, and thus useful for a wide range of instructional decisions."
Safer & Fleischman, 2005, 81	"Although student progress monitoring . . . was initially developed to assess the growth in basic skills of special education students, specific research has validated the predictive use of this method in early literacy programs and in the identification of general education students at risk for academic failure."
Safer & Fleischman, 2005, 83	". . . many teachers find this strategy worth the effort because it provides a powerful tool that can help them adjust instruction to ensure that all students reach high standards."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 187	"Classroom assessments can be one primary vehicle that teachers use to give students feedback. Ideally, scores from assessments should be used to determine next steps students must take to improve their learning."
Rose & Meyer, 2002, 121	"Providing ongoing, relevant feedback is critical when teaching skills. Learners need to know if they are practicing effectively, and if not, which aspects of the practice process they need to change."
Rose & Meyer, 2002, 121	"Software tools and digital networks can be an excellent source of ongoing feedback, particularly if students are shown how to take advantage of everything these tools offer."
Rose & Meyer, 2002, 154	"Embedded, flexible, ongoing assessments have the potential to resolve many of the problems with standardized, paper-and-pencil tests, particularly as tools for guiding teaching."
Mercer & Mercer, 2005, 88	"Teachers of students with learning problems encounter variable performances from many of their students. Teachers must know, however, whether a student is making adequate progress toward specified instructional objectives so that they can modify instructional procedures. Evaluation must be frequent and provide information for making instructional decisions."
Mercer & Mercer, 2005, 91	"Data-based instruction has roots in applied behavior analysis, precision teaching, direct instruction, and criterion-referenced instruction. Many educators . . . concur that it holds much promise for both current and future teaching practice."

Researcher(s)	Findings/Conclusions
Mercer & Mercer, 2005, 105-106	"Highlights of research on data-based approaches include the following: Considerable evidence supports the positive association between data-based monitoring and student achievement gains . . . . In a meta-analysis of formative evaluations, Fuchs and Fuchs found that data-based programs that monitored student progress and evaluated instruction systematically produced .7 standard deviation higher achievement than nonmonitored instruction. This represents a gain of 26 percentage points. Moreover, White reports outstanding gains for students involved in precision teaching programs."
Mercer & Mercer, 2005, 438-440	"Thornton and Toohey (1985) report substantial literature that indicates that modifying the sequence and presentation of learning tasks can improve basic fact learning among students with learning problems."
Wolfe, M. J., 2005, 1	"Diagnostic assessments, which occur before instruction, make teachers aware of their students' level of development. This information helps teachers create lessons and learning opportunities that build on their students' understandings and address individual students' needs. Real-time formative assessments allow teachers to continually monitor their students' progress. They can discover any difficulties their students are facing, and they can decide what assistance to provide."
Wolfe, M. J., 2005, 1	"Students, too, benefit from receiving the feedback provided by formative assessments. They learn what works and what doesn't; they can pursue successful strategies while rejecting unsuccessful ones. Feedback in the form of comments (but not grades or scores) helps students focus on their own learning instead of focusing on acquiring gold stars or collecting grades."
McEwan, 2000, 63	"Assess frequently enough so that the results can inform instruction, then be ready to change the game plan if necessary to achieve the goal."
Bohan, 2002, 15	"A critical component of the teaching and learning process that includes curriculum and instruction as partners, assessment should be used to guide and enhance instruction. With this broader view, assessment becomes a powerful and effective tool for making curricular and instructional decisions."
Popham, 2006, 82	"Classroom assessment for learning is a marvelous, cost-effective way of enhancing student learning. Solid research evidence confirms that it works, assessment experts endorse it, and teachers adore it."
Popham, 2006, 82	"In their 1998 article, Black and William drove home the significance of this assessment distinction by presenting a meta-analysis of previously reported empirical research regarding the effects of classroom assessment for learning. Their analysis indicated striking test-score improvements for students, not only on classroom assessments but on external examinations as well. Subsequent meta-analyses by other researchers have confirmed the idea that classroom assessment for learning can be a wonderful way of boosting students' scores on external achievement tests."
Popham, 2006, 82	"Unlike assessment of learning, which attempts to get a fix on what students know for the purposes of giving grades or evaluating schools, the array of test-like events in assessments *for* learning is always linked to the question 'What's next instructionally?'"

Self-assessment

According to Glasser (1990), educators should "Try to teach students this important lesson: The success or failure of our lives is greatly dependent on our willingness to judge the quality of what we do and then to improve it if we find it wanting" (p. 159). He stated that one of the two critical practices in a quality school is self-assessment, by staff and by students (p. 156). *MLS* overtly incorporates self-assessment in its Learn tasks in each sequence of concept lessons, and it encourages self-assessment in all performances. Numerous other researchers have also investigated the power of that process. A sample is included in Table 81.

Table 81: Research Findings on Self-assessment

Researcher(s)	Findings/Conclusions
National Research Council, 1999, 128	"Effective teachers also help students build skills of self-assessment. Such self-assessment is an important part of the metacognitive approach to instruction."
Alliance for Curriculum Reform, 1995, 14	"In the 1980's, cognitive research on teaching sought ways to encourage self-monitoring, self-teaching, or 'meta-cognition' to foster achievement and independence. Skills are important, but the learner's monitoring and management of his or her own learning has primacy. This approach transfers part of the direct teaching functions of planning allocating time, and review to learners. Being aware of what goes on in one's mind during learning is a critical first step to effective independent learning."
Marzano, Pickering, & Pollock, 2001, 99	"Students can effectively provide some of their own feedback."
Donovan & Bransford, 2005, 11	"Appropriate kinds of self-monitoring and reflection have been demonstrated to support learning with understanding in a variety of areas. In one study, for example, students were directed to engage in self-explanation as they solved mathematics problems developed deeper conceptual understanding than did students who solved those same problems but did not engage in self-explanation."
Donovan & Bransford, 2005, 17	"A number of studies show that achievement improves when students are encouraged to assess their own contributions and work."
Fuson, Kalchman, & Bransford, 2005, 238	"Ideally, we want students to monitor the accuracy of their problem solving, just as we want them to monitor their understanding when reading about science, history, or literature."
Fuson, Kalchman, & Bransford, 2005, 238	"One way to monitor the accuracy of one's computation is to go back and recheck each of the steps. Another way is to estimate the answer and see whether there is a discrepancy between one's computations and the estimate. However, the ability to estimate requires the kind of knowledge that might be called 'number sense."
Mercer & Mercer, 2005, 61	"Self-correcting materials give the student immediate feedback without the teacher being present. Self-correcting materials are especially useful with students with learning problems who have a history of academic failure. It is important to reduce their failure experiences, particularly public failures. When the student makes a mistake with a self-correcting material, it is a private event—no one else knows. Only the student sees the error, and the error can be corrected immediately."

Summary

This chapter discussed the scientific research behind the most powerful of the instruction strategies used in *MLS*, beginning with its unique utilization of multi-sensory processing. Also discussed were individualization/differentiation, practice/repetition, chunking/clustering, and engaged time-on-task—both the scientific studies relating to their efficacy with struggling learners and the documentation of *MLS'* application of the practices.

Included in Chapter VI, as well, was a discussion of *MLS'* comprehensive assessment system, corrective feedback, informed instruction (using assessment data for instructional decision-making), and self-assessment.

Chapter VII includes information about the implementation features provided with *MLS*: the role of the lab teacher/facilitator, professional development and follow-up coaching, motivation, and parental involvement.

Chapter VII: Research Findings that Ground *MLS'* Implementation Support

"Adoption of an innovation is simple. It is implementation that takes the time and effort."
(Erlauer, 2003, 2-3)

Overview

Chapters II-III provided research findings and discussion of the learning difficulties and disabilities most likely to affect negatively student performance in mathematics—the reasons that students struggle on their journey toward proficiency. In Chapter IV, the discussion focused on the content and skills that are most critical in a mathematics intervention: concept development in the foundational knowledge and fact fluency. Lesson steps and models were the topics explored in Chapter V. Research on direct instruction, mastery learning, and one-on-one tutoring was presented, along with descriptions of how components of these are used in *MLS*. In addition, the research on the concrete-semiconcrete-abstract sequence and use of manipulatives in effective teaching of mathematics was presented, followed by a comprehensive discussion of the research on computer-assisted instruction and effective screen designs for struggling learners.
Chapter VI included the scientific evidence on several powerful instructional strategies followed by documentation of how these strategies are utilized in the *MLS* design. Multi-sensory processing, individualization/differentiation, practice/repetition, chunking/clustering, and engaged time on task have an abundance of research behind them and unequivocal positive findings relating to their efficacy with struggling learners of mathematics. Because assessment is so important in an instructional program, the comprehensive assessment system that is a part of *MLS* was included, followed by the research and discussion of how *MLS* incorporates corrective feedback, informed instruction, and self-assessment in its tasks.

Chapters IV through VI, therefore, include a thorough description of all the component parts of *MLS*—its content, its lesson models and delivery through computer-assisted instruction, its instructional strategies, and its assessment system. The scientific evidence is abundant that each component is appropriate and effective, especially for the struggling learners for whom the program was designed. But regardless of how well a program is designed or how attractive its packaging is, the results it delivers frequently hinge on the effectiveness of implementation. Marzano (2003) notes that "Sadly, many, if not most, interventions are not fully implemented. In fact, it is not uncommon for an intervention to be considered ineffective or marginally effective when, in fact, the intervention was improperly or only partially implemented" (p. 166). Chapter VII explores four critical implementation topics: the role of the lab teacher/facilitator; professional development and follow-up coaching; student motivation for success; and parental involvement. The scientific evidence for each topic is presented, along with documentation of how *MLS* incorporates the findings. The chapter concludes with a general discussion about the importance of implementation.

Role of the Lab Teacher/Facilitator

MLS is not just educational software. It was designed initially and continues to emphasize the role of the lab teacher/facilitator in effective instruction, monitoring progress, coaching and encouraging students, diagnosing needs, and adapting the program as required for student success. The *MLS* program's approximately two dozen tasks include individual student work on the computer, but also one-on-one recitation to the teacher, self-assessments, presenting work for teacher assessment, and deliberate transitions between activities to allow time for processing of new information and skills.

Successful implementations of *MLS* labs are invariably a result of engaged, reflective teacher/facilitators who never turn responsibility over to the computers, but who think continually about ways to move student learning forward and who are continuously adapting lesson levels and parameters, as well as supplementing the software instruction with whatever is needed to ensure an individual student's success. Exemplary labs, for example, include a wealth of other materials appropriate to the age and skill levels of the learners. They include, as well, professional dialogue between and among the lab's teacher/facilitator and other teachers of the students being served and with the instructional leader(s) of the school—and, importantly, dialogue with parents.

There is a great deal of research on the importance of teacher mediation to facilitate student learning. Lev Vygotsky, a Russian psychologist, who lived in the early years of the 20th century, has led the way in this area. Rodriguez and Bellanca (1996) relied upon his research in advocating that role for teachers in their book aimed at urban educators. They defined mediation as "a mutual interaction between the mediator. . . and the student." They continued, "The mediator purposefully directs the interaction toward a specific goal by focusing attention, selecting, framing, interpreting, and cuing the student on specific stimuli. . . . With such mediation the child develops the internal controls that enable him to learn how to learn" (14-15). Some of the mediation is, of course, done by Digit, the computer tutor, but the program's effectiveness will be lessened without the active engagement of a reflective, caring teacher.

In a grant-funded research study on teacher engagement, Louis and Smith (1996) described four types of teacher engagement that are inferred in the literature. The one that matches CEI's vision for an effective teacher/facilitator is the "engagement with students as unique, whole individuals rather than as 'empty vessels to be filled'." The definition continues as follows:

> Teachers demonstrate this type of engagement when they listen to students' ideas, get involved in students' personal as well as school lives, and make themselves available to students who need support or assistance. Other examples of teacher engagement with students are formal and informal coaching, sponsoring, mentoring, and counseling activities (p. 126).

Thus, the suggested job description for an *MLS* teacher/facilitator included by CEI in its *MLS Implementation Toolkit* delineates the following roles:

- Preparing the classroom and the students for the program.
- Encouraging and motivating students.
- Administering and scoring third-party assessments (*DSTM* and *LET-II*)
- Administering and scoring the *MLS Placement Test* to select the appropriate *MLS* lessons.
- Using the *MLS* software—the *CEI Learning Manager*, the *MLS Player*, and *CEI Evaluate*.
- Training students to use the *MLS* software.
- Monitoring students as they work through lessons.

- Checking for mastery and reviewing as needed.
- Documenting and analyzing student progress.
- Modifying lessons to challenge, but not overwhelm the students.
- Planning and conducting focused mathematics instruction that encourages transfer to the regular classroom.
- Safeguarding equipment, software, materials, and supplies.
- Administering and scoring the *DSTM* post-test and submitting pre- and post-test data for graphical analysis.
- Communicating student progress to the principal, to other teachers of assigned students, and to parents.
- Participating in CEI Facilitator Training and advanced professional workshops.

This job description reflects a great many of the principles of effective teaching established by Brophy, Hunter, Berliner, Stallings, Rosenshine, Shulman, and others during the 1970s and 1980s and which have been synthesized by Crawford, Bodine, and Hoglund (1993). The principles that are embedded in this job description follow:

- Effective teachers establish rapport with their students and provide a pleasant and orderly environment that is conducive to learning (p. 223).
- Effective teachers maximize time on task by using minimum class time for noninstructional routines (p. 224). (See discussion of engaged time-on-task in Chapter VI.)
- Effective teachers clearly define expected behavior (p. 224).
- Effective teachers plan carefully and thoroughly for instruction (p. 224).
- Effective teachers continually monitor learners' behavior to determine whether they are progressing toward the stated objective (p. 225). (See discussion of assessment in Chapter VI.)
- Effective teachers heed the results of their monitoring and adapt their instructional strategies accordingly (p. 225). (See discussion of informed instruction in Chapter VI.)
- Effective teachers require all learners to practice new learning while under direct teacher supervision (p. 226). (See discussion of practice/repetition in Chapter VI.)
- Effective teachers expect learners to practice skills without direct teacher supervision but only after guided practice has shown that the learners understand what is expected (p. 225). (See discussion of practice/repetition in Chapter VI.)

Similar research-based discussions of the importance of the teacher, including those facilitating *MLS* labs, are provided in Table 82:

Table 82: Research Findings on Teacher/Facilitator Engagement

Researcher(s)	Findings/Conclusions
Darling-Hammond & Falk, 1997, 193	"Recent research has found that students experience much greater success in school settings that are structured to create close, sustained relationships between students and teachers."

Researcher(s)	Findings/Conclusions
Snow, Barley, Lauer, Arens, Apthorp, Englert, & Akiba, 2005, 76	"The research suggests that the role of the teacher in the computer-assisted intervention is significant."
Rose & Meyer, 2002, 7-8	"UDL [Universal Design for Learning] is also compatible with the concepts of teachers as coaches or guides (O'Donnell, 1998), learning as process (Graves, 1983, 1990), cooperative learning (Johnson and Johnson, 1989, 1999; Marr, 1997, Wood et al., 1993) and demonstrating learning in a wide variety of media (Sizer, 1992, 1996)."
Dixon-Krauss, 1996, 9	"For Vygotsky cognitive development was due to the individual's social interactions within the environment."
Dixon-Krauss, 1996, 20	"Teacher mediation is more than modeling or demonstrating how to do something. While the teacher is interacting with the student, he continuously analyzes how the student thinks and what strategies the student uses to solve problems and construct meaning. From this analysis the teacher decides how much and what type of support to provide for his students."
Dixon-Krauss, 1996, 26	"The social dialogue that occurs during literacy interactions is a key factor in learning. The ultimate goal for a teacher of young children should be to provide the assistance, through social dialogue, that is necessary for children to move from other-regulated to self-regulated reading and writing."
Dixon-Krauss, 1996, 16	"The teacher's role in supporting learning within the zone of proximal development involves three key elements: (1) The teacher mediates or augments the child's learning. She provides support for the child through social interaction as they cooperatively build bridges of awareness. (2) The teacher's mediational role is flexible. What she says or does depends on feedback from the child while they are actually engaged in the learning activity. (3) The teacher focuses on the amount of support needed. Her support can range from very explicit directives to vague hints."
Taylor, Pearson, Clark & Walpole, 2000, 157	"Although different terms have been used to describe what we have called coaching (e.g., use of structuring comments, probing of incorrect responses, scaffolded instruction), others have found this type of 'on the fly' instruction to be a characteristic of effective teachers. Our most accomplished teachers exhibited a general preference for coaching over telling or recitation, whereas the least accomplished teachers engaged more commonly in telling. We did find the practice of coaching during reading to provide word recognition instruction to be characteristic of both the most effective schools and the most accomplished teachers."
IRA, 2000, 3	"Excellent reading teachers interact with individual children frequently in the course of their daily teaching activities. As they help children solve problems or practice new skills and strategies, they 'coach' or 'scaffold' children by providing help at strategic moments. They are skilled at observing children's performance and using informal interactions to call children's attention to important aspects of what they are learning and doing. They often help children with a difficult part of the task so that the children can move forward to complete the task successfully. It is important to note that such teaching is neither incidental or unsystematic. Excellent reading teachers know where their children are in reading development and they know the likely next steps. They help children take these steps by providing just the right amount of help at just the right time."
Mercer & Mercer, 2005, 36	"It is well known that students learn more when the school and classroom environments are positive and supportive (Christenson et al., 1989). . . . The teacher is the key individual who influences the tone of a classroom."
Marzano, Pickering, & Pollock, 2001, 3	". . . the individual classroom teacher has even more of an effect on student achievement than originally thought."

Researcher(s)	Findings/Conclusions
Jones, Wilson, & Bhojwani, 1997, 156	". . . good teachers manage instruction so that students (a) spend the major portion of instructional time actively engaged in learning, (b) work with high levels of success, and (c) proceed through the curriculum while acquiring increasingly more complex skills and important generalizations. Thus, good teaching is indicated by students' responses to instruction. Obtaining high levels of achievement requires effective management of instruction."
Ontario Ministry of Education, 2005, 71	"To effectively teach mathematics to a classroom of learners that includes students with special needs, teachers need to understand the following: • The teacher plays a critical role in student success in mathematics. • There are general principles of children's learning of mathematics. • The mathematics curriculum is developmental. • The 'big ideas' are key concepts of mathematics. • There is an important connection between procedural knowledge and conceptual understanding of mathematics. • The use of concrete materials is fundamental to learning and provides a means of representing concepts and student understanding. • The teaching and learning process involves ongoing assessment.
Ontario Ministry of Education, 2005, 72	"More than in any other subject area, students' progress in mathematics is closely linked to the teacher's knowledge about children's mathematical development and teacher preparation in the teaching of mathematics (Ginsburg, Klein, & Starkey, 1998).
Ontario Ministry of Education, 2005, 72	"The teacher is the child's most important role model for mathematics learning, and so it is crucial that he or she adopt a knowledgeable, enthusiastic, and positive attitude toward mathematics and its applications (Mercer & Mercer, 1998).

MLS software provides research-based content, lesson steps and models, and instructional strategies. In themselves and without further embellishments, they are powerfully effective in assisting students in learning mathematics foundations. But *MLS* as a total program is far more than software, and its power cannot be fully realized without the engagement of quality teachers/facilitators—another component that is also firmly supported by research studies.

Professional Development and Follow-up Coaching

"A major strength of CEI is," according to Melinda Mace, CEI's sales coordinator, "the quality and intensity of its professional development program." The sales process, itself, is in part professional development for all those educators seeking to learn not only about the program, but about the causes of mathematics failure and what the research finds that works in teaching struggling learners. Then, as soon as a sale closes, CEI staff conduct a meeting with the school principal or other instructional leader to begin planning for implementation. He or she receives a copy of the *MLS Implementation Toolkit*, which contains ideas for targeting students, setting up the lab, test administration, choosing a facilitator, what to look for in classroom observations, and a model school improvement plan.

Two optional professional development sessions are then offered. CEI staff will, upon request, conduct faculty/department/grade-level awareness sessions on the *MLS* program, what results they can expect, the information available to them from the assessment system, and ways that they can support students assigned to the lab. These sessions are designed also to build support for the program in the school. A second option is a training session for district-level and school-level technical staff on software deployment, network issues, trouble-shooting, technical support services, and related issues. This level of staff is introduced to CEI's technical support manager and the various resources available to them for assistance.

CEI provides an initial two-day training session for *MLS* teachers/facilitators. On-going coaching and follow-up are provided by certified teachers who conduct visits to the lab and who are available via e-mail and telephone for consultation. CEI further provides expert consultation on a variety of educational topics, including student placement and assessment interpretations, through staff in its corporate office. Professionally written, research-based teacher manuals are provided to teachers/facilitators in the training sessions and then become handbooks for operating the labs throughout the school year. Teachers also have 24/7 access to expertise via CEI's webpage at www.ceilearning.com, where many publications are easily accessible. In the spring at least one day of advanced professional development is provided as a follow-up to the initial training and as an opportunity for teacher/facilitator networking and sharing of professional practices.

The *SHARE* newsmagazine is another vehicle for teacher and administrator growth. In each issue are articles written by CEI staff, as well as by teachers/facilitators or administrators in the schools that provide ideas for leveraging the power of the *MLS* lab, along with ideas about other populations of learners who can benefit from participation in the lab. *SHARE* is, therefore, a networking mechanism for teachers/facilitators and administrators.

A feature of *SHARE* is columns written by experts on *NCLB* compliance, meeting the needs of learners in the subgroups (such as English-language learners or special education), suggesting ways that the features of *MLS* can assist schools not only in improving student learning, but also in complying with various federal and state mandates, and in reviewing the scientific evidence behind the various components of the programs. Occasional book reviews and other useful information are included.

The emphasis on professional development and follow-up is similar to an insurance policy that CEI established for itself and its school partners so that, to every extent possible, a school receives the support it needs for effective implementation—to achieve the desired academic results for their students.

The research findings on professional development and follow-up coaching are provided in Table 83.

Table 83: Research Findings on Professional Development and Coaching

Researcher(s)	Findings/Conclusions
Snow, Burns, & Griffin, 1998, 331	"Staff development efforts are often inadequate for a number of reasons, including the lack of substantive and research-based content, the lack of systematic follow-up necessary for sustainability, and the one-shot character of many staff development sessions."
Snow, Barley, Lauer, Arens, Apthorp, Englert, & Akiba, 2005, 92	"The training of the teacher-tutor and the resulting intervention may have a significant effect on the quality of a given computer-assisted instructional session."

Researcher(s)	Findings/Conclusions
Biancarosa & Snow, 2004, 20	"Professional development does not refer to the typical one-time workshop, or even a short-term series of workshops, but to ongoing, long-term professional development, which is more likely to promote lasting, positive changes in teacher knowledge and practice."
Kamil, 2004, 30	"Research shows that a teacher's professional development can positively affect student achievement, which is sufficiently suggestive to warrant policies that encourage sustained, imbedded professional development for teachers in secondary schools."
Joyce & Showers, 2002, 77	". . . this is an important finding—a large and dramatic increase in transfer of training—effect size of 1.42—occurs when coaching is added to an initial training experience comprised of theory explanation, demonstrations, and practice."
Hawley & Valli, 2000, 9	"The content of professional development focuses on what students are to learn and how to address the different problems students may have in learning that material. . . . Professional development should be continuous and ongoing, involving follow-up and support for further learning, including support from sources external to the school and can provide necessary resources and outside perspectives."
Hawley & Valli, 2000, 9	". . . the ultimate test of the efficacy of the design principles is whether such teacher learning activities lead to changes in teaching that contribute to improved student learning."
Fullan, 1991, 91	"One of the reasons that peer coaching works so effectively is that it combines pressure and support in a kind of seamless way."
Sparks, 2002, 1-2	"Teacher expertise is one of the most important variables affecting student achievement."
Ontario Ministry of Education, 2005, 140	"Professional development should be accessible and relate directly to the realities of the classroom. It should also include follow-up support, such as mentoring, coaching, and/or lead teacher consultation, and address how the support will be sustained."
Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003, 219	"Mentoring, like coaching, is a teacher-to-teacher professional development strategy that sustains long-term, ongoing professional development embedded within the school culture. . . . It is within the coaching role that mentors assist new teachers in becoming more deliberate about effective teaching, learning, and assessing."
Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003, 236	"In the past decade, the use of the Internet, e-mail, online courses, CDs, chat rooms, real-time electronic conversations, bulletin boards, listservs, video- and audiotapes, and videoconferencing has exploded. Many of these are now used instead of face-to-face interactions and to provide follow up support after in-person learning events. Individual learning that comes from seeing background reading and resources has never been easier. Technology has put the world's libraries and databases at the fingertips of all who have access to the World Wide Web."
Ma, 1999, 130	"When asked how they [Chinese teachers] had attained their mathematical knowledge in 'a systematic way,' these teachers referred to 'studying teaching materials [textbooks, curriculum frameworks, and teacher's manuals] *intensively* when teaching it."
Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003, 236-237	"The 'just-in-time, technology-mediated environment (NSDC, 2001a) for teacher learning provides ample opportunity for teachers to participate from home on one's schedule, attend online workshops or courses from a university located across the country, engage in electronic networking with other teachers, or increase their content knowledge in science or mathematics through videoconference courses. Additional benefits are identified in the National Staff Development Council document, E-Learning for Educators (2001a, iv), including the following: • Job-embedded learning opportunities • Content-rich learning opportunities • Personalized professional development • Increased access to professional learning experiences • Reduction of the costs of professional development."

Researcher(s)	Findings/Conclusions
Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003, 241	"The use of technology can be effective in providing follow-up or enhancing other professional learning experiences. Workshop attendees can create an electronic network to continue discussing the ideas and information shared during the workshop. They can develop online study groups to collectively examine student work and engage in threaded discussions."
Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003, 242	"To address some of the disadvantages [of technology in professional development] . . ., many programs have learned the value of combining technology-based learning with in-person learning in which participants have the opportunity to develop relationships face-to-face, engage in activities and discussions at their leisure through online formats, and conduct collaborative study, such as examining student work in real-time, online formats."

Student Motivation

The critical need for a motivation component in an intervention for struggling learners is documented in Chapters II-III of this study. Chapter II included several causes of learning difficulties that have implications for motivation—including cultural attitudes about the value of mathematics, math phobia/anxiety, stereotype threat, and general issues relating to low motivation, especially issues of lack of self-efficacy or self-esteem. The disabilities described in Chapter III all have motivation implications as well, since it is difficult to be motivated to learn something when there has been repeated failure in the past in trying to do so. Marzano (1998) notes that "Researchers and theorists . . . have demonstrated that one of the most important aspects of one's sense of self is his beliefs about personal attributes" (p. 58). He explains that low motivation occurs when "There is a discrepancy between the desired status and the perceived status, but the individual has low efficacy beliefs relative to that personal attribute position within a group" (p. 61).

One of the startling findings in Hart and Risley's 1995 study related to the lack of positive feedback that many, many preschoolers from poverty households receive in their daily lives, as compared to the lives of children from professional families. Understanding these numbers makes it abundantly clear why schools must be very concerned about motivation of students.

> In a 5,200 hour year, the amount would be 166,000 encouragements to 26,000 discouragements in a professional family, 62,000 encouragements to 26,000 discouragements in a working-class family, and 26,000 encouragements to 57,000 discouragements in a welfare family. Extrapolated to the first 4 years of life, the average child in a professional family would have accumulated 560,000 more instances of encouraging feedback than discouraging feedback, and an average child in a working class family would have accumulated 100,000 more encouragements than discouragements (p. 199).

This same study found incredibly large gaps between the vocabularies of the children of professional parents and the child living in housing projects. Vast numbers of children, then, enter school each year severely disadvantaged in language acquisition and in familiarity with print and vocabulary—and with more than twice as many of their interactions with their parents being negative rather than positive. Those are major reasons that they come to school at risk of failure—not only in reading, but in mathematics and other academic subjects.

This information, coupled with the devastating effects on mathematics achievement of many of the conditions described in Chapter II on learning difficulties (such as mathematics phobia, stereotype threat, and low self-esteem) demand attention to motivation in any intervention. On top of these realities for growing numbers of America's children, a learner may also suffer with a learning disability, which too frequently also results in a loss of self-esteem and motivation for learning. The daunting challenge of the school is not only to close as much of the achievement gap as possible for these learners, but first to motivate them to believe in their own efficacy, to believe that effort makes a difference, to want to learn, especially to learn mathematics.

CEI is a part of a family of companies owned by Mr. Paul Meyer, who has devoted much of his career to teaching others about success motivation, so CEI staff are very cognizant of the important role of motivation in successfully teaching students mathematics. Included in the teacher/facilitator training, therefore, are many suggestions that go beyond the motivational strategies built into computer-assisted instruction.

Chapter VI included research on the motivational benefits of individualization/differentiation and working in the "zone of proximal development," with work that is adequately challenging, but enabling (with mediation) high levels of success, and the power of immediate corrective feedback. Tasks that are too easy have a negative effect on motivation, but Mercer and Mercer (2005) point out that "one of the primary findings in research" is that "learning improves most when students have a high percentage of correct responses" (p. 34). Additionally, the support program for *MLS* includes various recognition activities, such as certificates for mastery and completion, articles about outstanding students in *SHARE* (CEI's newsmagazine), and recognition for participation and achievement in the Creative Writing Contest.

In Meyer's (2002) *Unlocking Your Legacy: 25 Keys to Success*, he includes a chapter on self-image, where he identifies these six barriers to a positive self-image:

- Staying in the comfort zone and living at the present level of success is easier and less stressful than exerting effort to make needed changes.
- Fear of making a mistake or risking possible failure discourages trying anything new or different.
- The desire to avoid disapproval, either by themselves or by others, limits many to behavior that is calculated to please.
- Anxiety about changing the status quo convinces some that change is negative and not worth the risk.
- A poverty mentality, coupled with a false sense of inferiority, causes some people to believe they do not deserve the rewards of using their full potential.
- An illogical fear of success prevents many from breaking the success barrier. They feel unworthy or they fear they will not know how to handle success, so they subconsciously avoid it (pp. 90-91).

Students who fail academically every day, in public, no doubt suffer from negative self-image. Overcoming the barriers to a positive self-image outlined by Meyer is a part of the steps that they have to take in order to be motivated to learn to read, to learn mathematics and to be successful in school.

Meyer also feels strongly that success comes from desire coupled with effort, and that connection seems to be authenticated in the following:

> Purposely choosing to strengthen your self-image is an amazing possibility and the rewards and benefits will last for a lifetime, so keep pressing in and pressing on—then you can press through anything! Along the way, don't be discouraged if it takes effort and time. Nothing worth getting in life is ever free, but the payoff at the end will be worth every ounce of effort (p. 91).

Csikzentmihalyi (1991), one of the foremost authorities on motivation, says we all want more of what he calls "flow," or "the optimal experience" that is the result of a series of conditions:

> When people reflect on how it feels when their experience is most positive, they mention at least one, and often all of the following: First, the experience usually occurs when we confront tasks we have a chance of completing. Second, we must be able to concentrate on what we are doing. Third and fourth, the concentration is usually possible because the task undertaken has clear goals and provides immediate feedback. Fifth, one acts with a deep but effortless involvement that removes from awareness the worries and frustrations of everyday life. Sixth, enjoyable experiences allow people to exercise a sense of control over their actions. Seventh, concern for the self disappears, yet paradoxically the sense of self emerges stronger after the flow experience is over. Finally, the sense of the duration of time is altered; hours pass by in minutes, and minutes can stretch out to seem like hours. The combination of all these elements causes a sense of deep enjoyment that is so rewarding people feel that expending a great deal of energy is worthwhile simply to be able to feel it (p. 49).

CEI, of course, wants a learner's experience in an *MLS* lab to be a "flow" experience, so the features of "flow" are included in the design. The student is placed at a level where he or she can complete the tasks. The design of the lab, the use of headphones, and the engagement of the computer software make it possible to concentrate. The student's learning goals are clear to him or her, and immediate auditory feedback is provided at each step. Lessons are not so challenging as to cause frustration, and students have a great deal of support and control as they work through the phases. As students accumulate more and more success, their self-image improves and they are further motivated to keep working for mastery.

One of the stories, to illustrate these points, that is frequently told by Ben Rodriguez, CEI's senior vice president, is that he was visiting a lab early in the school year one fall and was particularly watching one small boy who was very engaged in his work at the computer. One feature of the computer-assisted instruction is the feedback provided after each student response, which is either praise for correct responses or encouragement to try again when the response has been in error. This small boy responded correctly, and the computer voice said, "Good job!" The boy looked around briefly, and then with a smile on his face, patted the computer monitor on its side and whispered, "Thanks!"

This student was perhaps having the first "flow" experience of his life—if he came from that economically disadvantaged home described by Hart and Risley and no doubt experienced some of the barriers articulated by Meyer. The story illustrates several ways in which *MLS* incorporates motivation to learn mathematics in the delivery of its instruction:

- Students are placed into the program at a level that assures a high degree of success, yet with sufficient challenge to maintain interest.
- Students have a great deal of support and choice in the *MLS* program design, allowing them a sense of control over the environment in which they learn.
- Students receive auditory praise when they respond correctly and encouragement when they do not, so that they will be willing to try again.
- Corrective feedback, whether auditory or written, is free of judgment and criticism.
- Students receive written feedback daily in their progress reports, which give them a feeling of accomplishment and a sense that their efforts are paying off.
- Teachers/facilitators are encouraged in their training and in the *Teacher's Manual* to provide positive and encouraging feedback to students as they monitor their performance.
- Practice exercises are varied to maintain interest, even though the lesson goal stays the same.
- CEI provides numerous opportunities for student recognition:
    - Articles in *SHARE* about outstanding students
    - Achievement certificates signed by the president of the company
    - Incentives and rewards

In reviewing the scientific studies on motivation, one sees recurring themes—many of which echo Meyer's emphasis on the importance of effort and many of which reflect the definition of "flow," as defined by Csikszentmihalyi. Table 84 includes that research.

Table 84: Research Findings on Motivation

Researcher(s)	Findings/Conclusions
Wakefield, 1999, 236	"It has always baffled me that some teachers work so hard to promote self-esteem in children yet simultaneously give them inappropriate tasks to perform. Some teachers fail to recognize and acknowledge that a child's response may be 'on the way to being right.' Children develop self-confidence from experiencing success. Teacher praise, sticker awards, and 'all about me' theme projects will not alter the perception children have of themselves if failure dominates their day."
Marzano, 1998, 61	". . . beliefs about purpose and efficacy are key to the processes of motivation and, subsequently, attention."
Marzano, 1998, 62	". . . beliefs in the categories of purpose and efficacy most probably generate the greatest changes in attention and motivation."

Researcher(s)	Findings/Conclusions
Marzano, 1998, 64	". . . if the self-system contains no beliefs that would render a given task important, then the individual will not engage in the task or will engage with low motivation."
Erlauer, 2003, 73	"Success breeds success. If we can make a student feel successful in learning and satisfied with life within the classroom and school, he or she will be motivated to continue striving to achieve. Part of making students feel successful is meeting their personal learning needs. When students find school and learning interesting, they want to learn. Making lessons interesting and the content and skills being taught meaningful and relevant to the students is one way of meeting students' needs. Another way to meet the needs of students is through recognizing their individual abilities and learning styles and implementing practices related to those individual differences."
Henderson, 1992, 80	"If one is working with pupils with learning difficulties it is very easy to lower one's aims and objectives; and if a pupil thinks that his teacher does not expect him to achieve very much, then there is a tendency for the pupil not to achieve—the low aspirations of the teacher somehow permeate to the pupil."
Pajares, 1996, 325	"According to Bandura's (1986) social cognitive theory, students' beliefs about their capabilities to successfully perform academic tasks, or self-efficacy beliefs, are strong predictors of their capability to accomplish such tasks. . . . Students' self-efficacy beliefs help determine what students will do with the knowledge and skills they possess. As a consequence, academic performances are highly influenced and predicted by students' perceptions of what they believe they can accomplish."
Pajares, 1996, 325	"Self-efficacy beliefs act as determinants of behavior by influencing the choices that individuals make, the effort they expend, the perseverance they exert in the face of difficulties, and the thought patterns and emotional reactions they experience."
Pajares, 1996, 326	"Pajares and Miller (1994) reported that self-efficacy to solve math problems was more predictive of that performance than were prior determinants such as sex or math background or other variables such as math anxiety, math self-concept, or perceived usefulness of mathematics."
Pajares, 1996, 340	"Some self-efficacy researchers have suggested that teachers should pay as much attention to students' perceptions of capability as to actual capability, for it is the perceptions that may more accurately predict students' motivation and future academic choices (see Hackett & Betz, 1989)."
Zeldin & Pajares, 2000, 2	"Bandura (1986, 1997) has argued that the most important source of information comes from the interpreted results of one's past performance, which he called mastery experiences. Authentic mastery of a given task can create a strong sense of efficacy to accomplish similar tasks in the future. Alternatively, repeated failure can lower efficacy perceptions, especially when such failures occur early in the course of events and cannot be attributed to lack of effort or external circumstances. Continued success, on the other hand, can create hardy efficacy beliefs that occasional failures are unlikely to undermine."
Zeldin & Pajares, 2000, 2	"Beliefs of personal competence are also influenced by the verbal persuasions one receives. Verbal messages and social encouragement help individuals to exert extra effort and maintain the persistence required to succeed, resulting in the continued development of skills and of personal efficacy."
Zeldin & Pajares, 2000, 3	"Individuals with strong self-efficacy beliefs work harder and persist longer when they encounter difficulties than those who doubt their capacities."
Pajares, 2004, 396	"People also form their self-efficacy beliefs through the vicarious experience of observing others perform tasks. This source of information is weaker than mastery experience in helping create self-efficacy beliefs, but, when people are uncertain about their own abilities, they become more sensitive to it. The effects of modeling are particularly relevant in this context, especially when the individual has little prior experience with the task."

Researcher(s)	Findings/Conclusions
Pajares, 2004, 397	"And, just as positive persuasions may work to encourage and empower, negative persuasions can work to defeat and weaken self-efficacy beliefs. In fact, it is usually easier to weaken self-efficacy beliefs through negative appraisals than to strengthen such beliefs through positive encouragement."
Pennington, 1991, xii	"A child can have poor school performance without having a learning disorder, when the poor school performance is due entirely to emotional, motivational, or cultural factors."
McGuinness, 1997, 285	"What children want most is to show that they are competent in all areas in which their age mates are competent."
Marzano, 1992, 25	"Current research and theory on motivation . . . indicate that learners are most motivated when they believe the tasks they're involved in are relevant to their personal goals."
Marzano, 1992, 27	"Learners who believe they have the inner resources to successfully complete a task attribute their success to effort; there is no task they consider absolutely beyond their reach."
Marzano, Pickering, & Pollock, 2001, 50	"Not all students realize the importance of believing in effort. . . . The implication here is that teachers should explain and exemplify the 'effort belief' to students."
Marzano, Pickering, & Pollock, 2001, 52	"A powerful way to help [students] make this connection [between effort and achievement] is to ask students to periodically keep track of their efforts and its relationship to achievement."
Marzano, Pickering, & Pollock, 2001, 55	"Rewards do not necessarily have a negative effect on intrinsic motivation."
Marzano, Pickering, & Pollock, 2001, 55-56	"Reward is most effective when it is contingent on the attainment of some standard of performance."
Marzano, Pickering, & Pollock, 2001, 57	"Abstract symbolic recognition is more effective than tangible awards."
Marzano, Pickering, & Pollock, 2001, 57-58	". . . it appears obvious that abstract rewards—particularly praise—when given for accomplishing specific performance goals can be a powerful motivator for students."
Marzano, Pickering, & Pollock, 2001, 58	". . . it is best to make this recognition as personal to the students as possible."
Bruer, 1993, 258	"If we want more students to thrive, we will have to restructure classrooms and schools to create environments where children believe that, if they try, they can learn."
Levin & Long, 1981, 8	". . . students in the mastery group develop higher levels of motivation for later units in the series. Since they have experienced success in the earlier units, they are more confident in their ability to learn well and to succeed in subsequent units."
Providing Appropriate Levels of Challenge, 2000, 1	"The right level of challenge is always a moving target. As skill improves, the next challenge tests new mastery to just the right extent. The same kind of incremental, responsive challenge can foster engagement in the classroom. Without new challenges, students become bored; impossible challenges frustrate and dishearten them. The right level of challenge at the right time can 'pull in' students the way video games do, building mastery a step at a time."
Kujala, Karma, Ceponiene, Belitz, Turkkila, Tervaniemi, & Naatanen, 2001, 7	"As previous studies have shown, attention and motivation are important factors in causing plastic neural changes in the brain."
Sousa, 2001, 209	"Look for abilities, not just disabilities. Sometimes we get so concerned about the students' problems that we miss the opportunity to capitalize on their strengths. Many studies indicate that using an individual's strengths to mitigate areas of weakness often results in improved performance and a well-needed boost to that person's self-esteem."

Researcher(s)	Findings/Conclusions
Csikezentmihalyi, 1991, 49	"When people reflect on how it feels when their experience is most positive, they mention at least one, and often all of the following: First, the experience usually occurs when we confront tasks we have a chance of completing. Second, we must be able to concentrate on what we are doing. Third and fourth, the concentration is usually possible because the task undertaken has clear goals and provides immediate feedback. Fifth, one acts with a deep but effortless involvement that removes from awareness the worries and frustrations of everyday life. Sixth, enjoyable experiences allow people to exercise a sense of control over their actions. Seventh, concern for the self disappears, yet paradoxically the sense of self emerges stronger after the flow experience is over. Finally, the sense of the duration of time is altered; hours pass by in minutes, and minutes can stretch out to seem like hours. The combination of all these elements causes a sense of deep enjoyment that is so rewarding people feel that expending a great deal of energy is worthwhile simply to be able to feel it."
Smey-Richman, 1988, 24-25	"Success at novel and challenging tasks is important to low achieving students . . ., but overly difficult tasks produce confusion and discouragement. According to Brophy, the degree of cognitive strain produced by tasks that allow students a 50 percent or less success rate is so great that it exceeds the tolerance level of the slow learner. In this regard, Harter has shown that students feel motivated when they experience success with what they perceive as reasonable effort, but are discouraged when they achieve success only with sustained effort."
Smey-Richman, 1988, 25	". . . the combination of high effort and failure is especially damaging, as it leads to suspicion of low ability. It is this self-realization of incompetency that triggers humiliation and shame."
Smey-Richman, 1988, 35	". . . continued success on easy tasks is ineffective in producing challenge-seeking and persistent behavior. . .; consistently easy tasks lower self-confidence."
Shaywitz, 2003, 284	"Motivation is critical to learning and can be strengthened by adhering to a few simple principles. First, any child, and particularly one who is dyslexic, needs to know that his teacher cares about him. Second, motivation is increased by a child's having a sense of control, such as a choice about assignments—which book he will read and what topic he will report on. Third, he needs some recognition of how hard he is working as well as tangible evidence that all his effort makes a difference; this can come in the form of improvement on a graph of his fluency rates or receiving a grade on the content of his written work rather than its form."
Levine, n.d., 3	"So a student can lose motivation because he doesn't like a goal, because he feels he could never get that goal, or because the goal would be much too hard to get. You can see how a student with learning disorders might lose motivation when it comes to getting a good report card."
Levine, n.d., 3	"A kid may look lazy or she has lost motivation. Some kids look lazy when they really have attentional difficulties that make it extremely hard for them to concentrate."
Levine, n.d., 3	"Most of the time, when kids are bored in school, it is either because they are having trouble with their attention or because they don't fully understand what is going on."
Tileston, 2000, 5	"Jenson believes that enrichment in the classroom comes primarily from challenge and feedback. He warns that too little challenge in the classroom breeds boredom and that too much can intimidate. Challenge should be filtered so that it provides stimulating and fun experiences that match the ability level of the student without causing frustration."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 95	"Research shows that students may not realize the influence effort has on their success in school, but they can learn that efforts helps them succeed. Simply teaching students that added effort pays off in terms of enhanced achievement actually increases student achievement. In fact, one study (Van Overwalle & De Metsenaere, 1990) found that students who were taught about the relationship between effort and achievement achieved more than students who were taught techniques for time management and comprehension of new material."

Researcher(s)	Findings/Conclusions
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 95	"Research . . . shows that rewards do not necessarily decrease student motivation and that reward is most effective when contingent on successfully completing a specific level of performance. We also know symbolic recognition is more powerful than tangible rewards."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 109	"When used properly, praise is highly effective. Generally, it is best to provide recognition for specific elements of an accomplishment."
Marzano, Norford, Paynter, Pickering, & Gaddy, 2001, 109	"Symbolic tokens, such as stickers or certificates, can be effective tools to recognize the successful completion of special learning goals. However, to keep students from losing their intrinsic motivation, teachers should avoid rewarding students for simply completing an activity."
Rose & Meyer, 2002, 35	"Understanding affective issues can help teachers support all learners more appropriately. Of the three learning networks, affective networks are perhaps intuitively the most essential for learning, yet they are given the least formal emphasis in the curriculum. All teachers know how important it is to engage students in the learning process, to help them to love learning, to enjoy challenges, to connect with subject matter, and to persist when things get tough. When students withdraw their effort and engagement, it is tempting to consider this a problem outside the core enterprise of teaching. We believe this is a mistake. Attending to affective issues when considering students' needs is an integral component of instruction, and it can increase teaching effectiveness significantly."
Rose & Meyer, 2002, 125	"Affect is the fuel that students bring to the classroom, connecting them to the 'why' of learning. The work of Goleman (1995) shares the UDL prospective that motivation is at least as important for school success as the capacity to recognize and generate patterns. Affect goes beyond simple enjoyment, and among other things, it plays a part in the development of persistence and deep interest in a subject. If we emphasize skills and knowledge to the exclusion of emotion, we may breed negative feelings toward learning, especially in students having difficulties."
Rose & Meyer, 2002, 127	"We know that students learn best in their 'zone of proximal development' (Vygotsky, 1962), where challenge is just beyond their current capacity but not out of reach. Students' comfort zones—the level of difficulty, challenge, and frustration optimal for them—vary considerably. Teachers who hope to sustain students' engagement must be able to continually adjust the challenge for and among different learners."
Rose & Meyer, 2002, 127	"Adjustable levels of challenge have advantages beyond the immediate power to engage. Providing such choices for students makes the process of goal-setting explicit and provides a structured opportunity for students to practice setting realistic goals and optimal challenges for themselves."
Mercer & Mercer, 2005, 45	"The need for students to experience high levels of success has substantial research support. In this research, success refers to the rate at which the student understands and correctly completes exercises (Borich, 1992). . . . Apparently, during high success more content is covered at the learner's appropriate instructional level. . . . Borish (1992) claims that research suggests that students need to spend about 60 to 70 percent of their time on tasks that allow almost complete understanding with occasional careless errors. Instruction that promotes high success not only contributes to improved achievement but also fosters increased levels of self-esteem and positive attitudes toward academic learning and school. . . . Lack of success can lead to anxiety, frustration, inappropriate behavior, and poor motivation. In contrast, success can improve motivation, attitudes, academic progress, and classroom behavior."
Mercer & Mercer, 2005, 139	"Once students learn that successes are the result of their own efforts, they are more likely to feel in control of their learning and develop more independent learning behaviors."

Researcher(s)	Findings/Conclusions
Marzano, Pickering, & Pollock, 2001, 50	". . . this body of research demonstrates that people generally attribute success at any given task to one of four causes: • Ability • Effort • Other people • Luck Three of these four beliefs ultimately inhibit achievement. . . . Belief in effort is clearly the most useful attribution."
Marzano, Pickering, & Pollock, 2001, 51	"An interesting set of studies has shown that simply demonstrating that added effort will pay off in terms of enhanced achievement actually increases student achievement."
Marzano, Pickering, & Pollock, 2001, 55	"Those who have carefully analyzed all the research on rewards, commonly came to the conclusion that they do not necessarily decrease intrinsic motivation."
Marzano, Pickering, & Pollock, 2001, 57	"Abstract symbolic recognition is more effective than tangible awards. . . . the more abstract and symbolic forms of reward are, the more powerful they are. . . . verbal reward seems to work no matter how one measures intrinsic motivation. Tangible rewards, on the other hand, do not seem to work well as motivators, regardless of how motivation is measured."
Marzano, Pickering, & Pollock, 2001, 58	". . . it is best to make this recognition as personal to the students as possible."
Marzano, Pickering, & Pollock, 2001, 59	"Reinforcing effort can help teach students one of the most valuable lessons they can learn—the harder you try, the more successful you are. In addition, providing recognition for attainment of specific goals not only enhances achievement, but it stimulates motivation."
Jones, Wilson, & Bhojwani, 1997, 152	"Chapman concluded that students who come to doubt their abilities (a) tend to blame their academic failures on those deficits, (b) generally consider their low abilities to be unchangeable, (c) generally expect to fail in the future, and (d) give up readily when confronted with difficult tasks. Unless interrupted by successful experiences, continued failure tends to confirm low expectations of achievement, which in turn sets the occasion for additional failure."
Jones, Wilson, & Bhojwani, 1997, 152	"Specific student estimates of self-efficacy were more accurate predictors of performance than prior experience in mathematics."
Jones, Wilson, & Bhojwani, 1997, 152	". . . negative expectations and motivational problems may be reduced by interventions to eliminate deficits in specific mathematics skills."
Balfanz, McPartland, & Shaw, 2002, 18	". . . learning activities need to be structured so that students can experience success, receive positive reinforcement, and exercise some control over their learning process. . . ."
Center for Development and Learning, 2005, 1	"Using diverse instructional strategies and tactics for diverse learners connects with, engages, and motivates students. That may sound simple, but the underlying knowledge a teacher must have to pull that off day after day, hour after hour—assessing his/her students and adjusting strategies and tactics moment by moment—requires a sophisticated skill level."
Center for Development and Learning, 2005, 4	"So whose job is it to motivate students? It is every teacher's job to motivate every student. Learning more about the brain and the development of the mind, studying the new information on learning, making learning meaningful and learning about learning, watching the learning process, monitoring closely for breakdowns, and applying specific strategies that directly address the breakdowns—these are teachers' challenges as they work to create classrooms that honor diversity."
Ontario Ministry of Education, 2005, 115	"Success on moderately difficult or challenging tasks that is attributed to personal effort and ability gives rise to feelings of pride, competence, determination, satisfaction, persistence, and personal control."

Researcher(s)	Findings/Conclusions
Ontario Ministry of Education, 2005, 116	"Positive reinforcements should outweigh negative reinforcements by a ratio of four to one (Gottfredson, 1997; Lipsky, 1996). Rules should be stated in terms of what students will do, rather than what not to do. . . ."
Sherman, Richardson, & Yard, 2005, 3	"Some students believe that their mathematical achievement is mainly attributable to factors beyond their control, such as luck. These students think that, if they scored well on a mathematics assignment, they did so only because the content happened to be easy. These students do not attribute their success to understanding or hard work. Their locus is external because they believe achievement is due to factors beyond their control and do not acknowledge that diligence and a positive attitude play a significant role in accomplishment. Students might also believe that failure is related to either the lack of innate mathematical ability or level of intelligence. They view their achievement as accidental and poor progress as inevitable. In doing so, they limit their capacity to study and move ahead (Beck, 2000; Phillips & Gully, 1997)."
National Research Council, 2001, 131	"Productive disposition refers to the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics."
Vaughn, Gersten, & Chard, 2000, 8	"Critical variables that influence intervention effectiveness are the use of strategies used to enhance task persistence and the moderation of task difficulty. . . . Controlling for task difficulty to ensure that students experience success and persist in learning activities has long been recognized as a critical feature of effective instruction for students with LD (Gersten, Carnine, & White, 1984). Furthermore, while academic engagement (Anderson, Evertson, & Brophy, 1979; Greenwood, 1999) has been established as an essential factor linked to enhanced academic outcomes, time on task and persistence with tasks is affected by students' motivation to learn. Students' working on tasks that are challenging and meaningful but not beyond their reach influence all of these. Students who experience some successes in school are much more likely to participate actively in educational or work experiences following school (Blackorby & Wagner, 1996). Conscious attention to task difficulty is likely to be linked to higher levels of student achievement."
Vaughn, Gersten, & Chard, 2000, 9	". . . a recent synthesis examining the effects of intervention research on the self-concept of students with LD indicates at the elementary level that academic interventions are the most effective means to improved self-concept (Elbaum & Vaughn, 1999)."
National Research Council, 2001, 339	"Students' motivation depends on both expectation and value. That is, students are motivated to perform the task successfully if they apply themselves and the degree to which they value the task or the rewards that performing it successfully will bring. Therefore, teachers can motivate students to strive for mathematical proficiency both by supporting their expectations for achieving success through a reasonable investment of effort and by helping them appreciate the value of what they are learning."
Marzano, 1998, 8	"If the presenting task is judged as important and the probability of success is high, positive affect is generated and the individual is motivated to engage in the presenting task. . . . If the presenting task is evaluated as low relevance and/or low probability of success, negative affect is generated and motivation for task engagement is low."

Parental Involvement

CEI staff provide a parent workshop, if requested by the school, for the parents of *MLS* students so that parents will know what their children will be doing in the *MLS* lab and how they can support their growth, as well as the kinds of growth they can expect to see. *MLS* teachers/facilitators are encouraged to involve parents as much as possible because of the abundance of scientifically-based research that predicts higher achievement when that occurs. Parent progress reports for *MLS* are on the drawing board for development in the near future, and they will soon be available in both English and Spanish.

The scientifically-based evidence on the importance of parental involvement is presented in Table 85.

Table 85: Research Findings on Parental Involvement

Researcher(s)	Findings/Conclusions
Rose & Meyer, 2002, 166	"All educators know that if you want to make an initiative happen, it's a good idea to get parents involved."
Alliance for Curriculum Reform, 1995, 9	"Dozens of studies in the U. S., Australia, Canada, England, and elsewhere show that the home environment powerfully influences what children and youth learn within and outside school. This environment is considerably more powerful than the parents' income and education in influencing what children learn in the first six years of life and during the 12 years of primary and secondary education."
National PTA, 2000, 12-13	"The most accurate predictors of student achievement in school are not family income or social status, but the extent to which a student's family is able to (1) create a home environment that encourages learning; (2) communicate high, yet reasonable expectations for the child's achievement and future career; and (3) become involved in the child's education at school and in the community."
Gray & Fleischman, Dec. 2004/Jan. 2005, 85	"When parents are involved, students tend to achieve more, regardless of socioeconomic status, ethnic/racial background, or the parents' educational level."
Gray & Fleischman, Dec. 2004/Jan. 2005, 85	"A final key component of serving the needs of English-language learners is establishing strong relationships with families."
Taylor, Pearson, et al., 2000, 158	"At the school level, the most effective schools made more of an effort to reach out to parents than the moderately and least effective schools. At the classroom level, the teachers in the most effective schools made more of an effort to communicate regularly with parents than teachers in the other schools."
National PTA, 2000, 12	"If parents do not participate in school events, develop a working relationship with their children's educators, or keep up with what is happening in their children's schools, their children are more likely to fall behind in academic performance."
National PTA, 2000, 17	"When parents receive frequent and effective communication from the school or program, their involvement often increases, their overall evaluation of educators often improves, and their attitudes toward the program are often more positive."
Sousa, 2001b, 213	"Frequent communication with parents is important so that you are all working together to assist the student in meeting expectations."
Neuman & Roskos, 1998, 12	"Communication between families and teachers built on mutual respect and the sharing of information creates bonds of continuity, purpose, and consistency in children's early literacy programs."
Walberg & Paik, 7	"Co-operative efforts by parents and educators to modify these alterable academic conditions in the home have strong, beneficial effects on learning. In twenty-nine controlled studies, 91% of the comparisons favoured children in such programmes over non-participant control groups."
ERS, 2000, 1	"The research base developed over many years has made it clear that meaningful family involvement is a powerful predictor of high student achievement."
Stein & Thorkildsen, 1999, 31	"Results of experimental studies reviewed here show statistically significant differences on measures of achievement between children whose parents participate in parent involvement programs and those who do not."
Stein & Thorkildsen, 1999, 32	"Of all aspects of parent involvement studied, parents' expectations of their children's achievement have the strongest relationship with children's actual level of achievement."

Researcher(s)	Findings/Conclusions
Stein & Thorkildsen, 1999, 32	"Schools must play a major role in encouraging involvement through regular invitations to school activities and social events."
Stein & Thorkildsen, 1999, 33	"All types of parents have been successful in parent involvement activities. Consequently, economic disadvantage should not be viewed as a barrier to getting parents involved."
National PTA, 2000, 12	"In programs designed to involve parents in full partnerships, student achievement for disadvantaged children not only improves, but can also reach levels that are standard for middle-class children. Children who are furthest behind are the most likely to make the greatest gains."
Ontario Ministry of Education, 2005, 114	"Educators and researchers increasingly recognize how important it is for teachers and parents to jointly guide the learning of children with special education needs (Williams & Cartledge, 1997). Educators need to establish open lines of communication so that everyone's experiences can be put to use."
Zemelman, S., Daniels, H., & Hyde, A., 1998, 95	"The Best Practices in mathematics teaching and learning . . . make frequent mention of manipulatives, concrete materials, and real-world situations for optimal learning. These are the contexts that make understanding of mathematical ideas possible and provide a bridge to the more abstract symbolism that has maximal power and usefulness. Parents and the home environment of children of all ages can provide the richness of materials and opportunities for latent mathematical thinking to flourish."
Zemelman, S., Daniels, H., & Hyde, A., 1998, 97	"Perhaps the best way for parents to help their children with mathematics is to send the clear message through their words and actions that mathematics is all around us, it is a vital part of our lives, and it is understandable with some effort. Let's do it together; it can be fun."
Zemelman, S., Daniels, H., & Hyde, A., 1998, 97	"Many principals hold Parents' Nights to present new mathematical ideas, methods, and materials to parents. Some make videotapes of these presentations and send them home to parents who did not attend. Principals arrange for programs on evenings or Saturdays, at which teachers develop activities for parents and their children to do together. Then parents continue these activities and extensions of them at home."

Implementation Effectiveness

Existing research on implementation and its role in achieving improved academic growth is becoming more and more important to practitioners in this era of high-stakes accountability and the mandate for scientifically-based programs. The principal or other instructional leader simply must assume the responsibility for ensuring that the program is not only initially well implemented, but that the staff involved continue to implement according to the design. Otherwise, students will not learn as much as they might have, and program evaluation is impossible. CEI's *MLS Implementation Toolkit*, plus the professional development and follow-up coaching, motivation program, and parental involvement activities are provided to support the school's efforts and to ensure, to the extent possible, a successful implementation of every *MLS* lab. The research findings on implementation are provided in Table 86.

Table 86: Research Findings on Implementation Effectiveness

Researcher(s)	Findings/Conclusions
Schmoker, 1999, 53	"Educators are hungry for both kinds of details: evidence of exactly how well a method works as well as concrete descriptions of how to make it work."

Researcher(s)	Findings/Conclusions
ERS, 2002, 68	"Research on early intervention programs has concluded that, to be effective, the approaches must be part of a comprehensive, schoolwide plan."
Erkayer, 2003, 2-3	"Adoption of an innovation is simple. It is the implementation that takes the time and effort. Even successful implementation of a change in a school setting is not enough. If lasting improvement is to occur, the new practices must be sustained over a long period of time in order to become part of 'the way we do things here.'"
Biancarosa & Snow, 2004, 21	"Without a principal's clear commitment and enthusiasm, a curricular and instructional reform has no more chance of succeeding than any other schoolwide reform."
Fullan, 1991, 54	"Initiation of change never occurs without an advocate."
Fullan, 1991, 76	". . . one of the best indicators of active involvement is whether the principal attends workshop training sessions."
Marzano, 2003, 165	"Once a specific intervention is identified, it must be thoroughly implemented if a school is to expect it to impact student achievement. . . . There are many stages of implementation. Just because a school has provided training in a new intervention does not mean that staff members are actually using it. Sadly, many, if not most, interventions are not fully implemented. In fact, it is not uncommon for an intervention to be considered ineffective or marginally effective when, in fact, the intervention was improperly or only partially implemented."
Marzano, 2003, 166	"The goal of any intervention is to positively impact student achievement. Therefore, not collecting data on student achievement (once there is some evidence that the intervention has been implemented) is a major mistake—one that can ultimately kill a school reform effort."
Bottoms, Presson & Han, 2004, 25	"The differences in achievement behind the high-implementation and low-implementation schools can be directly attributed to the depth to which the two groups of schools have implemented the HSTW [High Schools that Work] design."
Bottoms, Presson & Han, 2004, 25	"The high-implementation schools exemplify that the more completely the design is implemented, the higher the student achievement."
Rose & Meyer, 2002, 157	"The major components necessary to implement UDL at the local level within a district are technology infrastructure, administrative support, teacher training and support, redefined roles for special and regular education teachers, a new curriculum planning model, parent and community involvement, and creative funding."
McEwan, 2000, 90	"Lack of supervision and accountability are major stumbling blocks to successful change. Somebody has to care about these two critical issues, and the building principal is the go-to person for making sure and keeping track. If you are not organized, structured, and data-driven, find someone to help you who is (e.g., a lead teacher, a building secretary, or a school improvement coordinator)."
Stigler & Hiebert, 1999, 8	"Policymakers adopt a program, then wait to see if student achievement scores will rise. If scores do not go up—and this is most often what happens, especially in the short run—they begin hearing complaints that the policy isn't working. Momentum builds, experts meet, and soon there is a new recommendation, then a change of course, often in the opposite direction. Significantly, this whole process goes on without ever collecting data on whether or not the original program was even implemented in classrooms—or, if implemented, how effective it was in promoting student learning."

Summary

Chapter VII presented research on and discussed four specific program features developed by CEI to assist its partners in implementing *MLS* successfully and effectively—to get the results they need for improved student performance. These four features are (1) engaged role of lab teachers/facilitators; (2) professional development with follow-up coaching; (3) motivation of students to learn; and (4) parental involvement. The chapter concludes with the overall topic of implementation and the research base for effectiveness.

Chapter VIII, the concluding chapter, will include a summary discussion of the characteristics of effective mathematics interventions, an analysis of CEI's data on growth in *MLS* labs, and a summary and conclusions of the research examined and how it is reflected in the *MLS* design.

Chapter VIII: Insights and Conclusions

"It is not the case that a large number of children are simply 'bad at maths,' and that nothing can be done about it." (Dowker, 2004, 42)

Overview

Chapter I explained the federal requirements for programs to be based on scientific evidence and then explored the expert definitions of what that means. In summary, according to an *NCLB* guidance document (Jan. 7, 2004) published by the U. S. Department of Education, strategies that apply scientific research would themselves be considered as grounded in scientifically-based research. The purpose, then, of *Why MLS Works: Its Scientific, Theoretical, and Evaluation Research Base* has been, first, to deconstruct *MLS* so that its component parts can be identified, and, then, systematically to document the scientific evidence underlying each of those components.

Essential background on learning difficulties and learning disabilities and how they are manifested in mathematics performance were discussed in Chapters II and III. CEI staff report that it is easier to discuss why *MLS* is scientifically-grounded and the results that schools typically achieve with struggling learners if people understand the nature of the causes for weak mathematical performance. These learners with learning difficulties and disabilities, clearly, are the ones who are left behind in many, many schools, and they continue to get left behind if educators fail to understand their needs. Understanding the causes of learning problems, how they are manifested, and what kinds of interventions are needed are critical to understanding why *MLS* is worth consideration.

Chapter IV included a comprehensive description of *MLS'* content, along with the research on why that specific content is critical to mathematics success. *MLS'* major strands are concept development and fact fluency. The units and levels include foundational knowledge and skills that were selected as priorities in the design of the program because they are critical components in the foundational areas of mathematics, they are the areas where students with learning difficulties and disabilities tend to have problems, and they are the essential knowledge and skills for success in the next level of mathematics—algebra. Chapter IV ties in closely with the research documented in Chapters II and III.

An analysis of *MLS'* lesson models, sequences, and delivery was the focus of Chapter V. Documentation of ways that *MLS* incorporates scientifically-based components of direct instruction, mastery learning, and one-on-one tutoring was emphasized since these three models are the ones with a plethora of positive research findings as to their efficacy with struggling learners. The research cited in this chapter, along with that in Chapter II on inappropriate teaching and the mathematics wars, makes it clear that while discovery learning may work with some learners, it is absolutely the wrong approach to use in a mathematics intervention. Chapter IV also explored the research on the concrete-semiconcrete-abstract (CSA) lesson sequence found by many researchers to be effective in teaching concepts in mathematics, and it then documented the use of this lesson sequence in the *MLS* design, including the use of problem-solving in the abstract phase. The research on using manipulatives was also included (which fits, of course, with the concrete phase of the lesson). Finally, *MLS'* employment of computer-assisted instruction (CAI) was described, as well as the research on its effectiveness in teaching mathematics to struggling learners and the research on effective screen design for those kinds of students.

In Chapter VI, *MLS'* most powerful instructional strategies were identified, by task, and the research that grounds their inclusion in the program design was discussed and documented. Multi-sensory processing, probably the most powerful of these strategies—and the most unique—reflects cutting-edge research on how the brain, including the brain that struggles, learns mathematics. The research findings include primary research from various fields, as well as research syntheses from professional organizations, advocacy organizations, and such credible groups as the National Research Council. Individualization/differentiation, practice/repetition, chunking/clustering, and engaged time-on-task were also identified, by task, with research documentation. Finally, *MLS'* comprehensive assessment system was described, accompanied by the research studies that support its component parts, followed by discussions and analysis of corrective feedback, informed instruction, and self-assessment. An important point made in this chapter is how the lesson content, the lesson model, and the instructional strategies, including assessment and corrective feedback, are interwoven in the *MLS* lesson and have the appearance of seamlessness in their presentation, just as they are in a lesson taught by an expert teacher. They are discussed separately only for the purpose of analysis, not because they are ever observed discretely. The components also point back to the research findings identified in Chapters II and III on the manifestations of learning difficulties and disabilities. Instructional strategies are selected to address those learning problems so that foundational knowledge and skills are effectively moved to long-term memory for rapid and accurate retrieval and application as needed.

Most of the *MLS* design decisions center around content, lesson models and sequences, the use of manipulatives and computer-assisted instruction, and the choice of the most powerful instructional strategies. But *MLS* is much more than its software. Chapter VII presented the research behind four additional critical strategies that form a part of CEI's implementation support program provided with the *MLS* software. Implementation, after all, and again according to scientific evidence, is critical to the success of any program and certainly to the effectiveness of interventions for struggling students. CEI values highly the role of the engaged lab teacher/facilitator in a quality implementation. The research on teacher engagement, along with a job description derived from that research, was presented and discussed. The importance of professional development with follow-up coaching was also described, along with the research. A close reading of Chapters II and III makes it imperative that a mathematics intervention include attention to student motivation. Therefore, *MLS'* support of schools in this area was described in Chapter VII, along with the research. The fourth strategy is support for parental involvement—again with a description of services, resources, and documentation of research. The chapter concludes with a general discussion on leadership and accountability for effective implementation, along with the research on those topics.

In the final chapter, Chapter VIII, there are three sections. In the first section, the general research on effective mathematics interventions will be discussed with references to previous chapters that address the findings. Discussions of "opportunity-to-learn" standards, the urgency of early intervention, and how *MLS* implementation fits into a total quality environment are included. Next, a summary of the pre- to post-test score gains on the *DSTM* representing many *MLS* labs will be given, along with an analysis. The final section includes general summaries and conclusions about "why *MLS* works" as a therapeutic intervention.

Characteristics of an Effective Therapeutic Intervention for Mathematics

Virtually every school in every school district, public or private, in every state has students failing to attain the proficiency level in mathematics on the state accountability assessments or local assessments measuring performance. Educators have a variety of funding sources to serve struggling learners, and those funding sources drive the identification of students for various program initiatives, such as Title I, bilingual education or ESOL, dyslexia, special education, general at-risk programs, or test remediation programs. What this study indicates in the wealth of research cited on the reasons that students struggle, the manifestations of their difficulties or disabilities, and the appropriate curriculum and instruction for them is that the intervention needs are very similar, so the diversity of students failing, regardless of age or program label, are highly likely be served well in an *MLS* lab, especially since CEI's programs are, first and foremost, individualized. Table 87 cites important research findings relating to intervention program characteristics that are effective in improving student learning.

The third column of the table indicates the chapter(s) in this study where those research findings are addressed in *MLS*. A review of that third column will reveal that virtually every topic covered in Chapters IV-V-VI-VII is cited by at least one researcher as a component of a scientifically-based intervention, and many of those topics are cited multiple times, e.g., the critical importance of teaching both concepts and fact fluency and the essential role of multi-sensory processing and practice/repetition in moving new learning into long-term memory for retrieval. In addition, each *MLS* component was carefully documented with scientific research as it was introduced and described. These correlations of research findings with *MLS* components substantiate CEI's positioning of *MLS* as a therapeutic intervention for struggling learners in mathematics. If the components of an intervention are scientifically-research based, then the intervention can be determined to be scientifically-research based (see Chapter I for definitions of SBR).

Table 87: Effective Mathematics Interventions

Researcher(s)	Findings/Conclusions	*MLS* Design
Kroesbergen & Van Luit, 2003, 99	"An intervention is judged as effective when the students acquire the knowledge and skills being taught and thus appear to adequately apply this information at, for example, posttest."	Chapter I—see definitions of scientific-based research.
Balfanz, McPartland, & Shaw, 2002, 13	". . . it is perhaps more accurate to view the extra-help needed by the majority of high school students not as remediation (since in many cases it is the norm), but rather as effective means to accelerate their learning so they can be prepared for and supported in the mastery of rigorous intellectual work."	Chapter I—see definitions of scientific-based research and description of *MLS.*

Researcher(s)	Findings/Conclusions	*MLS* Design
Dowker, 2004, 42	"Research strongly supports the view that children's arithmetical difficulties are highly susceptible to interventions. It is not the case that a large number of children are simply 'bad at maths,' and that nothing can be done about it."	Chapter II—see discussions of mathematics difficulties. Chapter III—see discussions of mathematics disabilities.
Given, 2002, 85	"Identification of a learning disability is no excuse for academic failure. Only a small number of children are unable to learn within a satisfactory range when taught with alternative strategies based on learning-system strengths."	Chapter III—see discussions of learning disabilities.
Gardner, 1985, 31	"As I read current findings in the brain and biological sciences, they bear with particular force on two issues that concerns us here. The first issue involves the flexibility of human development. The principal tension here centers on the extent to which the intellectual potentials, or capacities of an individual or a group can be altered by various interventions. From one point of view, development may be viewed as relatively locked-in, preordained, alterable only in particulars. From an opposing perspective, there is far more malleability or plasticity in development, with appropriate interventions at crucial times yielding an organism with a far different range and depth of capacities (and limitations). Also pertinent to the issue of flexibility, are the related questions of the kinds of interventions that are most effective, their timing, the role of critical periods during which pivotal alterations can be brought about. Only if such issues are resolved will it be possible to determine which educational interventions are most effective in allowing individuals to achieve their full intellectual potential."	Chapter III—see discussions of various kinds of learning disabilities and their manifestations.
Darling-Hammond & Falk, 1997, 193	"Research on schools that have met high standards and maintained low retention rates with diverse student populations provides insights into successful teaching strategies. Teachers in these schools offer students challenging, interesting activities and rich materials for learning that foster thinking, creativity, and production. They make available a variety of pathways to learning that accommodate different intelligences and learning styles, they allow students to make choices and contribute to some of their learning experiences, and they use methods that engage students in hands-on learning. Their instruction focuses on reasoning and problem-solving rather than only recall of facts, fosters peer collaboration and extensive interaction between students and teachers, and stimulates internal rather than external motivation."	Chapter VIII—see discussion of *MLS* results in diverse schools. Chapter V—see discussion of lesson models, CSA sequence of lessons, use of manipulatives, and computer-assisted instruction. Chapter IV—see discussion of multi-sensory processing strategies. Chapter IV—see discussion of *MLS* content that includes both concept development and fact fluency. Chapters II-III-VI and VII—see discussions on motivation and feedback.
National Research Council, 2001, 19	"Although there is much 'remediation' done as part of school mathematics instruction in grades K to 8 and beyond, there are not nearly so many supplementary interventions in mathematics as there are in reading. There is very little in the way of 'mathematics recovery' that provides early targeted enrichment in mathematics to help students overcome special difficulties."	Chapter IV—see discussion of early school mathematics.

Researcher(s)	Findings/Conclusions	*MLS* Design
National Research Council, 2001, 19	"As in the case for reading, students develop some basic concepts and practices in mathematics outside of school, but a new and unfamiliar topic in mathematics—say, the division of fractions—usually cannot be fully grasped without some assistance from a text or a teacher."	Chapter IV-see discussions of concepts and procedures and, specifically, the discussion on fractions. Chapter VI—see discussion on CAI.
Lochy, Domahs, & Delazer, 2005, 469	"At the cognitive level, the aim of intervention should be adaptive expertise. . ., which allows a subject to apply meaningful knowledge to familiar and to unfamiliar tasks. . . . On the contrary, routine expertise is knowledge memorized by rote that can be used effectively with familiar tasks but not with novel ones. Thus, rehabilitation is not very successful if a patient learns to retrieve simple multiplication tables from memory but is unable to apply this knowledge in new situations, such as in complex calculation."	Chapter IV—see discussion of fact fluency.
Elmore, 2002, 7	". . . the usual remediation strategies we employ when kids fail to meet the statewide testing requirements are to give them the same unbelievably bad instruction they got in the first place, only in much larger quantities with much greater intensity. This is what we call the louder and slower approach."	Chapter V—see discussion on lesson models. Chapter VI—see discussions on instructional strategies.
Allington, 2005, 463	". . . no study has ever identified an educational treatment that has worked effectively for all participants."	Chapter VI—see discussion on individualization/differentiatio n.
Darling-Hammond & Falk, 1997, 193	"Effective alternatives to grade retention improve teaching by (1) ensuring that teachers have the knowledge and skills they need to teach to the new standards, (2) providing school structures and targeted services that support intensive learning, and (3) creating processes for school assessment that can evaluate opportunities to learn and leverage continuous change and improvement."	Chapter VI—see discussions on engaged time-on-task and on assessment. Chapter VII—see discussion on professional development and follow-up coaching.
Lochy, Domahs, & Delazer, 2005, 469	"What are the aims of rehabilitation in the field of number processing? Though it seems trivial at a first glance, agreement between patients, families, and therapists on the goals is a crucial precondition for successful intervention."	Chapter VII—see discussion on parental involvement.
Smith, 2002, 126	"Four Essential Conditions for Learning Mathematics 1. The mathematics must be interesting and comprehensible. 2. There's no fear of mathematics. 3. Inappropriate things aren't learned. 4. There's sufficient time."	Chapter IV—see discussion of *MLS* content. Chapter II & VII—see discussion on motivation, including the elimination of fear. Chapter II and VII—see discussions on motivation and valuing of mathematics. Chapter VI—see discussion on engaged time-on-task.
Miller & Mercer, 1997, 10	". . . a few additional components were identified (Mercer and Miller, 1992), including monitoring student progress on a frequent basis, teaching math skills to mastery, and teaching generalization."	Chapter VI—see discussions of practice/repetition, assessment for progress monitoring, and informed instruction. Chapter IV—see discussions of fact fluency and concept development.

Researcher(s)	Findings/Conclusions	*MLS* Design
Strickland, 2001, 328-330	"An examination of the characteristics of successful prevention and intervention programs reveals a fair degree of agreement regarding essential elements or components that receive attention . . . . Following is a list of principles based on consistent elements across these programs . . . . 1. Early intervention is preferable to extended remediation. 2. A systematic program of home support is essential. 3. Children considered at risk require more time on task than do others. 4. Students must be given materials they can handle successfully. 5. Careful consideration must be given to the content and nature of the learning experiences offered. 6. Individual progress monitored on a regular, ongoing basis. 7. The professional development of teachers, aides, and volunteers is a key component of success."	Chapter IV—see discussion of early school mathematics. Chapter VII—see discussion of parental involvement. Chapter VI—see discussion of engaged time-on-task. Chapter V—see discussion of manipulatives and computer-assisted instruction. Chapters IV-V-VI—see discussions of content, lesson models, and instructional strategies. Chapter VI—see discussions of assessments, progress monitoring, and informed instruction. Chapter VII—see discussion of professional development and follow-up coaching.
Swanson, Hoskyn, & Lee, 1999, 36	"Effective instructional approaches: combined approach of explicit, systematic instruction and strategic instruction: • Sequencing of instructional skills: breaking down of the task, fading of prompts or cues, sequencing short activities. • Difficulty or processed demands of task controlled: tasks are sequenced from easy to difficult. • Instructional routines (e.g., presentation of subject matter, guided and independent practice). • Modeling: teacher provides a demonstration of processes or steps to solve problem or explains how to do a task, makes use of 'think aloud.' • Drill-repetition and practice review: daily testing of skills, distributed review and practice, redundant materials or text. • Teaching to criterion."	Chapters II & VIII—see discussions on motivation and control of task difficulty. Chapter V—see discussions of lesson models, direct instruction, and mastery learning. Chapter VI—see discussion on practice/repetition. Chapter VI—see discussions on assessment and importance of 80% mastery or better.
Bryant, n.d. b, 7	"What do we know about effective instructional practices? • Modeling • Examples • Opportunities to respond • Correction procedures • Thinking aloud • Flexible grouping • Student progress monitoring • Scaffolded instruction • Strategy + automaticity interventions."	Chapter V—see discussions of lesson models, direct instruction, and mastery learning. Chapter VI—see discussion of assessment for progress monitoring and informed instruction. Chapter VI—see discussion on individualized/differentiated instruction (scaffolding). Chapter IV—see discussion on fact fluency.

Researcher(s)	Findings/Conclusions	*MLS* Design
Vaughn, Gersten, & Chard, 2000, 2	"The most interesting facet of the meta-analysis (Swanson, Hoskyn, & Lee, 1999) was that of instructional components analysis. The authors searched for factors associated with high effects—regardless of the model of instruction used or the content of instruction. . . . Through multiple regression analyses, . . . they discerned the three most critical factors: • Control of task difficulty (i.e., sequencing examples and problems to maintain high levels of student success). • Teaching students with LD in small interactive groups of six or fewer students. • Directed response questioning. . . . These three instructional components . . . have the potential to work in concert to influence, to the largest degree possible, student learning and students' independent functioning, regardless of instructional domain. . . . these aspects of instruction play a crucial role in virtually all areas of academic learning."	Chapters II & VIII—see discussions of motivation and control of task difficulty. Chapter V—see discussion of CSA sequence of lessons. Chapter V—see discussion of one-on-one tutoring and how computer-assisted instruction makes one-on-one possible even in a group. Chapter V—see discussion of direct instruction.
Vaughn, Gersten, & Chard, 2000, 3	"Similar to the findings in a broader meta-analysis (Swanson & Hoskyn, 1998), an instructional model that included only a few components predicted the magnitude of effects for higher-order processing. The important components for teaching higher-order skills to adolescents included a somewhat broader array of components than the K-12 analysis. They included: • Using extended practice with feedback. • Having small, interactive group instruction. • Using directed questioning and responses. • Breaking down tasks into component parts and fading prompts and cues. • Using technology. . . . The results of Swanson's meta-analysis suggest that providing many practice opportunities can minimize the difficulties with complex, cognitive activities experienced by students with LD."	Chapter V—see discussions of lesson models, direct instruction, and mastery learning. Chapter VI—see discussion on practice/repetition. Chapter VI—see discussion on corrective feedback. Chapter V—see discussions on one-on-one tutoring and computer-assisted instruction in lieu of small group instruction. Chapter V—see discussion on direct instruction. Chapter V—see discussion of CSA sequence and fading prompts and cues. Chapter V—see discussion on computer-assisted instruction.
Dowker, 2004, ii	"Despite such variable patterns of strengths and weaknesses, some areas of arithmetic do appear to create more problems than others for children. One of the areas most commonly found to create difficulties is *memory for arithmetical facts*. For some children, this is a specific, localized problem; for children with more severe mathematical difficulties it may be associated with exclusive reliance on cumbersome counting strategies. Other common areas of difficulty include *word problem solving*, *representation of place value* and the ability to solve *multi-step arithmetic problems"*	Chapter III—see discussions on fact fluency and dyslexia. Chapter IV—see discussion on fact fluency. Chapter V—see discussion on practice/repetition. Chapter IV—see discussion on counting. Chapter IV—see discussion on problem-solving and multi-step problems. Chapter IV—see discussion on place value.

Researcher(s)	Findings/Conclusions	*MLS* Design
Dowker, 2004, 15	"The componential nature of arithmetic is important in planning and formulating interventions with children who are experiencing arithmetical difficulties. Any extra help in arithmetic is likely to give some benefit. However, interventions that focus on the particular components with which an individual child has difficulty are likely to be more effective than those which assume that all children's arithmetical difficulties are similar (Weaver, 1954; Keogh, Major, Omari, Gandara, and Reid, 1980)."	Chapters II-III—see discussions on manifestations of difficulties and disabilities. Chapter IV—see discussion on problem areas in mathematics and how they are addressed in *MLS*. Chapter VI—see discussion on individualization/differentiation.
Brigham, Wilson, Jones, & Moisio, 1996, 2	". . . the error filled and tentative nature of competence demonstrated by typical students in this area (fractions) of achievement (Lankford Jr., 1974; Tourniaire & Pulos, 1985) suggests that carefully designed intensive instruction delivered, at least initially, by competent teachers will be the most profitable for students with LD. Additionally, students with mild disabilities require more intensive instruction to promote mathematical competence than is available given the spiral nature of many general education curricula (Parmar et al., 1994)."	Chapter III—see discussions of manifestations and treatment of learning disabilities. Chapter V—see discussions of lesson models, direct instruction, mastery learning, and one-on-one tutoring. Chapter VI—see discussions on engaged time-on-task. Chapter II—see discussions on inadequate instruction.
Lochy, Domahs, & Delazer, 2005, 476	"Findings of rehabilitation studies converge with the mentioned training studies: drill produced relative specific effects (i.e., little generalization or flexible application to other problems, in particular when improvement was measured in terms of latency. When accuracy was the critical variable, slightly better generalization effects were reported. In contrast, conceptual training allowed better transfer and generalization to untrained problems and new situations."	Chapter III—see discussions on fact fluency. Chapter IV—see discussions on fact fluency. Chapter VI—see discussions on practice/repetition. Chapter IV—see discussions on vocabulary and concept development.
Butterworth, 2005, 459	"Difficulty with basic arithmetic is a common characteristic, but dyscalculics appear to have a more fundamental problem in that they perform poorly on tasks requiring an understanding of basic numerical concepts, especially the concept of numerosity. This affects even very simple tasks such as counting or comparing numerical magnitudes."	Chapter III—see section on dyscalculia. Chapter III—see section on disabilities relating to mathematical concepts. Chapter IV—see discussion on teaching mathematical concepts and vocabulary and counting.
Siegler & Booth, 2005, 211	"Several plausible sources of difficulty have been hypothesized in all three areas of estimation: limitations of conceptual understanding, of component skills such as counting and arithmetic, and of working memory."	Chapter III—see discussion on working memory and language system disabilities. Chapter IV—see discussion on teaching estimation.
Sherman, Richardson, & Yard, 2005, 56	"Use simple numbers to explain a mathematical operation, then move to a more complex level."	Chapter IV—see description of *MLS* content. Chapter V—see discussion of CSA lesson sequence and use of manipulatives.

Researcher(s)	Findings/Conclusions	*MLS* Design
Sherman, Richardson, & Yard, 2005, 208-209	Reasons for poor achievement in mathematics: "Little Understanding of Mathematics Vocabulary. The meaning of terms such as *addend, sum, quotient, divisor, dividend, factor, numerator, denominator*, and *difference* is unknown or confusing. Limited Ability to Read Problems. This difficulty is related to students' reading levels. If the mathematics terms are unfamiliar or the vocabulary is beyond students' comprehension, the mathematics of the problem is lost. Students' abilities are also limited by difficulty with complex sentence structure and vocabulary. Limited Verbal Ability to Explain Thinking. Students who lack verbal skills have trouble expressing or explaining their thinking aloud or on paper. This error pattern has serious implications for current testing practices in which students must explain their reasoning. . . . Poor verbal skills often prevent students from getting started, and they become even more frustrated. Difficulty Focusing on Important Information. Students are unable to understand what is being asked in the problem because shapes, numbers, and/or symbols distract them. Students can neither choose the most important information nor detect a plan of action because they do not understand what is relevant and what information is unnecessary for the solution. Limited Ability to 'Picture' the Situation. The problem has no contextual meaning. Limited Self-Checking Ability. Some students have little experience in determining whether answers are reasonable. They have always asked the teacher if work is correct or accepted any answer to finish the problem. Limited Personal Appeal. Most students must want to solve problems. Motivation is limited if learners feel there is little connection and relevance to their experiences. Students then often question the usefulness of mathmatics in daily life situations or in their future. Limited Time to Solve Problems. When students feel rushed and do not take the time to think through a situation, errors result. Also, students may not check their work because they do not have enough time to thoroughly evaluate each step of the problem-solving process."	Chapter II—see discussion of language differences. Chapter IV—see discussion of vocabulary and concept development. Chapter IV—see discussion on strategies for English-language learners. Chapter III—see discussion on language system disabilities, including section on dyslexia. Chapter III—see discussion on central executive disabilities. Chapter IV—see discussion on problem solving and handling irrelevant information. Chapter V—see discussion on computer screen design. Chapter V—see discussion on CSA lesson sequences and use of manipulatives. Chapter VI—see discussion on self-assessment. Chapter III—see discussion on central executive disabilities. Chapter II—see discussion on motivation and cultural values. Chapter VII—see discussion on motivation. Chapter II—see discussion on inadequate instruction. Chapter V—see discussion on mastery learning. Chapter VI—see discussion on engaged time-on-task.
Cawley, Parmar, Foley, Salmon, & Roy, 2001, 323	"The mathematics vocabulary of students with mild disabilities is less developed than that of general education students. All available data relative to reading demonstrates that students with mild disabilities read at levels far below other students. Thus, mathematics programs that are rooted in the vocabulary-laden textbook are totally inappropriate."	Chapter III—see discussion of dyscalculia and on how dyslexia affects mathematics achievement. Chapter II—see discussion of inappropriate curriculum materials. Chapter IV—see discussion on teaching mathematics concepts and vocabulary.

Researcher(s)	Findings/Conclusions	*MLS* Design
National Research Council, 2001, 189	"Intervention studies indicate that teaching counting-on procedures in a conceptual way makes all single-digit sums accessible to U. S. first graders, including children who are learning disabled and those who do not speak English as their first language."	Chapter II—see discussion on English-language learning of mathematics. Chapter IV—see discussion on early school mathematics. Chapter IV—see discussion on teaching counting. Chapter IV—see section on teaching English-language learners.
Fazio, 1999, 429	"(a) Help the child to use alternative representations such as pictures and manipulatives as a scaffold to written calculation (paper and pencil worksheet activities). (b) Design techniques for promoting an understanding of the fundamental relationships between mathematical vocabulary and the accompanying numerical concepts such as the reciprocal nature of multiplication and division. (c) Provide instruction in compensatory strategies for retrieval of basic facts such as counting up or down from known number facts to unknown number facts. (d) Explore methods for fostering automatic retrieval of facts. (e) Assist in learning and rehearsing correct procedures for mathematical calculations. (f) Promote the use of calculators for computation once conceptual and procedural knowledge of a particular mathematical operation is demonstrated."	Chapter V—see sections on CSA lesson sequence and use of manipulatives. Chapter IV—see discussions on teaching mathematical vocabulary and concepts, as well as counting and algorithms. Chapter III—see section on difficulties in fact fluency. Chapter IV—see section on teaching fact fluency. Chapter VI—see section on practice/repetition. Chapter V—see section on computer-assisted instruction.
Fazio, 1999, 429	"A third intervention strategy suggested by the present research relies on the interaction between knowledge of arithmetic procedures, mathematical vocabulary, and mathematical concepts. Children with delayed language abilities who process information slowly need well-defined, integrated conceptual and procedural knowledge of arithmetic. Effective calculation methods require a strong understanding of math operations and number relationships."	Chapter III—see discussions of disabilities relating to mathematical concepts and procedures. Chapter III—see section on language system disabilities. Chapter IV—see discussions on teaching mathematics concepts and vocabulary, on algorithms, on the development of fact fluency.
Sherman, Richardson, & Yard, 2005, 65	"Subtraction of whole numbers is represented in the physical world by the complement of one set within a larger set. The cardinal numbers of the large set represents the minuend, one of the subsets is the subtrahend, and the cardinal number of the remaining set is the difference. There are at least two situations depicting subtraction. One involves 'take away,' where something is taken away; and the other involves 'how many more,' where two sets are compared to determine how many more are in the larger set. . . . Children need to understand both subtraction situations well before beginning work on subtraction algorithms."	Chapter IV—see discussions on teaching mathematical concepts and vocabulary and on teaching algorithms. Chapter IV—see discussion on fact fluency.

Researcher(s)	**Findings/Conclusions**	***MLS* Design**
National Research Council, 2001, 191	"Intervention studies with U. S. first graders that helped them see subtraction situations as taking away the first x objects enabled them to learn and understand counting-up-to procedures for subtraction. Their subtraction accuracy became as high as that for addition."	Chapter IV—see section on early school mathematics. Chapter IV—see section on concept development. Chapter IV—see section on counting. Chapter IV—see section on algorithms.
Vaughn, Gersten, & Chard, Fall 2000, 7	"Given the increasing numbers of students with LD who are provided instruction in the general education classroom, teachers and parents need not be concerned that the application of interventions that are effective for students with disabilities will provide less than effective outcomes for students without disabilities. Research conducted with individuals with LD has educational benefits for all learners, thus providing for generalizability of effective interventions for students with disabilities to a broader learning community."	Chapter II—see discussions on mathematics difficulties. Chapter III—see discussions on mathematics disabilities.
Jones, Wilson, & Bhojwani, 1997, 156	"Effective curricula provide for an economical or parsimonious, use of time and resources. Woodward (1991) contended that emphasis should be given to mastery of concepts, relationships, and skills that are essential for the subsequent acquisition and functional generalization of math skills. Curricula should be organized so that instruction of specific skills and concepts is tightly interwoven around critical concepts. Woodward's test for the parsimony of an instructional program is whether or not what is learned at one time will be used later."	Chapter IV—see description of *MLS* content and discussions on teaching to mastery the concepts and procedures critical to later success in mathematics.
Mercer & Mercer, 2005, 483	"Many students with learning problems experience math difficulties. However, if educational researchers can scientifically tap the potential benefits of research-driven principles and if teacher educators and publishers of commercial materials can place the products of these findings in the hands of teachers, educators have an opportunity to improve significantly the math learning of students and the math instruction of teachers."	Chapter II explains learning difficulties. Chapter III explains learning disabilities. Chapter II also includes a discussion of inappropriate mathematics instruction, including the use of inappropriate curriculum materials. Chapter VII—see discussions on lab teacher/facilitator role and on professional development and on-going coaching.
Erlauer, 2003, 81	"Our brains have a capacity to remember the equivalent of approximately 10 million books of 1,000 pages each. This incredible statistic would lead us to believe that remembering and learning should be an easy task. However, the rest of the story is that only one out of every hundred bits of information received by the brain makes it to long-term memory."	Chapter III—see section on information processing. Chapter VI—see discussion of multi-sensory processing.
Miller & Mercer, 1997, 5	"The information-processing model provides numerous perspectives for examining the math difficulties of students with learning disabilities. Information-processing theory focuses on which information is acquired and how. Its primary features include attention, sensation, perception, short-term memory, long-term memory, and response."	Chapter III—see section on information processing. Chapter VI—see section on multi-sensory processing. Chapter VI—see sections on chunking/clustering and on practice/ repetition.

Researcher(s)	Findings/Conclusions	*MLS* Design
Mercer & Mercer, 2005, 429	"Fuchs and Fuchs (2001) present four principles of prevention of math difficulties: instruct at a quick pace with varied instructional activities and high levels of engagement, set challenging standards for achievement, incorporate self-verbalization methods, and present physical and visual representations of number concepts or problem-solving situations. . . . Fuchs and Fuchs note that there should be a focus on the individual student as the unit for instructional decision-making, intensive instruction delivery, and explicit conceptualization of skills-based instruction."	Chapter V—see discussion of direct instruction. Chapter VI—see discussion of engaged time-on-task. Chapter VI—see discussion of assessment. Chapter IV—see discussion on teaching fact fluency. Chapter VI—see discussion on practice/ repetition (and self-verbalization) Chapter V—see sections on CSA lesson sequence and use of manipulatives. Chapter VI—see discussion on individualization/differentiation.
Miller & Mercer, 1997, 10	"Mastropieri, Scruggs, and Shiah (1991) conducted an extensive literature search and located 30 studies that validated instructional techniques for teaching mathematics to students with learning disabilities. Included among those techniques were (a) implementing demonstration, modeling, and feedback procedures; (b) providing reinforcement for fluency building; (c) using a concrete-to-abstract teaching sequence; (d) setting goals; (e) combining demonstration with permanent model; (f) using verbalization while solving problems; (g) teaching strategies for computation and problem solving; and (h) using peers, computers, and videodiscs as alternative delivery systems."	Chapter III—see discussions on learning disabilities. Chapter V—see discussions on lesson models, including direct instruction and mastery learning. Chapter VI—see section on corrective feedback. Chapter III and Chapter IV—see sections on development of fact fluency. See also section on practice/repetition in Chapter VI. Chapter V—see sections on concrete-to-abstract teaching (CSA). Chapters II and VII—see sections on motivation. Chapter III contains also a section on central executive. Chapter V—see section on use of manipulatives. Chapter IV—see sections on teaching algorithms and problem solving. Chapter V—see discussion of computer-assisted instruction.
Mercer & Mercer, 2005, 131	"The findings of mathematics research indicate that students can benefit from instruction that includes both explicit and implicit methods (Mercer, Jordan, & Miller, 1994). The literature supports explicit methods such as description of procedures, modeling of skills . . ., use of cues and prompts, direct questioning of students to ensure understanding, and practice to mastery."	Chapter II—see discussion on inappropriate instruction. Chapter V—see section on direct instruction. Chapters III and IV include sections on development of fact fluency. Chapter VI includes a discussion on practice/repetition.
Mercer & Mercer, 2005, 133	"Teachers and teacher educators have a responsibility to examine the research and apply the findings as they develop teacher practices. Greenwood, Arreaga-Mayer, and Carta (1994) found that students in classrooms in which teachers used research-based interactive teaching practices had higher academic engagement times and achievement scores than students in classrooms in which teachers used other methods."	Chapter V—see discussion of interactive nature of computer-assisted instruction. Chapter VI—see discussion on engaged time-on-task. Chapter VII—see discussion on the role of the *MLS* teacher/facilitator and on motivation.

Researcher(s)	**Findings/Conclusions**	***MLS* Design**
Sherman, Richardson, & Yard, 2005, 209-210	"Instructional Strategies *Establishing a Context for Interest*. An important instructional technique is to embed problem solving in mathematics lessons by relating the problem to students' interests.	Chapter II—see discussion on motivation. Chapter IV—see discussion of *MLS* content.
	Teaching a Variety of Heuristics. The teacher focuses on how to use a particular strategy, such as drawing a picture, using manipulatives, or finding patterns by carrying them out with students during lessons.	Chapter IV—see discussion on problem solving. Chapter V—see discussion on CSA lesson sequence and use of manipulatives.
	Grouping Similar Types of Problems that Call for Similar Types Together. This idea helps students find patterns in solution attempts.	Chapter IV—see *MLS* scope and sequence and discussion on problem solving. Chapter VI—see chunking/clustering.
	Starting with Simple Problems. Solutions are more easily found and confidence is built when students experience success quickly. They are more willing to take risks after knowing they are, in fact, able to find solutions correctly.	Chapter IV—see section on problem solving. Chapter V—see discussion on CSA sequence of lessons.
	Rewarding Students for Small Steps of Success. Frequent words of praise and positive comments on written work for step-by-step improvement are powerful tools for encouragement. Suggestions and hints are also encouraging because they spur students from first attempts throughout the problem-solving procedure.	Chapter VI—see section on corrective feedback. Chapter VII—see section on motivation. Chapter IV—see problem-solving.
	Compiling a Mathematics Dictionary Journal with Students. Important mathematical terms should be found in dictionaries and also discussed in class. The words should be defined and further identified with drawings. For example, have students write a definition for *dividend* and draw an arrow to the dividend in a long division problem.	Chapter IV—see section on teaching mathematics concepts and vocabulary.
	Provide Sufficient Time for Solving Problems. *Simplifying Numbers.* *Reduce Reading Difficulties*. Reduce the number of words and/or record the problems on a tape recorder."	Chapter VI—see section on engaged time-on-task. Chapter V—see discussion of computer-assisted instruction.
Sousa, 2001, 153	"Mathematical disorders often arise when students fail to understand the language of mathematics, which has its own symbolic representations, syntax, and terminology. Solving word problems requires the ability to translate the language of English into the language of mathematics. The translation is likely to be successful if the student recognizes English language equivalents for each mathematical statement. . . . Learning to identify and correctly translate mathematical syntax becomes critical to student success in problem solving."	Chapter II—see discussion on how language affects mathematical performance. Chapter III—see discussion on disabilities relating to learning mathematical concepts and on language system disabilities. Chapter IV—see discussion on teaching mathematical concepts, including problem solving. Chapter IV—see section on teaching English-language learners.

Researcher(s)	Findings/Conclusions	*MLS* Design
Sousa, 2001, 153	"Language can be an obstacle in other ways. Students may learn a limited vocabulary for performing basic arithmetic operations, such as 'add' and 'multiply,' only to run into difficulties when they encounter expressions asking for the 'sum' or 'product' of numbers. Teachers can avoid this problem by introducing synonyms for every function."	Chapter II—see discussion on how language affects mathematical performance. Chapter III—see discussion on disabilities relating to learning mathematical concepts and on language system disabilities. Chapter IV—see discussion on teaching mathematical concepts, including problem solving. Chapter IV—see section on teaching English-language learners.
Griffin, 2005, 266	" . . . several developmental principles that should be considered in building learning paths and networks of knowledge . . . for the domain of whole numbers have come to light. They can be summarized as follows: • Build upon children's current knowledge. . . . • Follow the natural developmental progression when selecting new knowledge to be taught. By selecting learning objectives that are a natural next step for children . . ., the teacher will be creating a learning path that is developmentally appropriate for children, one that fits the progression of understanding as identified by researchers. • Make sure children consolidate one level of understanding before moving on to the next. For example, give them many opportunities to solve oral problems with real quantities before expecting them to use formal symbols. • Give children many opportunities to use number concepts in a broad range of contexts and to learn the language that is used in these contexts to describe quantity."	Chapters III-IV—see discussions for teaching mathematical vocabulary and concepts. Chapter V—see discussion of CSA lesson sequence. Chapter VI—see discussion on individualization/differentiation. Chapter VI—see discussion on practice/repetition. Chapter VI—see section on assessment and importance of 80% mastery. Chapter IV—see section on teaching mathematical vocabulary and concepts.
Wakefield, 1999, 235	"Piaget said that children cannot see, hear, or remember that which they cannot understand. If the mental structures are not in place to support what is seen or heard, there will be no mental connection, and consequently it will not be remembered."	Chapter II—see discussions of learning difficulties. Chapter III—see discussions of diverse learning disabilities. Chapter III—see section on information processing and on language and visuo-spatial learning disabilities. Chapter V—see sections on CSA lesson sequences and use of manipulatives. Chapter VI—see sections on multi-sensory processing, chunking/clustering, and practice/repetition.
Whitehurst, n.d., 3	". . . there is research that suggests where some of practices and assumptions of both the constructivists and their critics may require more nuanced implementation."	Chapter II—see discussions on inappropriate instruction and the math wars.

Researcher(s)	Findings/Conclusions	*MLS* Design
Reys, 2001, 261	"Standards-based materials help students make sense of mathematics in several ways. Sense-making is promoted by spending substantial time on the fundamental ideas of a mathematical domain, such as rational numbers."	Chapter II—see discussions on inappropriate instruction. Chapter IV—see description of *MLS* content. Chapter IV—see section on fractions.
Brysbaert, 2005, 33	"A first robust finding is that the processing is more demanding for larger numbers than for smaller numbers."	Chapter IV—see discussions on mathematical cognition and early school mathematics.
Brysbaert, 2005, 33	"A second robust finding in Arabic numeral processing is that when two numbers are processed together, processing times are influenced by the distance between the numbers. This is particularly clear when both numbers have to be compared, as it is much easier to say which digit is the smaller for the pair 2-8 than for the pair 2-3. More precisely, decision times are a function of the logarithm of the distance between the two numbers. Another distance-related effect that has been described is the number priming effect. A target digit is recognized faster when it follows a (tachistoscopically presented) prime with a close value than when it follows a prime with a more distant value."	Chapter IV—see discussions on mathematical cognition and definitions of mathematics.
Brysbaert, 2005, 35	"A third major finding about the processing of Arabic numerals is that the semantic magnitude information of the numeral is activated more rapidly than is the case for verbal numerals."	Chapter IV—see discussions on mathematical cognition and definitions of mathematics.
Campbell & Epp, 2005, 357	"Kashiwagi, Kashiwagi, and Hasegawa (1987) studied Japanese aphasics with impaired performance for simple multiplication. Despite extensive practice, the patients could not relearn multiplication with verbal presentation and responses. They did, however, learn to generate the multiplication facts given visual presentation combined with written responses. Such findings support the theory that the representations underlying multiplication facts can involve multiple codes that are differentially involved as a function of surface form."	Chapter III—see discussions of learning disabilities. Chapter III—see section on information processing and language system disabilities. Chapter VI—see section on multi-sensory processing.
Campbell & Epp, 2005, 357	"Our review identified a variety of types of evidence for the conclusion that retrieval processes for simple arithmetic depend to some extent on surface format."	Chapters III-IV include sections on the development of fact fluency (including retrieval).
Lochy, Domahs, & Delazer, 2005, 473	"Calculation training took place twice weekly over 8 weeks. Problems were presented visually and at the same time read aloud by the therapist. The patients were allowed to answer in their preferred modality. Instant feedback was provided and errors were discussed with the patients if necessary. Training led to long-lasting improvements, evidenced by accuracy rates of more than 90%."	Chapter III—see discussion on information processing and on language and visuospatial disabilities. Chapter VI—see section on multi-sensory processing. Chapter VI—see section on corrective feedback. Chapter VI—see section on practice/ repetition.

Researcher(s)	Findings/Conclusions	*MLS* Design
Erlauer, 2003, 56	"Knowing how the brain chunks and categorizes information is useful to teachers in helping students connect new information to prior knowledge. For instance, demonstrating how the new skill or multiplication is related to the previously learned concept of addition can make it easier for the students' brains to make connections and learn the new concept. An important thing for teachers to keep in mind is that one student's brain may chunk or categorize information differently from another student's brain."	Chapter VI—see section on chunking/ clustering. Chapter VI—see discussion on individualization/differentiation.
Kibel, 1992, 45	"Strong kinesthetic and visual images should underlie mathematical terms. I always arrange for considerable overlearning of the language, and ensure that abstract terms are linked to a concrete base."	Chapter III—see section on information processing. Chapters III-IV include sections on development of fact fluency. Chapter VI—see section on practice/ repetition.
Kibel, 1992, 45	"Concepts should not be passed on ready-made. They should be allowed to grow in concrete situations and only later should formal written work take place."	Chapters III-IV include sections on teaching mathematical concepts and vocabulary. Chapter V—see sections on CSA lesson sequences and use of manipulatives.
Rose & Meyer, 2002, 17	"Because smoothly functioning recognition networks take advantage of both top-down and bottom-up processes, teaching to both processes rather than focusing exclusively on one or the other is the wisest choice. A positive example is the recent truce in the 'phonics wars.' Most programs have not adopted a form of reading instruction that incorporates both the top-down whole language method and bottom-up phonics. This balanced approach is consistent with the way the learning brain works."	Chapter II—see sections on inappropriate instruction and the math wars.
Mercer & Mercer, 2005, 139	". . . generalization to new situations occurs when a student demonstrates proficiency in math facts and continues to respond quickly and accurately when these facts are embedded in calculation problems."	Chapters III-IV include sections on the development of fact fluency. Chapter VI—see section on practice/ repetition.
Garnett, 1998, 2	"Several curriculum materials offer specific methods to help teach mastering of basic arithmetic facts . . . . Suggestions from these teaching approaches include: • Interactive and intensive practice with motivational materials such as games. • Distributed practice, meaning much practice in small doses. • Small numbers of facts per group to be mastered at one time. • Emphasis is on 'reverses,' or 'turnarounds' (e.g., 4+5/5+4, 6x7/7x6). • Student self-charting of progress. • Instruction, not just practice."	Chapters I and IV include descriptions of *MLS* content, including a fact fluency game called *Digit's Widgets*. Chapter VI—see section on practice/ repetition. Chapter VI—see section on chunking/ clustering. Chapter IV—see description of *MLS* scope and sequence; sections on teaching fact fluency and algorithms. Chapter VI—see section on self-assessment. Chapters IV-V-VI—see discussions of content and instructional strategies that go way beyond practice.

Researcher(s)	Findings/Conclusions	*MLS* Design
Furner & Duffy, 2002, 68	"Based on a culmination of research, Zemelman, Daniels, and Hyde (1998) have put together what is considered the best practice for teaching math: • use of manipulatives (make learning math concrete) • use of cooperative group work • use of discussion • use of questioning and making conjectures • use of justification of thinking • use of writing in math for thinking, expressing feelings, and solving problems • use of problem-solving approaches to instruction • making content integration a part of instruction • use of calculators, computers, and all technology • teachers serving as facilitators of learning • assessments of learning as a part of instruction."	Chapter V—see section on manipulatives and CSA lesson sequence. Chapter V—see descriptions of lesson model components used in *MLS.* Chapter IV—see description of *MLS* content—problem-solving. Chapter V—see section on computer-assisted instruction. Chapter VII—see discussion of role of *MLS* lab teacher/facilitator. Chapter VI—see sections on assessment and progress monitoring.
Lock, 1996, 1	"In general education classrooms, adaptations and modifications in mathematics instruction are appropriate for all students, not just students with LD."	Chapters II-III describe the diversity of learning difficulties and disabilities.
Lock, 1996, 5	". . . six problem-solving strategies: • Read and understand the problem. • Look for the key questions and recognize important words. • Select the appropriate operation. • Write the number sentence (equation) and solve it. • Check your answer. • Correct your errors."	Chapter IV—see section on problem solving.
Lock, 1996, 6	"Hammill and Bartel . . . offer many suggestions for modifying mathematics instruction for students with LD. They encourage teachers to think about how to alter instruction while maintaining the primary purpose of mathematics instruction: Competence in manipulating numbers in the real world. Their suggestions include: 1. Altering the type or amount of information presented to a student such as giving the student the answers to a story problem and allowing the student to explain how the answers were obtained. 2. Using a variety of teacher-input and modeling strategies such as using manipulatives during the instructional phase with oral presentations."	Chapter III—see discussion on learning disabilities. Chapter IV—see section on problem solving. Chapter V—see section on CSA lesson sequence. Chapter V—see discussions on modeling and on use of manipulatives.
Lock, 1996, 6-7	"Teaching key math terms as a specific skill rather than an outcome of basic math practice is essential for students with LD."	Chapters III-IV—see sections on teaching mathematics vocabulary.

Researcher(s)	Findings/Conclusions	*MLS* Design
Jones, Wilson, & Bhojwani, 1997, 153	"Students learn from examples. An important part of the business of education is selecting and organizing examples to use in instruction such that students will be able to solve problems they encounter outside of instruction."	Chapter IV—see discussions on *MLS* content, including problem-solving.
Ontario Ministry of Education, 2005, 72	"Children with special needs often require additional time, many concrete experiences in different contexts, and extra guidance from the teacher to understand and demonstrate their mathematical knowledge."	Chapter VI—see section on engaged time-on-task. Chapter V—see section on CSA lesson sequence and use of manipulatives. Chapter VII—see section on role of *MLS* lab teacher/facilitator.
Ontario Ministry of Education, 2005, 77	"Outcomes for children across ability levels and for children with specific difficulties in mathematics are improved when math problem-solving instruction is overt, systematic and clear, and scaffolded by the teacher and peers."	Chapter IV—see section on problem solving. Chapter V—see section on direct instruction. Chapter VI—see section on individualized/differentiated instruction.
McEwan, 2000, 43	"When students don't understand subject matter, they want coherent explanations, plenty of worked-out examples from which to draw conclusions, and problem-solving demonstrations, along with the strong sense than an adult is in charge. "Students feel uneasy and stressful when a classroom is chaotic and their classmates (particularly a few disruptive ones) are in control rather than the teachers. "Students (especially those with attention and learning disorders) grow agitated with noise and disturbances and fail to learn. "Many students have a feeling of urgency about how much they have to learn and grow impatient with pooling their ignorance in groups where there is neither individual nor group accountability."	Chapter IV—see description of *MLS* content, including problem solving. Chapter V—see discussion of lesson models. Chapter VII—see section on role of *MLS* lab teacher/facilitator. Chapter VII—see section on role of *MLS* lab teacher/facilitator. Chapter III—see section on central executive disabilities. Chapters II and VII—see sections on motivation. Chapter V—see sections on one-on-one tutoring and computer-assisted instruction.
McEwan, 2000, 50-51	"The following categories summarize the critical aspects of effective instruction: 1. Instruction is guided by a preplanned curriculum (Venezky & Winfield, 1979). . . . 2. There are high expectations for student learning (Phi Delta Kappa, 1980). . . . 3. Students are carefully oriented to lessons (Stallings, 1979). . . . 4. Instruction is clear and focused (Lortie, 1975). . . . 5. Learning progress is monitored closely (Evertson, 1982). . . . 6. When students don't understand, they are retaught (Rosenshine, 1983). . . . 7. Class time is used for learning (Stallings, 1980). . . .	Chapter IV—see description of *MLS* content. Chapter VI—see assessment section and expectation of 80% mastery. Chapter VII—see section on role of *MLS* teacher/facilitator. Chapter IV—see discussion of *MLS* content. Chapter VI—see section on assessment for progress monitoring and on informed instruction. Chapter I and IV—see description of *MLS* structure with automatic recycling when mastery is not achieved. Chapter VI—see discussion of engaged

Researcher(s)	Findings/Conclusions	*MLS* Design
	8. There are smooth, efficient classroom routines (Brophy, 1979)…. 9. The instructional groups formed in the classroom fit instructional needs (Stallings, 1979). 10. Standards for classroom behavior are explicit (Anderson, 1980). . . . 11. Personal interactions between teachers and students are positive (Rutter, Maugham, Mortimore, & Ouston, 1979) . . . . 12. Incentives and rewards for students are used to promote excellence (Emmer & Evertson, 1981).	time-on-task. Chapter VII—see teacher's role in classroom management. Chapter V—see discussions of one-on-one tutoring and computer-assisted instruction. Chapters II and VII—see discussions on student motivation. Chapter VII—see section on motivational rewards. Chapter VI—see section on corrective feedback.
Fuchs & Fuchs, 2003, 318	"With respect to the state of knowledge about 'effective instruction,' we offer the following observation. Within special education, research on instructional practices is strong on process variables that provide the foundation for instruction. These variables . . . include, but are not limited to, modeling, quick pace, frequent responding and high proportion of engaged time, overrehearsal, and guided feedback."	Chapter III—see discussions of learning disabilities. Chapter V—see discussions on lesson phases and the components of direct instruction, mastery learning, and one-on-one tutoring. Chapter VI—see discussions of practice/ repetition (overrehearsal), engaged time-on-task, and corrective feedback.
Fuchs & Fuchs, 2001, 85-86	". . . a substantial body of intervention studies provides the basis for specifying methods to prevent and treat mathematics difficulties. . . . Primary prevention focuses on universal design. With universal design, instruction for all students is formulated to incorporate principles that address the needs of specialized populations while benefiting (or at least not harming) others."	Chapter II—see discussions of learning difficulties. Chapter III—see discussions of learning disabilities. Chapter VI—see discussions of instructional strategies, especially multi-sensory processing, individualization/ differentiation, and practice/repetition.
Fuchs & Fuchs, 2001, 86-87	"The effective teacher incorporated a dramatically quicker pace, and this faster pace resulted in more activities in every lesson. . . . As might be expected, the effective teacher's quick instructional pacing and varied instructional formats led to more active student involvement."	Chapter V—see discussions of lesson phases and direct instruction. Chapter VI—see discussion of engaged time-on-task.
Fuchs & Fuchs, 2001, 87	"The effective teacher simply devoted more effort to motivating her students; she incorporated 6 times more motivating statements and activities into her lessons."	Chapter II—see discussion on motivation. Chapter VII—see discussion on motivation.
Fuchs & Fuchs, 2001, 87	"Research in mathematics has specifically identified cognitive strategy instruction as an effective instructional tool. Students are taught and memorize explicit steps for approaching and solving problems, and they apply these steps by verbalizing them, first overtly and gradually fading their overt use over time."	Chapter IV—see discussions of problem solving, algorithms, and fact fluency. Chapter VI—see discussions on practice/ repetition.
Fuchs & Fuchs, 2001, 88	". . . research demonstrates that using physical and visual representations to facilitate conceptual understanding helps children master and maintain mathematical competence."	Chapter V—see discussion of concrete—semiconcrete—abstract lesson sequence and discussion on use of manipulatives.

Researcher(s)	Findings/Conclusions	*MLS* Design
Fuchs & Fuchs, 2001, 91	"Individually referenced decision making is perhaps the signature feature of effective special education intervention. . . . Evidence documents how individually referenced decision making enhances learning for students with LD."	Chapter VI—see discussion on individualization/differentiation of instruction. Chapter III—see discussion of learning disabilities.
Fuchs & Fuchs, 2001, 91	"Results showed that students whose instructional decisions were tailored to their own ongoing assessment profiles achieved reliably better than their matched pairs, and that measurement alone contributed little to student achievement."	Chapter VI—see discussion on the use of assessment to inform instruction. Chapter IV—see discussion on individualization/differentiation.
Fuchs & Fuchs, 2001, 92	"First, the study illustrates how intensive instruction can produce excellent growth rates among students with LD. . . . intensive instruction refers to a broader set of instruction features including, but not limited to, (1) high rates of active responding at appropriate levels, (2) careful matching of instruction with the individual student's skill levels, (3) instructional cues, prompts, and fading to support approximations to correct responding, and (4) detailed task-focused feedback. . . ."	Chapter V—see discussion of mastery learning and CSA lesson sequence. Chapter VI—see discussions of engaged time-on-task, individualization/ differentiation, practice/repetition, and corrective feedback.
Elbaum & Vaughan, 2003, 235	". . . students with LD who have truly low self-concepts can benefit considerably from appropriate interventions. For these students the most effective interventions . . . may differ according to students' age. The fact that most effective interventions for younger students appear to be academic interventions suggests that improving these students' academic skills can have a collateral effect on their self-perceptions."	Chapter II—see discussions on math phobia and motivation issues. Chapter VII—see discussion on motivation.
McREL, 2002, 1	"In 2002, McREL conducted a synthesis of recent research on instructional strategies to assist students who are low achieving or at risk of failure. From this synthesis of research, McREL identified six general classroom strategies that research indicates are particularly effective in helping struggling students achieve success: • Whole-class instruction that balances constructivist and behaviorist strategies. • Cognitively oriented instruction that combines cognitive and meta-cognitive strategies with other learning activities. • Small groups of either like-ability or mixed-ability students. • Tutoring that emphasizes diagnostic and prescriptive interactions. • Peer tutoring, including classroom-wide tutoring, peer-assisted learning strategies, and reciprocal peer tutoring. • Computer-assisted instruction in which teachers have a significant role in facilitating activities (1).	Chapter V—see discussion on direct instruction. Chapter VI—see discussions on instructional strategies. Chapter VIII—see discussion on balanced curriculum. Chapter V—see discussions on mastery learning and one-on-one tutoring. Chapter V—see discussion on tutoring. Chapter VI—see discussion on individualization/differentiation. Chapter V—see discussion on computer-assisted instruction.

Researcher(s)	Findings/Conclusions	*MLS* Design
Balfanz, Legters, & Jordan, 2004, 4	"What is clear is that the type of accelerated learning required by poorly prepared students in high-poverty high schools needs to involve more than narrow test preparation. It is to be substantial and sustained and enable students to rapidly develop declarative, procedural, and meta-cognitive knowledge (Kilpatrick et al., 2001). It also has to motivate students to learn and take advantage of the strengths they bring to the classroom."	Chapter II—see discussion on cultural attitudes toward mathematics. Chapter IV—see discussion on content—both concepts and procedures. Also, see Chapter VIII discussion of balanced curriculum. Chapter VII—see discussion on motivation.
Texas Education Agency, 2003, 34	"Components of early mathematics curriculum are listed, along with examples of practices. Delivery of Instruction 1. Instruction based on students' informal mathematical knowledge 2. Instruction based on various activities that are active and rich in mathematical language 3. Explicit instruction using modeling and thinking aloud 4. Balanced instruction with conceptual understanding and procedural skills development 5. Corrective feedback and appropriate reinforcement 6. Guided practice and sufficient time to review prerequisite skills and practice new skills 7. Teaching skills to mastery Instructional Grouping 1. Small groups (3-5 students), similar-ability groups of students receiving 20 minutes of instruction identified for their needs 2. Student pairs with a higher performing student helping a struggling student 3. Instructional grouping based on assessment of instructional needs 4. Various grouping formats, depending on the purpose of the lesson and the needs of students Instructional Materials/Technology 1. Diverse activities of various levels of difficulty to meet students' needs 2. Classroom materials that cover and enhance early mathematics skills 3. Grade-appropriate mathematics texts that cover the critical components of a mathematics curriculum for the early grades and are based on real-life application 4. Concrete and visual manipulatives for understanding and communication 5. Classroom materials that include game-like activities to engage students 6. CD-ROM and activities that enable students to solve problems systematically and repeatedly for mastery and transfer."	Chapter IV—see discussions on early mathematics learning. Chapter IV—see discussions on teaching mathematics vocabulary. Chapter V—see discussion on lesson phases, including modeling. Chapter III—see discussion on importance of concepts and procedures. Chapter IV—see sections on concepts and procedures. Chapter VI—see discussion on corrective feedback. Chapter V—see discussion on lesson models. Chapter V—see discussion of mastery learning; also Chapter VI—discussion on assessment. Chapter V—see discussion on one-on-one tutoring; also Chapter VI on individualization/differentiation. Chapter VI—see section on using assessment to inform instruction. Chapters V and VI—see sections on one-on-one instruction, computer-assisted instruction, and individualization. Chapter VI—see discussion on individualization/differentiation. Chapter IV—see section on early mathematics learning. Chapter II—see discussion on inappropriate mathematics curriculum materials. Chapter V—see discussion of concrete—semiconcrete—abstract lesson sequence and on use of manipulatives. Chapter VI—see descriptions of *Digit's Widgets* and other math games in *MLS*. Chapter IV—see discussion on problem solving.

Balanced Curriculum: Both Concepts and Procedures

Given the intensity of the math wars and the continuing debate about what to teach, as well as how to teach, it seems prudent once again to review research on the importance of including both concepts and procedures in an intervention for learners who struggle. Table 88 includes additional findings to those cited in Chapters III and IV about a balanced curriculum approach.

Table 88: Importance of a Balance of Concepts and Procedures

Researcher(s)	Findings/Conclusions
Siegler, 2003, 226	"Although there are exceptions, procedural skill and conceptual understanding usually are highly correlated."
Siegler, 2003, 227	"It turns out that a substantial percentage of children first gain conceptual understanding and then procedural competence, but that another substantial percentage do the opposite (Hiebert & Wearne, 1996)."
Siegler, 2003, 227	"Studies aimed at improving teaching of multidigit addition and subtraction typically emphasize steps in the procedures to the concepts that support them. In general, these teaching techniques successfully increase both conceptual and procedural knowledge. Although not currently conclusive, they suggest that instruction that emphasizes conceptual understanding as well as procedural skill is more effective in building both kinds of competence than instruction that only focuses on procedural skill (Fuson & Briars, 1990; Hiebert & Wearne, 1996)."
Siegler, 2003, 227	"The conceptually-oriented instruction produced substantial gains in both kinds of knowledge; the procedurally-oriented instruction produced substantial gains in procedural knowledge and smaller gains in conceptual knowledge. To the degree that this result proves general, it suggests that conceptual instruction should be undertaken before instruction aimed at teaching procedures (Rittle-Johnson & Alibali)."
Jones, Wilson, & Bhojwani, 1997, 156	"Effective curricula provide for an economical or parsimonious, use of time and resources. Woodward (1991) contended that emphasis should be given to mastery of concepts, relationships, and skills that are essential for the subsequent acquisition and functional generalization of math skills. Curricula should be organized so that instruction of specific skills and concepts is tightly interwoven around critical concepts. Woodward's test for the parsimony of an instructional program is whether or not what is learned at one time will be used later."
Battista, 1999, 428	"Sound curricula must include clear long-range goals for ensuring that students become *fluent* in employing those abstract concepts and mathematical perspectives that our culture has found most useful. Students should be able to apply, readily and correctly, important mathematical strategies and lines of reasoning in numerous situations. They should possess knowledge that supports mathematical reasoning. For instance, students should know the 'basic number facts' because such knowledge is essential for mental computation, estimation, performance of computational procedures, and problem solving."
Daro, Feb. 15, 2006, 34	"Evidence from countries that perform well in mathematics shows that the war is phony. What's needed in mathematics is not one paradigm or another, but common-sense—and carefully engineered—changes in what we teach. The countries that do well in international comparisons do not choose between skills or problem-solving; they teach concepts and skills and problem-solving."
Fuchs & Fuchs, 2001, 87	". . . studies demonstrate the importance of conceptual understanding, not only to facilitate application of procedural knowledge, but also to accomplish long-term retention of procedural competence."

Researcher(s)	Findings/Conclusions
Fuchs & Fuchs, 2001, 92	"The notion of isolated skills instruction has been replaced with more contextualized presentations, where strategies for applying skills within generalized contexts are taught explicitly. Research documents the potential value of situating explicit skills instruction within structured, motivating, and authentic contextualized applications for knowledge application to occur."

In summary, designing an effective mathematics intervention is very complex work. Schoenfeld (2002) notes that "People need to understand that simple-minded solutions to complex problems won't work, and that progress is best made by building carefully on a well-established base" (p. 31). The well-established base on the root causes of learning difficulties and disabilities and the scientific evidence available on appropriate mathematics content, lesson models, instructional strategies, assessments, and implementation support all serve to ground the *MLS* program and to predict its effectiveness with learners who struggle.

The Imperative of Opportunity to Learn

Enough is known, clearly, to understand that just because a student manifests learning difficulties or disabilities does not mean that the student cannot learn mathematics. Effective interventions are not easy to deliver, motivating students to believe in the efficacy of effort is not easy either, and implementation presents its own challenges. However, educators have a moral imperative to teach all children to the best of their knowledge and ability, and that includes the adoption of appropriate curriculum and curriculum materials, such as *MLS*. The research on inadequate instruction presented in Chapter II is a major part of the problem that struggling learners must overcome, and when that inadequacy is coupled with inappropriate instruction (also discussed in Chapter II), then the problem is compounded, whether the student has learning difficulties or disabilities.

Not in every single case, but in almost every case, the student failing to acquire mathematical knowledge and skills has simply been denied the opportunity to learn. In the late 1980s and early 1990s, the federal government made much ado about opportunity-to-learn (OTL) standards. The expectation that those standards be developed was embedded in the establishment of Goals 2000, yet never really enforced. Noddings (1997) notes that the OTL standards "define the availability of programs, staff, and other resources that schools, districts, and states provide so that students are able to meet challenging content and performance standards" (p. 185). Some interpreted OTL standards to encompass social services, health care, parent education, and other initiatives that support student learning, but others confined their definitions to academic support and appropriate interventions. Cawelti (1995) points out, for example, that "The extent of students' opportunity to learn mathematics content bears directly and decisively on student mathematics achievement" (p. 102). OTL further means appropriate core instruction and appropriate interventions for students falling behind for whatever reason.

The National Research Council (2001) makes note of the concept of OTL in *Adding It Up*:

> The circumstances that allow students to engage in and spend time on academic tasks such as working on problems, exploring situations and gathering data, listening to explanations,

> reading texts, or conjecturing and justifying have been labeled as *opportunity to learn*. As might be expected, students' opportunity to learn affects their achievement (p. 333).

Schoenfeld (2002) adds this sobering conclusion:

> Hence conversations about the mathematical needs of American students must focus not only on what mathematics the students should learn, but also on how we as a nation can insure that all students have the opportunity to learn it (p. 214).

Becoming proficient in mathematics is also related to equity, as was evident in the discussion of the status of mathematics achievement in Chapter I. Elmore and Fuhrman (1995) state the following:

> Holding ambitious expectations for all students should promote equity, by providing a clear message to teachers, parents, and students about what constitutes successful performance in school. At the same time, schools and districts vary considerably in the opportunities they presently provide for students to be taught the content and to meet the performance expectations contained in the proposed standards. This variation in opportunity leads to proposals that content and performance standards should be augmented by standards that focus on students' opportunity to learn, standards that would assure equal delivery of instructional opportunities (p. 1).

Providing appropriate interventions, such as *MLS*, at the earliest possible time is one such way that the United States can insure that all students have an equitable opportunity to learn mathematics—even if they do have learning difficulties and/or disabilities.

Prevention and Early Intervention Programs

MLS, although usually thought of as an intervention program, may also be used as prevention. For instance, some schools use it along with regular classroom instruction in mathematics to reinforce the teacher's instruction on key concepts and procedures and to catch students up who may have transferred in with no previous schooling, with inadequate schooling, or with inappropriate schooling. Others use the fluency activities to provide the necessary varied practice sessions for students to develop rapid and accurate recall of prior learning, especially mathematics facts. Some schools even use *MLS* as a gifted/talented program—allowing advanced students to work ahead of their peers in learning critical mathematics concepts and operations.

Related to OTL standards is the urgency of early interventions, to every extent possible—not because later intervention cannot be effective, but because of the damage done to the individual because help did not arrive earlier. His or her self-esteem has suffered tremendously, and academic progress is delayed with its dire consequences, including dropping out. One theme in both reading and mathematics research is the importance of early interventions. The National Research Council (2001) points out that "relatively simple interventions may yield substantial payoffs in ensuring that all children enter or leave first grade ready to profit from mathematics instruction" (p. 173). Wolfe and Brandt (1998) add that "an intervention program for impoverished children could prevent children from having low IQs and mental retardation"

(p. 11). The stakes are indeed very high. Other research on early intervention is presented in Table 89.

Table 89: Urgency of Early Interventions

Researcher(s)	Findings/Conclusions
Dowker, 2004, 15	"It is desirable that interventions should take place at an early stage. This is not because of any 'critical period' or rigid timescale for learning. Age of starting formal education has little impact on the final outcome (TIMSS, 1996). People who, to varying degrees, lacked opportunity for or interest in learning arithmetic in school, may learn later as adults (Evans, 2000)."
Kasten, 2005, 1	". . . prompt intervention can help make up for insufficient early learning experiences. . . . while there does not seem to be a point beyond which intervention in mathematics will not help, prompt is still an important word. The earlier a student can have experiences that support his or her understanding of number and space, the better."
Lyon, Fletcher, Shaywitz, Shaywitz, Torgesen, Wood, Schulte, & Olson, 2001, 259, 260	"We contend that sound prevention programs can significantly reduce the number of older children who are identified as LD and who typically require intensive, long-term special education programs. Moreover, prevention programs will prove more effective than remedial programs. . . . We estimate that the number of children who are typically identified as poor readers and served through either special education or compensatory education programs (as well as children with significant reading difficulties who are not formally identified and served) could be reduced by up to 70 percent through early identification and prevention programs."
ERS, 1992, 65	". . . the cost effectiveness of successful programs becomes apparent when they are compared with the high costs of remediation, retention, and placement in special education programs."
Scruggs & Mastropieri, 2002, 164	"Students with learning disabilities who are identified and treated early have a brighter future than students with learning disabilities who are not identified early."

MLS is an effective intervention, beginning even in kindergarten and extending into adult classes for learners who lack essential concepts and fluency to move forward in mathematics. Schools use *MLS* for after-school tutoring programs; for second-period classes for students falling behind; for summer school and inter-sessions; and for pull-outs in special programs, such as Title I, ESOL, or dyslexia. As states begin the implementation of *IDEA*'s new initiative, Response to Intervention (RTI), schools are beginning to adopt *MLS* as a Tier II or III intervention in grades K-12, and especially at K-3, to comply with that reform. *MLS* is also used, of course, as an intervention in adult basic education, alternative schools, schools for adjudicated youth, prisons, developmental education programs in colleges, and adult English-as-a-second language classes.

Three-Tiered Mathematics Instruction

One promising model for prevention of mathematics failure and for intervention as early as possible is the three-tiered instruction model, employed as a major strategy in the federal government's Reading First initiative for grades K-3. It is also included in the recently reauthorized *IDEA* as an alternative strategy for the identification of students for special education—referred to as Response to Intervention (RTI). Among its proponents are Lynn Fuchs and Douglas Fuchs (2001), prominent researchers in the field of learning difficulties and disabilities. They write that "a substantial body of intervention studies provides the basis for specifying methods to prevent and treat mathematics difficulties" (p. 85).

Tier I or "primary intervention," they explain, "focuses on universal design. With universal design, instruction for all students is formulated to incorporate principles that address the needs of specialized populations while benefiting (or at least not harming) others" (p. 86). An example would be the scaffolding that a regular classroom teacher might do to make instruction accessible for individual students who otherwise might fall behind. They note that "although the goal of universally designed primary prevention is to preclude the development of disorders, primary prevention does sometimes fail" (p. 86). Part of the model is the inclusion of assessment and progress monitoring to ensure that failure is identified as early as possible.

"Secondary prevention" or Tier II "is offered to arrest the seriousness of the disorder or to reverse its course" (p. 86). Educators may see this step as analogous to what was formerly called pre-referral strategies for special education, "whereby general education is modified in ways that are feasible for the teacher and unobtrusive for classmates" (p. 86). The goal, state Fuchs and Fuchs, is "to effect better student progress with minimal invasiveness to target children and with minimal disruption to others" (p. 86).

In contrast, they write, "tertiary prevention," or Tier III, "is reserved for disorders that prove resistant to lower levels of prevention and require more heroic action to preclude serious complications" (p. 86). They add that "tertiary prevention is synonymous with intervention, whereby intensive, individualized attention requiring special resources is brought to bear to alleviate an individual student's difficulties" (p. 86). "Individually referenced decision making is perhaps the signature feature of effective special education intervention," they explain (p. 91).

The table below includes definitions of the three-tier intervention model from a paper prepared by the National Research Center on Learning Disabilities (NRCLD) for the United States Department of Education (n.d.). The three-tier model is not exclusive to the United States. Table 90 includes also the recommendations for adoption of these approaches in Canada, with some minor variations on the definitions of the tiers.

Table 90: Three-Tier Intervention Model

Researcher(s)	Findings/Conclusions
US Dept. of Ed., n.d.,	"RTI is a multi-tiered delivery intervention. . . . RTI is meant to be applied on a school-wide basis, in which the majority of students receive instruction in Tier One (the general classroom), students who are at risk for reading and other learning disabilities are identified (such as through school-wide screening) for more intense support in Tier Two, and students who fail to respond to the interventions provided in Tier Two may then be considered for specialized instruction in Tier Three. . . . " Tier One instruction is designed to provide for the majority of students' needs and consists of three elements: a. Research-based core instructional programs provided by the general education teacher. b. Progress monitoring of students such as through curriculum-based measurement(CBM).

Researcher(s)	Findings/Conclusions
	c. Analysis of the progress monitoring results to determine which students are at risk and require more intense instructional support. . . . Tier Two intervention is for those students for whom Tier One is insufficient and who are falling behind on benchmark skills and require additional instruction to achieve grade-level expectations. . . . Tier Two includes programs, strategies, and procedures designed and employed to supplement, enhance, and support Tier One instruction to all students. . . . The progress of students in Tier Two is monitored to determine whether they are responding to the intervention. . . . “Although no clear consensus exists on the duration of Tier Two interventions, in general, the research supports 8 to 12 weeks for each round of intervention. At the end a decision should be made about the student’s instructional needs. The options to be considered include the following: 1. Return to the general education classroom if the student has made sufficient progress. 2. Receive another round of Tier Two intervention if the student is achieving progress but still remains behind his/her grade-level expectations (e.g., perhaps repeat the intervention or change to another scientific, research-based intervention depending on progress monitoring results). 3. Consider for more intensive intervention in Tier Three. “Tier Three intervention is intensive, strategic, supplemental and often considerably longer in duration than the 10 to 12 weeks of supplemental instruction provided in Tier Two. In most schools, Tier Three might be synonymous with special education. Tier Three is for students who fail to make sufficient progress after receiving Tier Two interventions. . . . Progress monitoring is a continual part of Tier Three and is used to carefully observe student response to the intervention, report his/her progress to parents, and determine future instructional placements. As a general guideline, a student is ready to exit the intervention when he/she has reached benchmark on the targeted skills (pp. 3-5).
Ontario Ministry of Education, 2005, 60	“An extremely effective approach to assessment and intervention is the ‘tiered’ approach, which sequentially increases the intensity of instructional interventions (Vaughn & Fuchs, 2003). It promotes and facilitates early identification of students who are at risk, and therefore prevents learning disabilities. In addition, this approach ensures adequate interventions for students exhibiting persistent learning difficulties (Vaughn et al., 2003). For best outcomes, it should begin in Kindergarten, as students who are at risk can be identified early and provided with the appropriate intensity of instruction to prevent later persistent difficulties (Vaughn, Linan-Thompson, & Hickman, 2003). The use of the ‘tiered’ approach in the early years has been shown to dramatically reduce the number of students in the later grades who would meet criteria for learning disabilities (O’Connor, 2000; Vaughn et al.). “The first tier consists of sound classroom instruction, based on successful practice for all students. Assessment in this tier is classroom-based and involves the teacher monitoring the progress of the class and flagging any at-risk students.

Researcher(s)	Findings/Conclusions
	The classroom and individual learning profiles . . . would be useful tools for the teacher to use to monitor student progress, plan differentiated instructional strategies, and identify at-risk learners. "The second tier requires teachers to identify students who have failed to progress satisfactorily in tier 1 instruction. Tier 2 involves more intensive instruction (individually or in small groups) in addition to the tier 2 programming. This level of instruction may include other members of the school staff (e.g., special education teacher, teacher's assistants). "The third tier is for students who do not respond to instructional efforts in tiers 1 and 2. These students may need to be referred for more extensive psycho-educational assessment. This type of assessment information, coupled with classroom observations and teacher assessment of the students' previous responses to intervention strategies, can then be used to guide more specialized instruction."

CEI sees *MLS* as a potential prevention program at an early grade level in Tier I. *MLS* is not, of course, a core or comprehensive mathematics curriculum, but its use with all students in an early grade level as a supplement to the core would provide appropriate instruction to reinforce the regular classroom teacher's presentations, to re-teach key concepts, and to provide adequate and varied practice in procedural fluency to help prevent failure. Such an application could help prevent the identification of students for Tier II and/or Tier III levels and could, potentially, reduce the numbers of students who would require Title I targeted assistance or special education services. *MLS* is also of benefit to English-language learners (see Chapters II and IV for those discussions). Some schools even use it now to accelerate gifted/talented students who are ready to move ahead.

MLS is also perfect for a Tier II intervention—and more effective in preventing failure than the strategies usually recommended. A student assigned to an *MLS* lab would receive intensive instruction (30-45 minutes more instruction daily), and that instruction would be totally individualized/differentiated to meet his or her unique needs—not delivered in a small group of learners who might have different reasons for failure and need different content and strategies. The use of computer-assisted instruction would ensure individualization, and it also makes possible the effective use of multi-sensory processing strategies and enough varied practice/ repetition activities to move the learning into long-term memory—with little burden on the teacher. The scientifically-based evidence presented in earlier chapters of this study provide the rationale for the effectiveness of this approach to Tier II. Tier II students participating in *MLS* might include dyslexic students (see Chapter III for a discussion on how dyslexia affects mathematics performance) and English-language learners (see Chapters II and IV).

The evidence presented in Chapter III on learning disabilities and their manifestations, along with the correlation of the *MLS* components with research findings on what works, makes it clear that *MLS* is also effective as a Tier III intervention, and many schools use it this way.

Continuous Improvement Model

Many schools are adopting some version of quality management models as their focus for school improvement. Indeed, many of the more prominent models are themselves variations of the business models; they just use different language to talk about the processes. In any case, a major priority in a quality school is to prevent as much failure as possible. School implementation models incorporate many strategies that are similar to the three-tier model, including frequent and ongoing progress monitoring, early interventions, and progressive use of intense strategies to ensure student success.

The following table utilizes the Fourteen Points developed by Dr. W. Edwards Deming, internationally renowned authority in the field of statistical quality control, as interpreted for schools by John Jay Bonstingl (1992, pp. 77-82). *MLS* program features and services are correlated with that interpretation in the table below, developed by Dr. J. B. Berryhill, a retired administrator from the Brazosport Independent School District in Texas:

Table 91: *MLS* Correlation with Deming's Fourteen Points

POINT	SCHOOLS	*MLS*
1. Create constancy of purpose for improvement of product and service.	School must focus on helping students to maximize their own potentials through continuous improvement of teachers' and students' work together. Maximization of test scores and assessment symbols is less important than the progress inherent in the continuous learning process of each student.	*MLS* focuses on the very core of learning problems – faulty sensory processing. It gives educators the tools necessary to monitor student progress to maximize the continuous learning process of each student.
2. Adopt the new philosophy.	School leaders must adopt and fully support the new philosophy of continuous improvement through greater empowerment of teacher-student teams. Cynical application of the new philosophy, with the sole intent of improving district-wide test scores, destroys interpersonal trust, which is essential to success.	CEI strongly believes that human interaction and involvement are key elements in lifelong learning and the ability to make life-changing differences. Therefore, learning solutions are developed to strengthen effective student and educator interaction.

POINT	SCHOOLS	*MLS*
	Reliance on tests as the major means of assessment of student production is inherently wasteful and often neither reliable nor authentic. It is too late at the end of the unit to assess students' progress if the goal is to maximize their productivity. Tests and other indicators of student learning should be given as diagnostic and prescriptive instruments throughout the learning process. Learning is best shown by students' performance, applying information and skills to real-life challenges. Students must be taught how to assess their own work and progress if they are to take ownership of their own educational processes.	*MLS* provides an assessment that helps determine a student's primary learning modality, as well as his or her strengths and weaknesses. This learning system helps the lab facilitator build a prescriptive and sequential lesson plan based on each student's individual learning strengths and deficiencies. *MLS* uses skill level mastery so each student works on lessons that address his/her individual needs. In the event that a student does not meet the criteria for mastery on a test lesson, the program provides systematic review called "recycling" until the student does achieve mastery.
4. End the practice of doing business on price tag alone.	Build relationships of trust and collaboration within the school, and between school and the community. Everyone's roles as supplier and customer must be recognized and honored. Work together whenever possible to maximize the potentials of students, teachers, administrators, and the community.	CEI is committed to forming a partnership with each client that not only helps those with educational differences achieve academic, social, and professional success, but also gives clients the best support for their development dollar by providing: • Professional workshops; • Customized in-services and staff development presentations; • Ongoing coaching and follow-up; • Faculty and parent orientations; • On-site visits; and • Professional training.
5. Improve constantly and forever the system of production and service.	School administrators must create and maintain the context in which teachers are empowered to make continuous progress in the quality of their learning and other aspects of personal development, while they learn valuable lessons from (temporary) failures.	Each year CEI provides clients with updates, testing materials, software upgrades, resource manual updates, and supplementary materials. CEI also provides clients with toll-free educational and technical support.

POINT	SCHOOLS	*MLS*
6. Institute programs of training.	School leaders must institute programs of training for new employees unfamiliar with the specific culture and expectations of the school. Effective training programs show new teachers how to set goals, how to teach effectively, and how to assess the quality of their work with students. Teachers must also institute programs in which students learn how to set learning goals, how to be more effective in their school work, and how to assess their own work. Teachers should show students by attitude and actions what a good *learner* is all about. (Educators learn how to be educators from the modeling they receive as students.)	All lab personnel who work directly with students receive training in the implementation and operation of the CEI software. CEI's Professional Services team conducts annual workshops so that all CEI clients can share their ideas with other lab personnel. Attendees receive in-depth training on technical issues, current research, practical lab application and motivation. Replacement personnel are trained at no charge. Experienced trainers conduct the workshops.
7. Institute Leadership.	School leadership consists of working with teachers, parents, students, and members of the community as coach and mentor so that the organizational context in which all students' growth and improvement is valued and encouraged can be maximized by teachers and students, parents, and community members who support the common effort. Leading is helping, not threatening or punishing.	CEI provides administrators with training and an *MLS Implementation Toolkit* designed to assist them in planning for and implementing CEI's learning solutions in their school. This document includes information regarding material, technical, and staffing needs; program implementation; staff development opportunities; and school improvement planning.
8. Drive out fear.	Fear is counterproductive in school, as it is in the workplace. Fear is destructive of the school culture and everything good that is intended to take place within it. Institutional changes must reflect shared power, shared responsibilities and shared rewards.	CEI provides a sample School Improvement Plan in the "Toolkit," along with a school planning guide. Utilizing this process, schools can include teachers, students, parents,and administrators in the planning process to instill a collaborative atmosphere in the school. Computer-assisted instruction provides students with a risk-free environment with ongoing feedback and encouragement in which to learn.

POINT	SCHOOLS	*MLS*
9. Break down barriers between staff areas.	Teacher and student productivity is enhanced when departments combine talents to create more integrated opportunities for learning and discovery. Create cross-departmental and multi-level quality teams to break down role and status barriers to productivity.	Students of all ages and levels of education and ability have found success through the use of CEI products. *MLS* provides opportunities for students with diverse needs to improve knowledge and skills. Lab facilitators are encouraged to work collaboratively with other teachers of lab students—sharing assessment results, for instance.
10. Eliminate slogans, exhortations, and targets for the workforce.	Teachers, students, administrators, families, and community members may collectively arrive at slogans and exhortations to improve their work together, as long as power, responsibility, and rewards are equitably distributed. When educational goals are not met, fix the system instead of fixing blame on individuals.	CEI Results and Recognition. • CEI provides graphic representations of annual pre- and post-test scores to school and district contacts to show overall success, as well as improvements in specific populations and individuals. • *SHARE* newsmagazine contains success stories and profiles of CEI schools, labs, students, and educators. • Students and facilitators receive awards for performance.
11. Eliminate numerical quotas.	Assignments and tests that focus attention on numerical or letter symbols of learning and production often do not fully reflect the quality of student progress and performance. When the grade becomes the bottom-line product, short-term gains replace student investment in long-term learning, and this may prove counter-productive in the long run.	The *CEI Learning Manager* and *MLS* provide many features that enable facilitators to work more effectively. In addition to simplifying the tasks of organizing and maintaining student information, *CLM* offers more flexibility and control in analyzing and documenting student progress. By allowing the lab facilitator to select options that best fit the lab environment, *CLM* makes the job of communicating student progress easier and more efficient, allowing more time for personal interaction in the lab.

POINT	SCHOOLS	*MLS*
12. Remove barriers to pride and joy of workmanship.	Teachers and students generally want to do good work and feel pride in it. Schools must dedicate themselves to removing the systemic causes of teacher and student failure through close collaborative efforts.	The CEI approach focuses on mastery, positive reinforcement and motivation, all of which can result in significantly improved grade-level equivalents, test scores, self-esteem and overall performance.
13. Institute a vigorous program of education and retraining.	All of the school's people benefit from encouragement to enrich their education by exploring ideas and interests beyond the boundaries of their professional and personal worlds.	CEI provides initial training for lab facilitators, administrators, school staff, and parents and schedules reviews during the year and annual retraining programs for all personnel needing it. The key to CEI success is having well-trained school staff supporting the learning system.
14. Take action to accomplish the transformation.	School personnel at all levels (including students) must put this new philosophy into action so it becomes imbedded into the deep structure and culture of the school. Teachers and students alone cannot put the plan into effect. Constant top-level dedication to full implementation must be supported by a critical mass of school and community people to implement the plan and make it stick.	CEI manifests that "singular, vigilant, and collective focus on results" noted by Schmoker.

MLS Results

Just as with *Essential Learning Systems (ELS),* CEI collects data from participating *MLS* labs for analysis to determine the value-added gains that students achieve from their engagement in the *MLS* program. In 2000-2001 CEI analyzed the data representing 648 students. The staff annually analyzes existing data to determine if the 2000-2001 data are consistent, and they have been, within hundredths of a point on each subtest.

The *Diagnostic Screening Test for Mathematics (DSTM)* (Gnagey & Gnagey, 1982), a third-party assessment, is administered at the beginning of the school year as a pre-test and then near the end of the school year as a post-test. The test assesses student knowledge and skills relating to the basic concepts of addition, subtraction, multiplication, and division and the students' skills in using that knowledge to solve problems. Scores are provided for each area, and then the four subtopics are combined and averaged to attain a "Basic Processes" score. (See Chapter VI for the discussion on CEI's comprehensive assessment program.)

Average Gains. The mean (average) gain made by the 648 student records examined was as follows:

Addition	**2.57 years**
Subtraction	**1.98 years**
Multiplication	**1.92 years**
Division	**2.69 years**

On average, participating students gained 2.29 years in the *Basic Processes* in one year or less of participation in *MLS* labs in diverse schools. That accelerated gain in achievement was an average gain and included those excellent labs and the ones that were never implemented appropriately. The average was also calculated based on the pre- and post-scores of all students participating in the lab during a given school year, even though many enrolled late or exited early and did not engage in a full year of *MLS* instruction. It is not at all unusual for CEI to receive scores from labs where students gain four or even more years in one year. *MLS* truly accelerates student learning, both in the understanding of foundational concepts, but also in the attainment of fact fluency.

One especially remarkable finding is the gain in division. As discussed in Chapter IV, division concepts and fluency in operations are essential prerequisites for students to succeed in understanding fractions—and in algebra. This one component of *MLS,* therefore, makes it more than a worthwhile investment. If students can sail through the study of fractions without significant difficulty, and if they can pass algebra on the first try, then the *MLS* program is not only cost effective, it is a major bargain.

One of the stories in CEI lore is the story of a lab with Macintosh computers that had great difficulty running the original version of *MLS* software because of the computer language in which it was written. In spite of the school's and CEI's best efforts, the lab experienced chronic problems. Finally, the Chief Executive Officer of CEI offered the school a return of the money they had paid for the program. "No!" they exclaimed. "Not if it means we have to give the software back." They explained that they had never seen anything more effective in teaching division and fractions than *MLS* and that they wanted to keep the software in spite of the technical difficulties they were experiencing. (Version 3.0 of *MLS* corrected this problem for Macintosh users and includes multi-user versions for both PCs and Macs.) CEI staff hear student success stories regularly, such as this one, and many are printed in *SHARE*, the bimonthly newsmagazine.

Summary and Conclusions

Several powerful insights and conclusions emerged in the process of conducting this study. A few are included.

Struggling Learners Are Diverse

Learners who struggle in learning mathematics come in all ages and with a variety of causes—some the result of cultural attitudes, especially when mathematics is not valued; some the result of stereotype threat and mathematics anxiety/phobia; some the result of language differences; and some the results of various motivational issues, including low sense of self-efficacy or self-esteem. A major problem identified by researchers was the area of inadequate instruction, or lack of an opportunity to learn, along with the whole realm of inappropriate instruction. All these learners are said to have "learning difficulties," not disabilities since their problems in learning mathematics have nothing to do with a brain disorder.

On the other hand, six to ten percent of the population suffers from mathematics learning disabilities. These disabilities are also diverse—ranging from the catch-all term of dyscalculia to more specific disabilities in the central executive, in the language system, and in the visuospatial system. Many people with reading disabilities only, e.g., dyslexia, still have problems with mathematics. And there are other disabilities caused by genetic diseases such as Turner syndrome, Fragile X syndrome, Gerstmann's syndrome, and spinal bifida.

Knowing something of the research on learning difficulties and disabilities, as well as how these problems are manifested, is important to educators in determining which learners need interventions and the intensity of those interventions. An intervention should be considered for any learner not meeting curriculum standards as early as kindergarten or grade 1 and should be mandated thereafter. The costs of ignoring these needs are astronomical for the individual learner and his or her family and for society as a whole. It is a "911" situation—an urgent call for help.

Although the research adds a sense of urgency for early intervention, it is never too late. The older the learner, however, the longer it may take to bring him or her to an acceptable level since the standards for adult performance are so much higher than they are for young children. Evidence that older learners can learn mathematics is found in CEI's data, as well as in the scientifically-based evidence reported in this study.

Dyslexics Also Struggle in Mathematics

Many educators, including some on the CEI staff, were unaware until the research in this study was shared that dyslexia learners typically also have multiple problems in learning mathematics. The findings are so strong and so clear that states should include in their dyslexia program guidelines a requirement for mathematics interventions, as well as reading, spelling, and writing interventions for this group of learners. Schools that now have access to the research findings can implement *MLS* for these students and get ahead of the mandated practice. To diagnose the need, a school could simply track the mathematics performance of their identified dyslexic students and determine if they are making adequate progress toward proficient performance.

English-Language Learners Also Struggle in Mathematics

Just as schools typically attend to the literacy (but not the mathematics) needs of dyslexic students, so do they focus on the literacy needs of English-language learners (ELLs), many times simply mainstreaming those students in mathematics classes. Just as with dyslexia, there is ample research available now to know that ELLs frequently struggle with mathematics, especially the language of mathematics with its highly specialized vocabulary. They may also be confused by the difference in the way algorithms are taught in the United States. And again, just as with dyslexia, a school can assess the need for using *MLS* with these students by analyzing ELLs' mathematics performance to determine if they are making adequate progress.

College Students May Also Struggle with Mathematics

Both two- and four-year colleges all over the United States are currently under pressure both to increase enrollments and to reduce their dropout rates. Increasing enrollment results sometimes in more and more students entering college who are not college ready; that is, they lack the prerequisite background in reading and mathematics to be successful in college-level courses (Armington, 2003, p. 1). Indeed, some of these students are high school dropouts and/or recent immigrants. Others simply managed to get a high school diploma without developing essential knowledge and skills. And others may have learning disabilities, such as dyslexia or dyscalculia. Their problems include the same mathematical difficulties and disabilities as were described in Chapters II-III of this study. A growing department in many if not most colleges, therefore, is developmental education, where those students go to engage in academic remediation, as well as for counseling and coaching.

The problem of college dropouts is enormous. For example, the National Center for Education Statistics (NCES) reported that in 2000 there are 1,126 two-year, publicly funded colleges in the United States. The percentage of students graduating was only 29.3. There is a wide range of variation. South Dakota, for instance, graduates 65.1 percent of its students; Florida, 53.9 percent; Mississippi, 33.6 percent; California, 33.2 percent; Arkansas, 21.5 percent; Texas, 17.2 percent; New Mexico, 12.4 percent; and Nevada, 9.7 percent (NCES, 2003, Table 24a).

Developmental education serves, of course, a wide range of needs, so multiple programs will be required to improve the graduation rates. *MLS* can be an effective solution for a diversity of those who need help—those with inadequate education, those who recently immigrated to the United States, those who have mathematics anxiety, those who are the victims of inappropriate curriculum and/or instruction, and those with mathematics and/or reading disabilities. The earliest possible intervention is critically important, but it is never too late.

Alignment Mandates Make No Sense for Struggling Learners

Although there is certainly a “curriculum” of knowledge and skills in a scientifically-based mathematics intervention, an intervention curriculum, such as the one designed for *MLS*, cannot be expected to correlate or align with state curriculum content standards at every grade level. Rather, an educator can expect to see a rather tight alignment with state standards at some grade levels with a specific set of topics, but the *MLS* curriculum should be seen as the “prerequisite” knowledge and skills necessary for students to be able to access the grade-level curriculum standards. An intervention would not be needed if the student could do the grade-level work. Some states and districts seriously handicap schools when they insist that all curriculum materials must be aligned with standards. That mandate totally ignores the needs of all students who struggle with mathematics, regardless of their reasons, and it also ignores the findings of the scientifically-based research.

Math Wars Make No Sense if One Reads the Research

Another important insight is that both sides of the mathematics wars are right—depending on the individual learner’s needs. CEI’s reading of the research is that the literature on learning difficulties and disabilities is very clear: those learners have to have something different—a research-based therapeutic intervention that incorporates, among other practices, multisensory processing and direct instruction strategies. It must also include in its content both concept development and fact fluency. The National Research Council (1997) made this observation: "The assumption that mastery of basic skills is not a prerequisite for advanced learning appears tenuous for many students with cognitive disabilities" (p. 127). Askey (n.d.) adds to the discussion:

> Is it possible to teach standard algorithms so that students not only learn how to do the calculations correctly, but build a foundation for later study of mathematics? Of course it is, and it should be done. What happened in the U.S. is that mathematics educators looked at a system which did not work, and tried to build one which they thought would. However, this problem was too hard for them, and they have failed to build such a system. As one small instance of this, consider division of fractions in Connected Mathematics Project. It is completely missing (p. 8).

However, discovery learning and other more creative approaches to learning mathematics may be entirely appropriate for those learners who already have a solid grounding in foundational concepts and who already have developed fact fluency. The strategies selected must be appropriate to each learner's developmental stage and needs. It is gratifying to see of late a growing consensus of views in how best to teach mathematics.

Content Matters Greatly

Educators learned a great deal when the research on reading was synthesized by the National Reading Panel (NRP) (2000). What is taught in beginning reading is very important, as well as how it is taught. The NRP identified five critical components in a reading curriculum: phonics, phonemic awareness, fluency, vocabulary, and comprehension. Even though the design of *Essential Learning Systems* pre-dated the NRP's findings, CEI was not surprised with those findings since they are all incorporated in the *ELS* design.

Neither does CEI expect to be surprised when the findings of the National Mathematics Panel are revealed in 2007. The clarity of the research, as well as its abundance, identifies the importance of teaching foundational concepts (e.g., counting, base-ten, place value, addition, subtraction, multiplication, division, estimation, and fractions) upon which students can build additional understandings and skills. The research also makes it clear without a doubt that learners simply must develop fact fluency for rapid and accurate retrieval. Without fluency, the learner's working memory is consumed with determining the mathematics facts, and there is no room left for the steps of problem solving. Fluency is also required to the point of automaticity for standard algorithms. It is further evident that students must acquire a deep understanding of long division and fractions, as well as procedural fluency, if they are to be successful in algebra. *MLS'* inclusion of critical mathematics concepts and procedures is a great part of its effectiveness as a therapeutic intervention.

Lesson Models and Lesson Delivery Are Important

Much of the research cited on lesson models such as direct instruction, mastery learning, and one-on-one tutoring, as well as the research on the concrete-semiconcrete-abstract lesson sequence, the use of manipulatives, and the employment of computer-assisted instruction in a mathematics intervention directly refutes one side of the arguments in the mathematics wars. What is clear from the scientific evidence is that students with learning difficulties and disabilities have to have these kinds of instructional designs if they are to learn effectively. Again, they are critical to a therapeutic intervention. Discovery approaches, as do cluttered computer screens, result in ineffectiveness with these students and, therefore, constitute malpractice.

Instructional Strategies Can Be Powerful

CEI carefully selected the instructional strategies with large bodies of scientifically-based research behind them on what works with struggling learners. The most unique and, perhaps, the most powerful of these strategies in the *MLS* program is multi-sensory processing. These strategies directly tie to the research on information processing and on the appropriate treatments for students who have weak or non-existent neural pathways in one or more areas of the brain. Other strategies that are strongly advocated by cognitive neuroscientists are chunking/clustering learning, as *MLS* does in the fact fluency component, as well as in concept development, and, very importantly, in *MLS'* reliance on varied and adequate practice or repetition (in the various modalities) so that new learning is firmly embedded in long-term memory.

The research also is clear in that effective interventions must be individualized and differentiated and that students must have intensive and engaged time-on-task if they are to accelerate their learning. Computer-assisted instruction is a major enabler of individualized and differentiated instruction, as well as of multi-sensory processing.

Frequent Assessment Used to Inform Instruction Is Critical

Even though accountability systems typically focus entirely on summative, once-a-year assessments, the research is clear that the kind of assessment that makes a difference in effective learning is dynamic, formative, ongoing assessment—but only if the results of those assessments are provided in feedback to students and teachers—and only if the results inform instructional decision-making. CEI's comprehensive assessment system includes all the parts of good assessment: diagnosis, progress monitoring, and summative. The *MLS* program uses two third-party assessments (for diagnosis and summative determinations) and several kinds of curriculum-based measurements (CBMs) for ongoing use.

Implementation Requires Leadership and Attention

Implementation is likewise critical to success in improving mathematics achievement. That is why CEI used research in its inclusion of an interactive role for the *MLS* teacher/facilitator in its program design. Also, very important is the professional development with follow-up coaching through a variety of methods. That is also why research-based strategies involving student motivation to learn and parental involvement activities are included. The research on the importance of implementation makes CEI's support services both meaningful and essential in a school's achieving the results it needs. Labs that do not achieve invariably are labs that ignore the training provided to the instructional leaders and to the lab facilitators: they do not implement appropriately.

Scientific Research Validates *MLS'* Pre/Post Scores

When CEI staff decided to make this study of *MLS'* scientific research base, they agreed that there is no question about whether *MLS* works. Data abound to validate *MLS'* efficacy in improving mathematics achievement. However, CEI did not have documentation of the science behind why it works, and so the paper was commissioned. Every effort was made to survey the existing research base as widely as possible and to provide those findings to potential customers as well as long-time clients. As this study verifies, every component of the *MLS* design is scientifically-research based. These findings also substantiate the accelerated learning gains evident from pre- to post-test scores.

MLS Can Reduce the Dropout Problem and Improve Graduation Rates

An unanticipated insight gained in doing the research was the important role that mathematics achievement has in determining graduation rates. In surveys of high school dropouts, one of the chief reasons given is "math." In studies of high school failure rates, those rates are usually highest in algebra, and, as a result, some students never move beyond freshman-level mathematics—and so never graduate from high school. In studies of college students requiring remediation, the highest need is, again, mathematics.

NCLB uses as one accountability indicator for high schools the graduation rate, and many schools are now finding themselves identified for "school improvement" because that rate is unacceptably low. Ideally, students have appropriate mathematics interventions such as *MLS* in elementary school and certainly no later than middle school. However, increasing numbers of high schools are turning to *MLS* for students lacking foundational knowledge and skills—the prerequisites for learning algebra. *MLS*, therefore, has become a tool for improving graduation rates and lowering the dropout rates—not only in high schools, but also in colleges.

MLS Is More Than a Sum of Its Parts

A final, and most important conclusion is that *MLS* is truly more than a sum of its parts. All of its components are validated through scientifically-based research, so it is thoroughly grounded in scientific evidence. The chapters in this study have documented an abundance of research findings that support the inclusion of all the component parts of *MLS*, as well as the validity of its use as a mathematics intervention for the diversity of struggling learners. To recount, *MLS* is a true therapeutic intervention and is research-based in all the following areas:

- Aligns with the mandates of federal programs that require scientifically-based evidence (see Chapter I discussion and correlations);

- Aligns with the research on manifestations of learning difficulties (see Chapter II);
- Aligns with the research on manifestations of learning disabilities (see Chapter III);
- Aligns with the research on the importance of concept development and fact fluency as priorities in a mathematics intervention for struggling learners (see Chapter IV);
- Emphasizes development of fraction concepts and procedures essential for later success in algebra (see Chapter IV);
- Utilizes research-based lesson steps and models, including components of direct instruction, mastery learning, and one-on-one tutoring (see Chapter V);
- Incorporates the research-based concrete-semi-concrete-abstract lesson sequence (see Chapter V);
- Incorporates the use of manipulatives and working mats in concept development lessons (see Chapter V);
- Exploits the possibilities of computer-assisted instruction for lesson delivery and management of student data (see Chapter V);
- Aligns with the research on computer screen design for struggling learners (see Chapter V);
- Incorporates in all software-based tasks the power of multi-sensory processing strategies (see Chapter VI);
- Enables total individualized and differentiated lessons, adhering to the Universal Design for Learning (see Chapter VI);
- Utilizes research-based, varied, and adequate practice/repetition activities to ensure movement of knowledge and skills to long-term memory—the goal of all instruction (see Chapter VI);
- Employs research-based principles of chunking/clustering to develop fact fluency for accurate and rapid retrieval and application (see Chapter VI);
- Reflects the research on the efficacy of engaged time-on-task in intervention programs (see Chapter VI);
- Includes a comprehensive, research-based assessment system (diagnostic, progress-monitoring or formative, and summative instruments—two of them third-party) (see Chapter VI);

- Incorporates corrective feedback and encouragement after each student response that address research findings on effective teaching and motivation (see Chapter VI);

- Trains teachers to use assessment data to inform instruction so that frequent adaptations or modifications of lessons can occur so that students are adequately challenged, but not overwhelmed (see Chapter VI);

- Incorporates the important component of self-assessment (see Chapter VI);

- Reflects the value of engaged lab teachers/facilitators in the implementation of an *MLS* lab (see Chapter VII);

- Includes research-based professional development with follow-up coaching for all staff involved in implementation (see Chapter VII);

- Embeds in the *MLS* design and in teacher training a research-based student motivation component (see Chapter VII);

- Encourages educators to develop parental involvement and supports their work (see Chapter VII);

- Reflects the research on the importance of sound implementation for effectiveness in improving student achievement (see Chapter VII);

- Reflects the research on the components of a therapeutic intervention in mathematics for struggling learners, including the importance of teaching both concepts and procedures (see Chapter VIII);

- Provides an analysis of pre- and post-test data collected by CEI from participating *MLS* labs to measure value-added gains (see Chapter VIII);

- Correlates to the research on opportunity-to-learn standards, the urgency of early intervention, the three-tier model recommended in Response-to-Intervention, and continuous improvement model (see Chapter VIII).

Simply stated, for people who struggle to learn, there is some kind of difficulty or dysfunction in the brain neurons or neural pathways that results in faulty sensory processing. Instruction must be designed so that the difficulty or disability is somehow repaired. With *MLS* as a therapeutic intervention, the student's brain builds new pathways or strengthens weak ones so that they learn mathematics. In the National Research Council's (1999) synthesis of research on how people learn, they identified key findings from neuroscience and cognitive science as follows:

a. Learning changes the physical structure of the brain.
b. These structured changes alter the functional organization of the brain; in other words, learning organizes and reorganizes the brain.
c. Different parts of the brain may be ready to learn at different times (p. 103).

The Dana Foundation (2003), which invests much of its resources on brain research, describes the process of building new pathways in the following way:

> Systems neuroscience helps explain how people such as victims of stroke or head trauma, whose brains have been injured in a discrete site, can, over time, redevelop the functions lost as a result of the injury. Nerve cells in their brains in effect forge new pathways, bypassing the injured site and forming new connections, as if finding a new route to get to work after discovering that is bridge is out on the usual route. This ability to adapt, which scientists call *plasticity*, seems to be particularly strong in young brains, but "old" brains routinely learn new tricks, scientists have found (p. 13).

CEI program designers understand this research and apply it. The result is a scientifically-based, therapeutic mathematics intervention with proven effectiveness in improving student achievement—*Mathematical Learning Systems (MLS).*

As Ron Edmonds (1979) remarked, "We can whenever and wherever we choose successfully teach all children whose schooling is of interest to us. We already know more than we need in order to do this. Whether we do must finally depend on how we feel about the fact that we haven't so far." *MLS* provides a delivery system for the effective teaching of those students who struggle to learn mathematics, regardless of reason or age, K-adult. "We already know more than we need in order to do this" because the scientific, theoretical, and evaluation evidence is plentiful and clear.

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Dictionary of Acronyms

ADHD	Attention deficit hyperactivity disorder
AIP	Academic improvement plan
AMS	American Mathematical Society
CAI	Computer-assisted instruction
CBM	Curriculum-based measurement
CD	Cognitive disabilities
CEI	Creative Education Institute, publisher of *ELS* and *MLS*
CLM	*CEI Learning Manager*
CSA	Concrete—semi-concrete—abstract (lesson sequence)
DD	Developmental dyslexia
DI	Direct instruction
DSTM	*Diagnostic Screening Test for Mathematics*, published by Slosson
DSTR	*Diagnostic Screening Test for Reading*, published by Slosson
ELLs	English-language learners
ELS	*Essential Learning Systems*, CEI's learning-to-learn and learning-to-read program
ESL	English-as-a-second language
ESOL	English for speakers of other languages
FSIQ	Full-scale IQ
HSTW	High Schools That Work (Southern Regional Education Board program)
IDA	International Dyslexia Association
IDEA	*Individuals with Disabilities Education Act*
IEP	Individual education plan
ILS	Integrated learning system
IQ	Intelligence quotient
IRA	International Reading Association
LD	Learning disability

LEP	Limited-English proficient
LET—II	*Learning Efficiency Test*, published by Academic Therapy Publishing
LTM	Long-term memory
McREL	Mid-continent Research on Education and Learning, Aurora, CO
MD	Mathematics disability
MDOE	Mississippi Department of Education
MI	Multiple intelligences
MLD	Mathematics learning disability
MLS	*Mathematical Learning Systems*, CEI's learning foundational mathematics program
MTV	Music television
NADE	National Association for Developmental Education
NAEP	National Assessment of Educational Progress
NAEYC	National Association for the Education of Young Children
NASA	National Aeronautic and Space Agency
NCES	National Center for Education Statistics
NCLB	No Child Left Behind, also known as Elementary and Secondary Education Act (ESEA)
NRC	National Research Council
NCTM	National Council of Teachers of Mathematics
NMP	National Mathematics Panel
NRCLD	National Research Center on Learning Disabilities
NRP	National Reading Panel
NSB	National Science Board
NSDC	National Staff Development Council
NSF	National Science Foundation
OTL	Opportunity-to-learn
RD	Reading disability
RD/MD	Reading disability and mathematics disability (comorbidity)
RLD	Reading learning disability

RTI	Response to Intervention
SAIP	Student academic improvement plan
SBM	Spinal bifida myelomeningocele
SBR	Scientifically-based research
SHARE	See, Hear, and REspond—the name of instructional lessons in *ELS*; also the name of CEI's bimonthly newsmagazine
SI	Sensory integration
SIT	Sensory integration training
SLD	Specific learning disability
SLI	Specific language impairment
STM	Short-term memory
TAKS	Texas Assessment of Knowledge and Skills
TEA	Texas Education Agency
TEKS	Texas Education Knowledge and Skills (curriculum standards)
TIMSS	Third International Mathematics and Science Study
TMDS	Texas Mathematics Diagnostic System
TSI	Texas Success Initiative
UDL	Universal Design for Learning
USDE	United States Department of Education
WAC	Web-based activity center (CEI's practice exercises for *ELS* and *MLS*, available on the web)
ZPD	Zone of proximal development

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